# Pick Interpolation and Hilbert Function Spaces

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ABSTRACT. We develop an operator theoretic approach to interpolation problems of Pick type, wherein a function of smallest norm in some given algebra is to be found with certain prescribed values. The algebras we consider can all be realized as multiplier algebras for reproducing kernel Hilbert spaces. We pay particular attention to the bounded analytic functions on the disk and on the bidisk, and to a certain "universal Pick algebra" of analytic functions on the unit ball of a Hilbert space.

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# Preface

This book is about an operator theory approach to the Pick interpolation problem.

The original Pick problem is to determine, given $N$ points $\lambda_1, \ldots, \lambda_N$ in the unit disk $\mathbb{D}$ and $N$ complex numbers $w_1, \ldots, w_N$, whether there exists a holomorphic function $\phi$ on $\mathbb{D}$ that maps each node $\lambda_i$ to the corresponding value $w_i$ and such that

$$\|\phi\| \;:=\; \sup_{z \in \mathbb{D}} |\phi(z)| \;\leq\; 1.$$

This problem was first solved by G. Pick in 1916.

The operator theory approach, pioneered by D. Sarason, is to view this as a question about the multiplier algebra of a particular Hilbert space, namely the Hardy space $H^2$ of holomorphic functions on $\mathbb{D}$ whose Taylor coefficients at 0 are square-summable. It can be shown that the multiplier algebra of $H^2$ is the algebra $H^\infty(\mathbb{D})$ of bounded analytic functions on $\mathbb{D}$.

Let $\mathcal{N}$ be the subspace of $H^2$ consisting of functions vanishing on $\lambda_1, \ldots, \lambda_N$, let $\mathcal{M}$ be the orthocomplement of $\mathcal{N}$, and let $P$ be the orthogonal projection from $H^2$ onto $\mathcal{M}$. It is easy to see that if $M_\phi$ is the operator on $H^2$ of multiplication by $\phi$, then $PM_\phi|_\mathcal{M}$ is an operator on the *finite-dimensional* space $\mathcal{M}$ that depends only on the values $\phi(\lambda_1), \ldots, \phi(\lambda_N)$. Indeed, if $\phi$ vanished on all $N$ points, then $M_\phi f$ would be in $\mathcal{N} = \mathcal{M}^\perp$ for any $f$, so $PM_\phi = 0$.

So, if $\phi$ is any function that interpolates the given data, $\|PM_\phi P\|$ is a lower bound for the norm of $\phi$, and this lower bound can be calculated in terms of $\lambda_1, \ldots, \lambda_N$ and $w_1, \ldots, w_N$. Pick's theorem can be interpreted as saying that this lower bound is always achieved. That this theorem holds is a property of the Hardy space; it does not hold for example on the Bergman space. We say a space in which Pick's theorem holds has the *Pick property.*

The main themes of this book are

(1) analyzing what spaces have the Pick property;

(2) if a function algebra is not the multiplier algebra of a space with the Pick property, solving Pick's problem by considering a family of spaces simultaneously.

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This book is based on a course the second author gave at Washington University in the fall semester of 1999 to an audience of nine graduate students and five faculty members. We have attempted to make the book accessible to graduate students interested in operator theory or holomorphic spaces, by starting at the beginning of the subject. A reader familiar with holomorphic spaces will probably want to skip Chapters 0–4; Chapter 10 will be well known to any reader familiar with operator theory. Most of the material in Chapters 7, 8 and 11–15 appears here for the first time in book form.

Our goal is to expose the reader to a connected set of ideas and to bring him or her up to the current frontiers of research. We include many questions and problems that are, to the best of our knowledge, currently unsolved, and of interest at least to the authors. The ideal reader we have been fondly imagining as we wrote the book is a graduate student who has completed his or her qualifying exams and is looking for a dissertation problem. This intelligent and hard-working person will solve many of the problems raised in this book, transform the field, and become famous and happy. We will settle for the last.

The exercises come in two flavors. The unstarred exercises are routine. The starred ones are results that can be proved with the techniques developed in the book up to the point the exercise is presented, but are challenging. They often constitute published results that we chose not to treat in depth because of considerations of space.

The prerequisites for the book are a basic knowledge of functional analysis (Lebesgue integration, the closed graph theorem, Hahn-Banach theorem, Banach-Alaoglu theorem; some operator theory on Hilbert spaces, such as knowing what the strong operator and weak-star topologies are, knowing what a unitary operator is) and complex analysis (knowing Schwarz's lemma and what the Poisson kernel is). Chapter 0 is a crib-sheet for the prerequisites. Any reader missing some of the prerequisites should not be deterred; one can pick them up as one goes along.

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Here is a chapter-by-chapter summary of what is included.

Chapter 0. This establishes notation and lists some basic facts that we shall use throughout the text.

Chapter 1. We describe the Pick problem and what our approach to it will be.

Chapter 2. Hilbert spaces of holomorphic functions constitute a very large and well-studied area of mathematics. The kernel function for these spaces is critical. We describe the bijection between Hilbert function spaces and kernels. We prove the elementary but very useful fact that every kernel can be represented as a Grammian.

Chapter 3. We prove all the results we shall need about the Hardy space, in particular describing the passage between holomorphic functions on the disk and $L^2$ functions with vanishing negative Fourier coefficients on the circle. This material appears in many other books; we include it for completeness of our treatment, as the Hardy space is central to the whole subject of Pick interpolation.

Chapter 4. We show that $H^\infty(\mathbb{D})$ can be represented, isometrically and weak-star homeomorphically, as the multiplier algebra of many different spaces. (However only the Hardy space has the Pick property.)

Chapter 5. We prove that the Hardy space has the Pick property. Our proof is long, but the method generalizes to other spaces.

Chapter 6. We prove the realization formula, a way of representing functions in the unit ball of $H^\infty(\mathbb{D})$ as the transfer function associated with a unitary operator. This allows another proof of the Pick theorem.

Chapter 7. We characterize all those spaces that have the complete (*i.e.* matrix-valued) Pick property. We use this characterization to show that the Dirichlet space and the Sobolev space have the Pick property.

Chapter 8. We show that there is a universal kernel with the Pick property, in the sense that all kernels that have the complete Pick property are restrictions of this one kernel. We prove the Toeplitz-corona theorem for complete Pick kernels. We give Nevanlinna's parametrization of the set of all solutions to the Pick problem.

Chapter 9. We introduce the notion of an interpolating sequence. We prove Carleson's interpolation theorem and prove some partial generalizations to complete Pick kernels.

Chapter 10. We prove the standard results of model theory — the Sz.-Nagy dilation theorem, von Neumann's inequality, Andô's dilation theorem and the commutant lifting theorem.

Chapter 11. We consider the Pick problem for $H^\infty(\mathbb{D}^2)$. We give generalizations to $\mathbb{D}^2$ of Pick's theorem, the realization formula, the Toeplitz-corona theorem and Nevanlinna's theorem. We give a partial description of interpolating sequences for $H^\infty(\mathbb{D}^2)$.

Chapter 12. We analyze in detail Pick's problem with three points on the bidisk.

Chapter 13. Many algebras do not have the Pick property when thought of as the multiplier algebra of a single space, but if treated as the multiplier algebra of many spaces simultaneously they do have an analogue of the Pick property. We consider this phenomenon. We include a treatment of the Cole-Lewis-Wermer approach to the Pick problem in uniform algebras. We give a necessary and sufficient condition for a kernel structure to have the Pick property.

Chapter 14. We prove Stinespring's theorem and the Arveson extension theorem. We develop the hereditary functional calculus. We use this to characterize which operators can be modeled by the adjoint of a given multiplication operator. We also extend these ideas to commuting $m$-tuples of operators.

Chapter 15. We prove that the complete Pick property is equivalent to a certain localization property for dilations.

Appendices. These contain results that are needed in the text but are somewhat tangential to our development of the theory. In Appendix A, we discuss Schur products of matrices. In Appendix B, we prove Parrott's lemma on completing a matrix. Appendix C discusses interpolation of Banach spaces, in the Riesz-Thorin sense. Appendix D proves the spectral theorem for a finite set of commuting normal operators.

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The authors will maintain a web page with corrections to errors that are brought to their attention and with information on progress on the open problems in the book. It can be accessed at http://www.math.wustl.edu /~mccarthy/Pick.html.

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