Foliations I

Alberto Candel
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ABSTRACT. This book is the first of two volumes on foliations. It treats the foundations of the theory, foliated manifolds of codimension one, and selected topics about higher codimension and abstract foliated spaces.

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To our wives, Juana and Jackie,
and in loving memory of
Edna Conlon

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Contents

Preface

This is the first of two volumes on the qualitative theory of foliations. It is divided into three parts, the first being a primer on foliated manifolds. The material here could be the basis of a short graduate course or learning seminar, preparing students to begin exploring the literature. The remainder of this volume is a "sampler" of more advanced topics. In Part 2, we focus on foliated manifolds of codimension one. Here, hands–on geometric methods yield a structure theory (the theory of levels) which is reminiscent of the classical Poincaré–Bendixson theory of flows on surfaces. In arbitrary codimension (Part 3) these methods break down and we turn to the techniques of ergodic theory. Again the theory of flows, now in higher–dimensional manifolds, is the model. Specifically, we consider foliations to be generalized dynamical systems and extend to these systems the theory of invariant measures (following J. F. Plante [**107**], D. Sullivan [**127**], *et al.*) and the theory of topological entropy (following E. Ghys, R. Langevin and P. Walczak [**50**]). For the most part, these methods apply equally well to laminations (partial foliations of manifolds) and even to abstract laminations (foliated metric spaces). In Chapter 11, we introduce this abstract setting and most of the subsequent results are proven for foliated metric spaces.

The measures studied in Part 3 are invariant under holonomy. There are interesting conditions guaranteeing the existence of such measures (*cf.* Theorem 12.3.1 and Theorem 13.4.2) and the corresponding ergodic theory has beautiful applications. The foliation cycles of D. Sullivan (Chapter 10) exemplify this. The limitation of this approach is that holonomy–invariant measures do not always exist. On the other hand, the harmonic measures of L. Garnett [**46**] always exist and the ergodic theory for foliations based on them also has profound applications. We plan to treat this in the second volume.

Foliation theory has developed to the point that an encyclopedic treatment is out of the question. Our choice of topics is quite subjective, depending largely on the authors' tastes and expertise. As to topics omitted, we call special attention to the quantitative theory of foliations, culminating in the stunning existence and classification theorems of W. Thurston in the 1970s [**134, 135**]. Likewise, the applications of noncommutative geometry to foliations, pioneered by A. Connes (see [**31**]), and the beautiful study of transverse geometry (see P. Molino [**91**] and P. Tondeur [**137, 138**]) are not treated. Our hope is that the material we do present will whet the reader's appetite for more. In the second volume, in addition to harmonic measures, we will offer introductory treatments of a few other specialized topics, including the exotic characteristic classes of foliations and foliated 3–manifold theory.

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