

4-Manifolds and Kirby Calculus

Robert E. Gompf
András I. Stipsicz

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ABSTRACT. This text is intended to be an introduction and reference for the differential topology of 4-manifolds as it is currently understood. It is presented from a topologist's viewpoint, often from the perspective of handlebody theory (Kirby calculus), for which an elementary and comprehensive exposition is given. Additional topics include complex, symplectic and Stein surfaces, applications of gauge theory, Lefschetz pencils and exotic smooth structures. The text is intended for students and researchers in topology and related areas, and is suitable for an advanced graduate course. Familiarity with basic algebraic and differential topology is assumed.

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Preface

The past two decades represent a period of explosive growth in 4-manifold theory. From a desert of nearly complete ignorance, the theory has flourished into a virtual rain forest of ideas and techniques, a lush ecosystem supporting complex interactions between diverse fields such as gauge theory, algebraic geometry and symplectic topology, in addition to more topological ideas. Numerous books are appearing that discuss smooth 4-manifolds from the viewpoint of other disciplines. The present volume is intended to introduce the subject from a topologist's viewpoint, bridging the gaps to other disciplines and presenting classical but important topological techniques that have not previously appeared in expository literature.

For a better perspective on the rise of 4-manifold theory, it is useful to consider the history of topology. Manifolds have been a central theme of mathematics for over a century. The topology of manifolds of dimensions ≤ 2 (curves and surfaces) has been well understood since the nineteenth century. Although 3-manifold topology is much harder, there has been steady progress in the field for most of the twentieth century. High-dimensional manifold topology was revolutionized by the s -cobordism and surgery theorems, which were developed in the 1960's into powerful tools for analyzing existence and uniqueness questions about manifolds of dimension ≥ 5 . The resulting theory has long since matured into a subject with a very algebraic flavor. In dimension 4, however, there was not enough room to apply the fundamental "Whitney trick" to prove these theorems, and as a result, very little was known about 4-manifold topology through the 1970's. The first revolution came in 1981 with Michael Freedman's discovery that the Whitney trick could be performed in dimension 4, provided that we ignore smooth structures and work with the underlying topological manifolds up

to homeomorphism (and provided that the fundamental group is suitably “small”). The resulting theory [FQ] led quickly to a complete classification of closed, simply connected topological 4-manifolds, and topological 4-manifold theory now seems closely related to the theory of high-dimensional manifolds. Freedman’s revolution was immediately followed by the 1982 counterrevolution of Simon Donaldson. Using gauge theory (differential geometry and nonlinear analysis), Donaldson showed that smooth 4-manifolds are much different from their high-dimensional counterparts. In fact, the predictions made by the s -cobordism and surgery conjectures for smooth 4-manifolds failed miserably, resulting in a dramatic clash between the theories of smooth and topological manifolds in this dimension. For example, this is the only dimension in which a fixed homeomorphism type of closed manifold is represented by infinitely many diffeomorphism types, or where there are manifolds homeomorphic but not diffeomorphic to \mathbb{R}^n . (In fact, there are uncountably many such “exotic \mathbb{R}^4 ’s”.) One might think of dimension 4 as representing a phase transition between low- and high-dimensional topology, where we find uniquely complicated phenomena and diverse connections with other fields. Donaldson’s program of analyzing the self-dual Yang-Mills equations [DK] was central to smooth 4-manifold theory for 12 years, until it was superseded in 1994 (several revolutions later) by analysis of the Seiberg-Witten equations [KKM], [Mr1], [Sa], which simplifies and expands Donaldson’s original approach and results.

The results of gauge theory, from Donaldson through the Seiberg-Witten equations, are primarily in a negative direction, and require balance by positive results. That is, gauge theory proves the nonexistence of smooth manifolds satisfying various constraints, the nonexistence of connected-sum splittings, and the nonexistence of diffeomorphisms between pairs of manifolds. One needs a different approach for the corresponding existence results. While many useful examples come from algebraic geometry [BPV] and symplectic topology [McS1], perhaps the most powerful general technique for existence results (particularly for manifolds with small Betti numbers) is Kirby calculus. This technique, which allows one to see the internal structure of a 4-manifold (or its boundary 3-manifold) without loss of information, was created and developed into a fine art in the late 1970’s by topologists such as Akbulut, Fenn, Harer, Kaplan, Kirby, Melvin, Rourke, Rolfsen and Stern. However, the theory was handicapped by the pre-Donaldson absence of any way to prove negative results. Much time was spent on ambitious goals that gauge theory now shows are impossible. Eventually, the theory was abandoned by all but the most stalwart practitioners. Since the advent of gauge theory, however, Kirby calculus has entered a Renaissance. Armed with the knowledge of what *not* to attempt, topologists are using

the calculus to construct new manifolds with novel gauge-theoretic properties, some of which are nonalgebraic or even nonsymplectic, and to show that other examples are diffeomorphic or to decompose them into simple pieces. The insight provided by the calculus into the internal structure of manifolds meshes with gauge theory to create an even more powerful tool for analyzing 4-manifolds. In addition, surprising connections have emerged with affine complex analysis and contact topology [G13], [G14] since a discovery of Eliashberg led to a theory of Kirby diagrams for representing Stein surfaces.

One of the main goals of the present book is to provide an exposition of Kirby calculus that is both elementary and comprehensive, since there appears to be no previous reference in the literature that satisfies either of these conditions. We have attempted a complete exposition, providing careful proofs of the main theorems and constructions, either directly or through references to the literature (notably to [M4] and [RS] for careful treatments of handlebody theory in general dimensions). This is at least partly to avoid conveying a false impression of Kirby calculus as being “just pictures and not proofs”. For easy reference, we have included an index of important diagrams, following the glossary of notation in Chapter 13. The reader should note that we have included Kirby diagrams representing all of the main types of closed, simply connected 4-manifolds (as viewed from the current perspective of the theory), namely complex surfaces of rational, elliptic and general type, a symplectic but noncomplex manifold and an irreducible nonsymplectic one. (We have also included an example with even b_2^\pm that might be irreducible.) Chapter 13 also provides an index for Kirby moves and related operations such as Rolfsen moves, Gluck twists and logarithmic transformations. The text has been liberally sprinkled with exercises intended to increase the reader’s comprehension; many of these are labelled with an asterisk and solved in Chapter 12.

The remaining goal of the book is to introduce 4-manifold theory in its current state. There are many books available on the subject, but ours is almost unique in describing the theory from the point of view of differential topology. The other reference from this viewpoint is Kirby [K2]; our text is intended to be complementary to it. Parts of the text were inspired by Harer, Kas and Kirby [HKK]; where overlap occurs we have tried to choose a more elementary and leisurely approach. There are many references for gauge theory as applied to 4-manifolds, notably [DK] (one of the most recent references from the viewpoint of the self-dual equations), and [KKM], [Mr1], [Sa] on Seiberg-Witten theory. These provide detailed treatments, so our approach to gauge theory is to sketch the main ideas and applications with references for details. Similarly, the theory of complex surfaces is covered in detail in [BPV], and symplectic topology is carefully treated in

[**McS1**], so we again focus on the main applications to 4-manifold topology while avoiding unnecessary coverage of other aspects of these theories. For topological 4-manifolds, the reader is referred to [**FQ**] after our brief discussions. Although we treat Rolfsen calculus in some detail, the reader is also referred to [**Ro**] for this 3-dimensional technique related to Kirby calculus. One other noteworthy reference is Kirby's latest list [**K4**] of problems in low-dimensional topology; many of these problems are directly related to 4-manifolds and Kirby calculus.

This book is divided into four parts. The first part covers introductory material and basic techniques for later use, as well as an outline of the current state of the theory of 4-manifolds and surfaces contained in them. Part 2 is our main exposition of Kirby calculus. It is essentially independent of Part 1, except for such elementary notions as intersection forms. The logical dependence of the sections of Part 2 is approximately given by Figure 0.1. (Dashed arrows indicate only occasional or minor dependence.) Part 3 ties together the two previous parts by presenting more advanced applications of Kirby calculus, and consists of five mostly independent chapters intended to cover current research areas within 4-manifold theory and their connections to other disciplines. While we have attempted to include the most recent developments, such a goal is inevitably doomed by the rapid change of the field. Solutions to exercises and the tables described above comprise Part 4. The book can be used as a graduate text, with each of the first two parts providing enough material for nearly a semester. The topics in the third part provide supplementary material intended to introduce a student to research in 4-manifold topology.

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Robert E. Gompf and András I. Stipsicz

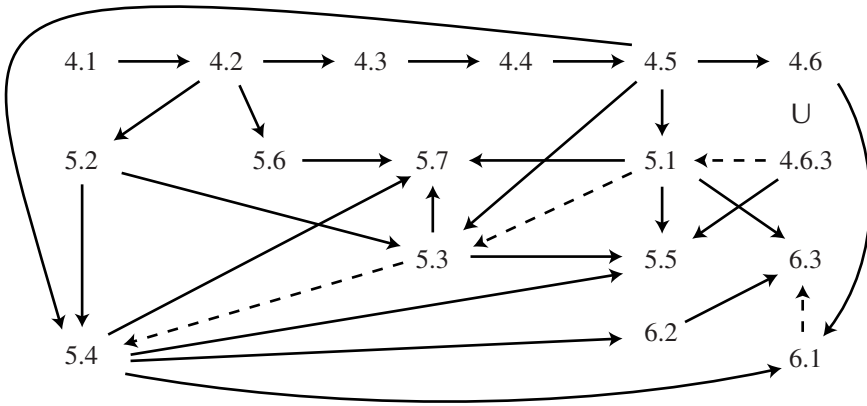


Figure 0.1. Logical dependence of the sections of Part 2

Notation, important figures

13.1. List of commonly used notation

\mathbb{N}	the set of positive integers
\mathbb{Z}	the ring of integers
$\mathbb{C}, \mathbb{R}, \mathbb{Q}$	the fields of complex, real and rational numbers
\mathbb{H}	the field of quaternions
\mathbb{Z}_n	the ring of integers modulo n
\mathbb{R}_+^n	the closed upper half space of \mathbb{R}^n
$\text{gcd}(p, q)$	greatest common divisor of p and q
$[X]$	the fundamental class of the manifold X
$\text{int } X$	the interior of X
$\text{cl}(X)$	the closure of X
\overline{X}	the manifold X with the opposite orientation
∂X	boundary of the manifold X
$\partial_{\pm} X$	part of the boundary of X
\cup_{∂}	gluing along a boundary
$\chi(X)$	the (topological) Euler characteristic of the manifold X
$\sigma(X)$	signature of the 4-manifold X
Q_X	intersection form of the 4-manifold X
\mathcal{C}_X	the set of characteristic elements in $H^2(X^4; \mathbb{Z})$

$b_2^+(X)$ ($b_2^-(X)$ resp.)	the dimension of the maximal positive (negative) definite subspace of $H_2(X; \mathbb{Z})$ with respect to the given intersection form Q_X
PD	Poincaré duality isomorphism
E_8, H	two important intersection forms
$\chi_h(S)$	holomorphic Euler characteristic of the complex surface S
$\kappa(S), \kappa(X)$	the Kodaira dimension of the complex surface S (or symplectic 4-manifold X)
K_S	the canonical line bundle of the complex surface S
D^n	n -dimensional disk
S^n	n -dimensional sphere
T^n	n -dimensional torus
$\mathbb{R}P^n$	n -dimensional (real) projective space
$\mathbb{C}P^n$	n -dimensional (complex) projective space
$[z_0 : \dots : z_n]$	homogeneous coordinates in $\mathbb{C}P^n$ or $\mathbb{R}P^n$
$E(n)$	the simply connected elliptic surface (with section) with $\chi_h(E(n)) = n$
$E(n)_{p_1, \dots, p_k}$	the above elliptic surface after k logarithmic transformations
$M(p, q, r)$	Milnor fiber
Σ_g	Riemann surface of genus g
$g(\Sigma)$	genus of the Riemann surface Σ
$(\mathcal{M}_g, *)$	mapping class group of Σ_g , with multiplication $\varphi * \psi = \psi \circ \varphi$
$\nu\Sigma$	tubular neighborhood of the submanifold Σ
\mathbb{F}_n	Hirzebruch surface
$\mathbb{G}_{n,g}$	geometrically ruled surface over the Riemann surface Σ_g
\approx	orientation-preserving diffeomorphism of manifolds
\sim	orientation-preserving diffeomorphism of Kirby diagrams
$\overset{\partial}{\sim}$	orientation-preserving diffeomorphism of boundary 3-manifolds in a Kirby diagram
\cong	isomorphism of groups
$\#$	connected sum of manifolds
\natural	boundary sum, end sum

$\#_f$	fiber sum
\amalg	disjoint union
\sim_c	cobordant
$\langle n \rangle$	surgery coefficient of $\partial_- X$ (in Kirby diagrams); also used to denote the bilinear form on \mathbb{Z} with matrix $[n]$
$P_G \rightarrow X$	principal G -bundle over X
$P_G \times_\rho F$	the associated fiber bundle (with fiber F) via the representation $\rho: G \rightarrow \text{Aut}(F)$.
$\Gamma(X; E)$	the vector space of C^∞ sections of the vector bundle $E \rightarrow X$
Λ^i	the bundle of i -forms
Λ^\pm	the bundle of self-dual and anti-self-dual forms over a Riemannian 4-manifold
F_A	the curvature of the connection A
F_A^+	the self-dual part of the curvature of the connection A
$O(n), SO(n)$	n -dimensional orthogonal and special orthogonal group
$U(n), SU(n)$	n -dimensional unitary and special unitary group
$GL(n; R), SL(n; R)$	n -dimensional general and special linear group over the ring R
$Spin(n)$	n -dimensional spin group
$Spin^c(n)$	n -dimensional $spin^c$ group
$Lie(G)$	Lie algebra of the Lie group G
\mathcal{S}_X	the set of spin structures on the manifold X
\mathcal{S}_X^c	the set of $spin^c$ structures on the manifold X
$\mathcal{S}_{X,\xi}^c$	the set of $spin^c$ structures on the manifold X inducing the contact structure ξ on ∂X
S^\pm	spinor bundles
$Met(X)$	the space of metrics on the manifold X
\not{D}	the Dirac operator on a spin Riemannian manifold
\not{D}_A	the twisted Dirac operator on a $spin^c$ Riemannian manifold
W^\pm	$spin^c$ spinor bundles
SW_X	Seiberg-Witten invariant of a closed 4-manifold X

$SW_{X,\xi}$	Seiberg-Witten invariant of a 4-manifold X with contact boundary $(\partial X, \xi)$
$\mathcal{P}ert(X)$	the space of perturbations on the 4-manifold X
$\mathcal{B}as_X$	the set of basic classes of a 4-manifold X
Cl_n (and $\mathbb{C}l_n$)	the n -dimensional real (and complexified) Clifford algebra
$\mathbb{C}l(X)$	the complex Clifford bundle over the spin manifold X
Ω_n	n -dimensional cobordism group
Ω_*	cobordism ring
(X, ω)	symplectic manifold with symplectic form ω
(M, ξ)	manifold with contact structure (or plane field) ξ
$lk(K_1, K_2)$	the linking number of the knots K_1, K_2
$w(K)$	writhe of a knot
$tb(K)$	the Thurston-Bennequin invariant of the Legendrian knot K
$r(K)$	the rotation number of the Legendrian knot K

13.2. Index of important diagrams

Akbulut cork: Figures 9.5, 9.7

Branched covers: Section 6.3

Bundles

D^2 -bundle over S^2 : Figure 4.22

D^2 -bundle over T^2 : Figures 4.36, 6.1

with Stein structure: Figure 11.7

D^2 -bundle over $\mathbb{R}P^2$: Figures 4.38, 6.2

with Stein structure: Figure 12.75

D^2 -bundle over Klein bottle: Figure 5.3

D^2 -bundle over genus-3 surface: Figure 12.5

D^2 -bundle over arbitrary closed surface: Figure 6.4

$S^2 \times S^2$: Figure 4.30, Figure 4.34 with n even

$S^2 \tilde{\times} S^2$: Figure 4.34 with n odd

S^2 -bundle over $\mathbb{R}P^2$: Figure 5.46

T^4 : Figure 4.42

Casson handles: Figures 6.14, 6.15

- Closed 4-manifolds (see also Bundles (S^2 -bundles and T^4), Elliptic surfaces, Lefschetz fibrations, lens spaces ($S^1 \times L(5, 1)$))
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- Complex surface $X(m, n)$: Figures 8.33, 8.34
- Horikawa surfaces: $H(n) = X(3, n) = X(n, 3)$, $H'(n) = U(3, n)$
- Irreducible, nonsymplectic manifold X_K : Figure 10.2
- Simply connected manifold $K3 \#_2 K3$ with b_2^\pm even: Figure 10.4
- Symplectic, noncomplex manifold P_1 : Figure 12.71
- Covers: Section 6.3
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- Cusp neighborhood: Figure 8.9
- Logarithmic transform N_p of cusp neighborhood: Figure 8.28
- $E(n)$: Figures 8.11, 8.15, 8.16, 8.31 and 8.32 ($m = 2$), 8.33 and 8.34 ($X(2, n)$ or $X(n, 2)$)
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- Generalized: Figure 7.5
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$L(p, q)$ (Surgery): Figure 5.24

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$S^1 \times L(5, 1)$: Figure 4.41

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with Stein structure: Figure 12.81

Generalized: Figure 7.5

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– on a sphere and torus: Figure 12.62

– on a pair of tori: Figure 12.6

– on a nonsimply connected graph: Figure 6.8

Self-plumbing: Figures 6.10, 6.11

Poincaré homology sphere $\Sigma(2, 3, 5)$: Figures 4.33, 5.22, 8.21

Equivalence of first two descriptions: Figure 12.9

$I \times \Sigma(2, 3, 5)$: Figure 12.36

See also Milnor fiber $M_c(2, 3, 5)$

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$S^2 \times S^2$: Figure 4.30, Figure 4.34 with n even

$S^2 \tilde{\times} S^2$: Figure 4.34 with n odd

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Covers/branched covers: Section 6.3

Doubling: Examples 4.6.3, 5.5.4

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Since the early 1980s, there has been an explosive growth in 4-manifold theory, particularly due to the influx of interest and ideas from gauge theory and algebraic geometry. This book offers an exposition of the subject from the topological point of view. It bridges the gap to other disciplines and presents classical but important topological techniques that have not previously appeared in the literature.

Part I of the text presents the basics of the theory at the second-year graduate level and offers an overview of current research. Part II is devoted to an exposition of Kirby calculus, or handlebody theory on 4-manifolds. It is both elementary and comprehensive. Part III offers in-depth treatments of a broad range of topics from current 4-manifold research. Topics include branched coverings and the geography of complex surfaces, elliptic and Lefschetz fibrations, h -cobordisms, symplectic 4-manifolds, and Stein surfaces.

The authors present many important applications. The text is supplemented with over 300 illustrations and numerous exercises, with solutions given in the book.

I greatly recommend this wonderful book to any researcher in 4-manifold topology for the novel ideas, techniques, constructions, and computations on the topic, presented in a very fascinating way. I think really that every student, mathematician, and researcher interested in 4-manifold topology, should own a copy of this beautiful book.

—Zentralblatt MATH

This book gives an excellent introduction into the theory of 4-manifolds and can be strongly recommended to beginners in this field ... carefully and clearly written; the authors have evidently paid great attention to the presentation of the material ... contains many really pretty and interesting examples and a great number of exercises; the final chapter is then devoted to solutions of some of these ... this type of presentation makes the subject more attractive and its study easier.

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