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Preface

The study of representations of p-adic groups was born over fifty years ago when Friedrich Mautner published a paper on spherical functions of p-adic PGL(2). In this 1958 paper he introduced principal series representations "as induced representations following the by now classical procedure of Frobenius, Schur, Bargmann, Gelfand, Naimark, Mackey and others" [Mau58, §3]. The companion paper to [Mau58], which was not published until 1964, exhibited the first examples of supercuspidal representations [Mau64]. Taken together, these two papers represent the first steps taken toward the solution of a problem which is deceptively simple to state: classify and construct all irreducible admissible representations of reductive p-adic groups.

By 1970, Harish-Chandra's work on the Plancherel formula for reductive p-adic groups had appeared in the Séminaire Bourbaki [HC70a], for which he developed the idea of the character of an admissible representation as a distribution on the Hecke algebra of the p-adic group. On the other hand, in the intervening years, Robert Langlands introduced L-parameters of admissible representations of p-adic groups as part of the celebrated Local Langlands Correspondence [Lan68].

The Leitfaden for this volume is based directly on the remarkable work done in this early period and follows three threads: the study of smooth representations as begun by Mautner (Part 1); character theory beginning with the work of Harish-Chandra (Part 2); and the Local Langlands Correspondence, framed by Robert Langlands (Part 3).

The first part begins with Alan Roche's chapter, in which he develops the Bernstein decomposition, thus setting up the theory of types. Part 1 continues with Jiu-Kang Yu's introduction to Bruhat-Tits theory — a theory which is indispensable in p-adic representation theory. Ju-Lee Kim's chapter provides an elementary introduction to supercuspidal representations, and goes on to describe her work proving that Jiu-Kang Yu's construction of supercuspidal representations [Yu01] is complete in many cases. The material in Part 1 beautifully frames Mautner's two original papers in the field.

The chapter by Paul Sally and Loren Spice gives an introduction to character theory beginning with Harish-Chandra and focuses on the history of calculations of character values. On the same theme of tools for computing character values, Part 2 continues with Julia Gordon and Yoav Yaffe's chapter on arithmetic motivic integration as an alternative to classical p-adic integration. This chapter also serves as a segue to some current research on character value computations.

The final part of this volume picks up the thread introduced in 1968 by Robert Langlands. Paul Mezo's chapter reviews local class field theory and sketches the Local Langlands Correspondence, which is now a theorem in many cases. Jiu-Kang

Yu's chapter in Part 3 then describes the Local Langlands Correspondence as it applies to algebraic tori.

The tapestry woven with the ideas found in the early papers by Mautner, Harish-Chandra and Langlands is remarkably rich and represents a testament to the inventiveness of the many mathematicians who have worked in this area during the last half-century. Nevertheless, much work remains to be done. For example, the Local Langlands Correspondence, when taken together with the structure of local L-packets, offers a parametrization of admissible representations which is, *a priori*, very different from that described in Part 1. The reconciliation of these two perspectives on admissible representations is an area of active research.

This volume evolved from two Fields Institute Workshops held at the University of Ottawa: the *Workshop on the Representation Theory of p-adic Groups* held in May 2004, organised by Jason Levy and Monica Nevins; and the *Workshop on the Representation Theory of Reductive Algebraic Groups*, held in January 2007, organised by Clifton Cunningham and Monica Nevins. Each workshop was constructed around three mini-courses, as well as a two-day conference. The goal was to present some of the main themes and results of the representation theory of p-adic groups to a broad audience.

The first workshop began with three six-hour mini-courses:

- Alan Roche (University of Oklahoma), *The Bernstein Centre and Types*;
- Jiu-Kang Yu (Purdue University), *Bruhat-Tits Theory and Buildings*;
- Paul Mezo (Carleton University), *The Local Langlands Program.*

It also featured a colloquium lecture by Paul Sally, Jr., University of Chicago, entitled *Characters for Reductive p-adic Groups.* Each mini-course lecturer graciously provided meticulous notes. The value of these notes as introductions to some of the most beautiful aspects of the subject was soon apparent and many participants encouraged their publication.

Nevertheless, it was not until some two and a half years later that steps were taken to ensure the longevity of these excellent resources. The second workshop included three four-hour mini-courses:

- Julia Gordon (University of British Columbia), *Motivic Integration and its Applications to p-adic Groups*;
- Ju-Lee Kim (University of Illinois at Chicago, now Massachusetts Institute of Technology), *Recent Progress in the Classification of Supercuspidal Representations*;
- Phil Kutzko (University of Iowa), *Plancherel Measure and Reducibility of Parabolic Induction via Types and Covers*;

as well as a colloquium talk by A. Raghuram (Oklahoma State University) entitled *Arithmetic of L-functions.* It was in fact Raghuram who finally convinced the organisers to undertake the creation of this volume even though, regretfully, his own talk did not fall under its scope.

We offer our profound thanks to all the mini-course lecturers for their diligence and generosity. They were instrumental in making these workshops a great success and this volume would not have been possible without their dedication to this project.

Some Background

Each chapter begins with an introduction to the necessary tools and terminology for the area. Nevertheless, a few words are in order regarding fundamental concepts and definitions in the field. Perhaps the most important reference for the representation theory of p-adic groups is Part 1 of [BC79], commonly called the *Corvallis Proceedings*, with which readers of the present volume are assumed to have some familiarity.

Fields

Although global fields play a minor role in these notes, they provide an important point of departure for the subject. Global fields come in two flavours: *number fields*, which are finite extensions of $\mathbb{Q}$ and therefore have characteristic zero, and *function fields*, which are finite extensions of $\mathbb{F}_p(t)$ and thus have non-zero characteristic. Global fields admit countably many equivalence classes of valuations and these too come in two flavours: *Archimedean valuations* and *non-Archimedean valuations*. All valuations of function fields are non-Archimedean, but number fields admit both Archimedean and non-Archimedean valuations.

The completion of any global field with respect to any non-trivial valuation yields a *local field* and all local fields arise in this manner. Thus, local fields come in two flavours also: *Archimedean local fields* and *non-Archimedean local fields*. Some authors reserve the term 'local field' for local non-Archimedean field; an excellent reference for the study of these fields is Jean-Pierre Serre's book [Ser79].

Since the completion $\mathbb{Q}_p$ of the global field $\mathbb{Q}$ with respect to a p-adic valuation is known as the field of *p-adic numbers*, it is common to refer to finite extensions of $\mathbb{Q}_p$ as *p-adic fields*; these are precisely the non-Archimedean local fields of characteristic zero. Unfortunately, there is no consensus on the use of this term, as some authors also use it as a synonym for 'non-Archimedean local field', thus including the so-called equal-characteristic fields $\mathbb{F}_q((t))$ as well; this is the case in Chapter 5, for example. In all other chapters of this volume, p-adic fields have characteristic zero. To remove any possibility of confusion, the arguably redundant term 'p-adic field of characteristic zero' is often used to refer to non-Archimedean local fields of characteristic zero with residual characteristic p.

Groups

Recall that *linear algebraic groups* are, by definition, affine algebraic varieties (that is, reduced affine schemes of finite type over algebraically closed fields) that are also group schemes.

A linear algebraic group $\mathbb{G}$ is *reductive* if the unipotent radical of the connected component $\mathbb{G}^0$ of the identity element of $\mathbb{G}$ is trivial. Some of the most important examples of reductive linear algebraic groups are provided by *classical* *algebraic groups*, amongst which are general linear, unitary, symplectic and orthogonal groups.

A few important classes of linear algebraic groups deserve special mention, as they appear in several places in this volume. A linear algebraic group is: *connected* if $\mathbb{G}^0 = \mathbb{G}$; *semi-simple* if it has no non-trivial closed, connected, solvable normal subgroups (so its radical is trivial, see [Spr98, 6.4.14]); and *simply connected* if it is connected and does not admit any non-trivial isogenies.

Connected reductive linear algebraic groups admit *parabolic subgroups*, *Borel subgroups*, *algebraic subtori* and *Weyl groups*, which are all indispensable to the general theory. We refer the reader to [Spr98, Bor91, Hum75] or [Spr79] for the definitions of these terms.

The classification of connected reductive linear algebraic groups is obtained by studying the *root data* (donnée radicielle) associated to these groups [SGA3, Exposé XXI]; more precisely, there is a canonical bijection between isomorphism classes of connected reductive linear algebraic groups and isomorphism classes of root systems. A lovely presentation of the details of this bijection may be found in [Spr98]; it is also briefly discussed in [Spr79].

A *linear algebraic group over* k, where k is an arbitrary field, is a (reduced, affine) k-scheme $\mathbb{G}_k \to \mathrm{Spec}(k)$ of finite type such that $\mathbb{G}_k \times_{\mathrm{Spec}(k)} \mathrm{Spec}(\bar{k})$ is a linear algebraic group, where $\bar{k}$ is an algebraic closure of k. A connected reductive linear algebraic group $\mathbb{G}_k$ over k is said to *split over* k if $\mathbb{G}_k$ admits a maximal subtorus which is defined over k and split over k. At another extreme, if $\mathbb{G}_k$ has no non-trivial subtori that are defined over k and split over k, then $\mathbb{G}_k$ is *anisotropic over* k. In between lie the quasi-split groups: a connected reductive linear algebraic group defined over a field k is *quasi-split over* k if it admits a Borel subgroup which is defined over k.

The group $\mathbb{G}_k(k)$ of closed k-rational points of a connected reductive linear algebraic group over a p-adic field k (of characteristic zero) is commonly called a *p-adic group* or a *reductive p-adic group*; this is the case in the title of this volume, for example.

One class of p-adic groups is particularly important to applications to automorphic representations: a connected reductive linear algebraic group $\mathbb{G}_k$ over a non-Archimedean local field k is said to be *unramified over* k if it is quasi-split over k and splits over a finite unramified extension of k [Car79, p.135].

A detailed classification of some classes of reductive linear algebraic groups over local fields is given in [Tit79]. This chapter of the Corvallis Proceedings also includes a very readable introduction to Bruhat-Tits buildings and Iwahori subgroups and constitutes some of the required background for Chapter 2.

It should be noted that [Tit79, §§3.4–3.5] includes important results on smooth group schemes which are integral models for reductive linear algebraic groups over local fields and their relation with certain classes of compact subgroups of p-adic groups; these results are amplified in [BT84a]. Chapter 2 provides a portal to the extensive literature on Bruhat-Tits theory, such as [Tit79, BT72, BT84a, BT84b, BT87a, BT87b].

Representations

A *representation* of a p-adic group G is a pair (π, V) consisting of a complex vector space V and a homomorphism π from G into the group $\mathrm{Aut}(V)$ of linear automorphisms of V. A representation (π, V) of G is *irreducible* if V has no non-trivial G-invariant subspaces.

A representation (π, V) of a p-adic group G is *smooth* if for every $v \in V$ there is a compact open subgroup K of G such that $\pi(g)v = v$ for every g in K; in other words, (π, V) is smooth if $V = \cup_K V^K$ where K runs over all compact open subgroups of G and where V^K denotes the set of elements of V fixed by K [Car79, Def.1.1].

The *Hecke algebra* $\mathcal{H}(G)$ of a p-adic group G is the convolution algebra of locally constant, compactly supported functions $f : G \to \mathbb{C}$ [Car79, p.116]. Hecke algebras are generally not unital, but they do admit many idempotent elements. In particular, for every compact open subgroup K, the convolution algebra $\mathcal{H}(G, K)$ of bi-K-invariant, compactly supported functions $f : G \to \mathbb{C}$ is a subalgebra of $\mathcal{H}(G)$ and the normalised characteristic function e_K of K in G is the unit in $\mathcal{H}(G, K)$ and an idempotent in $\mathcal{H}(G)$.

The category of smooth representations of p-adic groups is equivalent to the category of non-degenerate Hecke modules, with irreducible smooth representations of G corresponding to simple $\mathcal{H}(G)$-modules. Each smooth representation (π, V) of G determines a $\mathcal{H}(G)$-module structure for V according to the vector-valued integral $\pi(f)v := \int_G f(g)\,\pi(g)v\,dg$ with $f \in \mathcal{H}(G)$ and $v \in V$. The linear operator $\pi(f) : V \to V$ so defined has the property $\langle w, \pi(f)v\rangle = \int_G f(g)\ \langle w, \pi(g)v\rangle\,dg$ for every w in the vector space V^* of linear functionals on V. For each such pair $(w, v) \in V^* \times V$, the locally constant function $g \mapsto \langle w, \pi(g)v\rangle$ is called a *(matrix) coefficient of* π. Chapter 1 provides a detailed study of the category of smooth representations of a p-adic group G.

A representation (π, V) of a p-adic group G is *admissible* if it is smooth and if V^K is finite-dimensional for every compact open subgroup K of G [Car79, Def.1.2]. By a theorem of Harish-Chandra and Jacquet, all irreducible smooth representations are admissible.

One of the most important concepts in the theory of admissible representations begins with the following observation by Harish-Chandra: if (π, V) is an admissible representation of a p-adic group G then, for each $f \in \mathcal{H}(G)$, the operator $\pi(f) : V \to V$ is of finite rank. (In fact, this condition distinguishes admissible representations from smooth representations [Car79, 1.5(d)].) Accordingly, if (π, V) is an admissible representation, then the *character of* π is the distribution Θ_π defined by $\Theta_\pi(f) = \operatorname{trace}(\pi(f))$ for $f \in \mathcal{H}(G)$. By Harish-Chandra's theorem, this distribution is representable by a locally L^1 function, also denoted Θ_π and also called the *character of* π (see Chapter 4).

In fact, the word "character" has earned multiple meanings in representation theory. In Bruhat-Tits theory, a *character* is an element of $X^*(\mathbb{G}) = \mathrm{Hom}_k(\mathbb{G}, \mathbb{G}_m)$, that is, a homomorphism of algebraic groups into the multiplicative group, defined over the field k (see Chapters 2 and 7). In the theory of topological groups, a character is a complex 1-dimensional representation of G; this is sometimes called a *quasi-character* in which case a *character* is a unitary quasi-character. An *additive character* is a complex-valued representation of the additive group of the field. A representation π admits a *central character* when its restriction to the centre of the group is a 1-dimensional complex representation. The *infinitesimal character* of a representation is defined for real Lie groups via the Harish-Chandra isomorphism but also has interpretations for p-adic groups; see Remark 1.9.1.2

Various classes of representations of p-adic groups appear in this volume and one of the most important is that of supercuspidal representations: an admissible representation is *supercuspidal* if its matrix coefficients are compactly supported modulo the centre $Z(G)$ of G. These representations are the focus of Chapter 3.

Some excellent introductory references to the representation theory of p-adic groups include [Car79, Cas74].

Local Langlands Programme

The Langlands Programme gives the study of representations of p-adic groups context and relevance, without which it almost certainly would not have enjoyed such a rich half-century of research.

As we are reminded in Chapter 4, one should never forget that the study of representations of p-adic groups is inextricably linked with harmonic analysis. The Langlands Programme gives us a beautiful illustration of this fact. Efforts to stabilise the trace formula led to a conjecture about local orbital integrals, known as the Fundamental Lemma. This is now a theorem for a large class of groups over non-Archimedean local fields of characteristic p thanks to the work of Ngô Bao Châo [NBC08] building on work with Gérard Laumon [LN08]. Moreover, this result implies the Fundamental Lemma over non-Archimedean local fields of characteristic zero. Chapter 5 gives an introduction to motivic integration and provides the tools used to 'transfer' the Fundamental Lemma from characteristic p to characteristic zero non-Archimedean local fields in [CHL07].

One of the most important open problems in the field is that which goes by the name of the Local Langlands Correspondence, which is discussed in Chapters 6 and 7. The Local Langlands Correspondence is now a theorem in the case of p-adic general linear groups by the work of Michael Harris, Richard Taylor and Guy Henniart, and also in the case of a handful of low-rank groups. However, the full version of the Local Langlands Correspondence, which involves a deep understanding of the structure of L-packets of admissible representations, remains an open problem to this day.

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