Problems in Operator Theory

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To the Memory of our Parents

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Contents

Foreword

This book contains complete solutions to the more than six hundred exercises in the authors' book: **An Invitation to Operator Theory**, American Mathematical Society, 2002. The problems have been spread over eleven chapters following the format of that book. Each problem is identified by a triplet of numbers $x.y.z$; x designates the chapter, y the section, and z the exercise. For instance, Problem 3.4.7 indicates Exercise 7 in Section 4 of Chapter 3.

All solutions are based on the material covered in the text with frequent references to the results in the text. For example, a reference to Theorem 5.9 refers to Theorem 5.9 and a reference to Example 6.21 refers to Example 6.21 in the book **An Invitation to Operator Theory**. We have added an extra amount of material to many solutions in order to make this book as self-contained as possible.

This problem book will be beneficial to students only if they use it "properly," that is to say, if students look at a solution of a problem *only after* trying very hard to solve the problem. Students will do themselves great injustice by reading a solution without any prior attempt on the problem. It should be a real challenge to students to produce solutions which are different from the ones presented here.

Due to the extra material incorporated into the problems, the book can be used as a companion supplement to any text used for the standard functional analysis graduate courses. In addition, this solution book can be used as a reference not only for mathematical subjects but also for other disciplines that rely on functional analytic or measure theoretic techniques.

We would like to express our most sincere thanks to all people who made constructive comments and corrections regarding the text and the problems. Special thanks are due to Professors Arkady Kitover and Vladimir Troitsky who read the solutions and made numerous suggestions and corrections. A final thank you goes to Arlene O'Sean, the AMS Copy Editor, for her excellent job in editing the manuscript.

Y. A. Abramovich and C. D. Aliprantis

Indianapolis and West Lafayette, May 2002

Bibliography

1. Y. A. Abramovich, Some theorems on normed lattices, *Vestnik Leningr. Univ. Mat. Meh. Astronom.* no. 13 (1971), 5–11. (English translation: *Vestnik Leningr. Univ. Math.* **4** (1977), 153–159.)
2. Y. A. Abramovich, C. D. Aliprantis, and O. Burkinshaw, Local quasinilpotence, cycles and invariant subspaces, in: *Interaction between Functional Analysis, Harmonic Analysis, and Probability* (N. Kalton, E. Saab, and S. Montgomery-Smith, eds.), Lecture Notes in Pure and Applied Mathematics, **175**, Marcel Dekker, New York, 1995, pp. 1–12.
3. Y. A. Abramovich, C. D. Aliprantis, and W. R. Zame, A representation theorem for Riesz spaces and its applications to economics, *Economic Theory* **5** (1995), 527–535.
4. C. D. Aliprantis and K. C. Border, *Infinite Dimensional Analysis: A Hitchhikers Guide*, 2nd Edition, Springer–Verlag, Heidelberg and New York, 1999.
5. C. D. Aliprantis and O. Burkinshaw, *Locally Solid Riesz Spaces*, Academic Press, New York and London, 1978.
6. C. D. Aliprantis and O. Burkinshaw, *Positive Operators*, Academic Press, New York and London, 1985.
7. C. D. Aliprantis and O. Burkinshaw, *Principles of Real Analysis*, 3rd Edition, Academic Press, New York and London, 1998.
8. C. D. Aliprantis and O. Burkinshaw, *Problems in Real Analysis*, 2nd Edition, Academic Press, New York and London, 1998.
9. C. D. Aliprantis, O. Burkinshaw, and P. Kranz, On lattice properties of the composition operator, *Manuscripta Math.* **36** (1981), 19–31.
10. T. Andô, Contractive projections in L_p-spaces, *Pacific J. Math.* **17** (1966), 391–405.
11. T. Andô, Banachverbände und positiver Projektionen, *Math. Z.* **109** (1969), 121–130.
12. S. I. Ansari, Essential disjointness and the Daugavet equation, *Houston J. Math.* **19** (1993), 587–601.
13. W. Arendt, Über das spektrum regularer operatoren, Ph.D. Dissertation, University of Tübingen, 1979.
14. N. Aronszajn and K. T. Smith, Invariant subspaces of completely continuous operators, *Ann. of Math.* **60** (1954), 345–350.

15. W. B. Arveson and J. Feldman, A note on invariant subspaces, *Michigan Math. J.* **15** (1968), 61–64.
16. A. Benedek and R. Panzone, The spaces with mixed norm, *Duke Math. J.* **28** (1961), 301–324.
17. A. R. Bernstein and A. Robinson, Solution of an invariant subspace problem of K. T. Smith and P. R. Halmos, *Pacific J. Math.* **16** (1966), 421–431.
18. C. Bessaga and A. Pelczynski, On bases and unconditional convergence of series in Banach spaces, *Studia Math.* **17** (1958), 151–164.
19. E. Bishop and R. R. Phelps, The support functionals of a convex set, in: V. Klee ed., *Convexity* (Proceedings of Symposia in Pure Mathematics, **7**, Amer. Math. Soc., Providence, RI, 1963), 27–35.
20. A. V. Bukhvalov, V. B. Korotkov, A. G. Kusraev, S. S. Kutateladze, and B. M. Makarov, *Vector Lattices and Integral Operators*, Nauka, Novosibirsk, 1992. (English translation edited by S. S. Kutateladze was published by Kluwer Academic Publishers, Dordrecht and Boston, 1996.)
21. V. Caselles, On irreducible operators on Banach lattices, *Indag. Math.* **48** (1986), 11–16.
22. J. A. Clarkson, Uniformly convex spaces, *Trans. Amer. Math. Soc.* **40** (1936), 396–414.
23. J. Diestel, H. Jarchow, and A. Tong, *Absolutely Summing Operators*, Cambridge Studies in Advanced Mathematics, **43**, Cambridge University Press, 1995.
24. J. Diestel and J. J. Uhl, Jr., *Vector Measures*, Math. Surveys, **15**, Amer. Math. Soc., Providence, Rhode Island, 1977.
25. P. G. Dodds and D. H. Fremlin, Compact operators in Banach lattices, *Israel J. Math.* **34** (1979), 287–320.
26. P. G. Dodds, C. B. Huijsmans, and B. de Pagter, Characterizations of conditional expectation-type operators, *Pacific J. Math.* **141** (1990), 55–77.
27. R. G. Douglas, *Banach Algebra Techniques in Operator Theory*, Academic Press, New York and London, 1972.
28. N. Dunford and J. T. Schwartz, *Linear Operators I*, Wiley (Interscience), New York, 1958.
29. Z. Ercan and S. Onal, Invariant subspaces for positive operators acting on a Banach space with Markushevich basis, *Positivity*, forthcoming.
30. V. Gantmacher, Über schwache totalstetige operatoren, *Mat. Sb. (N.S.)* **7** (49) (1940), 301–308.
31. S. A. Gershgorin, Über die Albrenzung Eigenwerte einer Matrix, *Izv. Akad. Nauk SSSR Ser. Fiz.-Mat.* **6** (1931), 749–754.
32. W. T. Gowers and B. Maurey, The unconditional sequence problem, *J. Amer. Math. Soc.* **6** (1993), 851–874.
33. W. T. Gowers and B. Maurey, Banach spaces with small spaces of operators, *Math. Ann.* **307** (1997), 541–568.
34. J. J. Grobler, Band irreducible operators, *Indag. Math.* **48** (1986), 405–409.
35. A. Grothendieck, Une characterisation vectorielle metrique des espaces L^1, *Canad. J. Math.* **7** (1955), 552–561.
36. A. Grothendieck, Produits tensoriels toplogiques et espaces nuclaires, *Mem. Amer. Math. Soc.*, **16**, 1965.
37. F. L. Hernández and B. Rodriguez-Salinas, On ℓ^p-complimented copies in Orlicz spaces II, *Israel J. Math.* **68** (1989), 27–55.

38. R. B. Honor, Density and transitivity results on ℓ^∞ and ℓ^1, *J. London Math. Soc.* **32** (1985), 521–527.

39. R. C. James, Weakly compact sets, *Trans. Amer. Math. Soc.* **113** (1964), 129–140.

40. V. M. Kadets, Some remarks concerning the Daugavet equation, *Quaestiones Math.* **19** (1996), 225–235.

41. V. M. Kadets and M. M. Popov, On the Liapunov convexity theorem with applications to sign-embeddings, *Ukrainian Math. J.* **44** (1992), 1192–1200.

42. V. M. Kadets and M. M. Popov, The Daugavet property for narrow operators in rich subspaces of $C[0,1]$ and $L_1[0,1]$, *St. Petersburg Math. J.* **8** (1997), 571–584.

43. V. M. Kadets, R. V. Shvidkoy, G. G. Sirotkin, and D. Werner, Banach spaces with the Daugavet property, *Trans. Amer. Math. Soc.* **352** (2000), 855–873.

44. G. K. Kalisch, On similarity, reducing manifolds, and unitary equivalences of certain Volterra operators, *Ann. of Math.* **66** (1957), 481–494.

45. L. V. Kantorovich and G. P. Akilov, *Functional Analysis*, Pergamon Press, Oxford and New York, 1982.

46. L. V. Kantorovich, B. Z. Vulikh, and A. G. Pinsker, *Functional Analysis in Partially Ordered Spaces*, Gostekhizdat, Moscow and Leningrad, 1950.

47. R. Khalil, The Daugavet equation in vector-valued function spaces, *Panamer. Math. J.* **6** (1996), 51–53.

48. J. Kim, The characterization of a lattice homomorphism, *Canadian J. Math.* **27** (1975), 172–175.

49. V. B. Korotkov, *Integral Operators*, Nauka, Novosibirsk, 1983.

50. C.-S. Lin, Generalized Daugavet equations and invertible operators on uniformly convex Banach spaces, *J. Math. Anal. Appl.* **197** (1996), 518–528.

51. J. Lindenstrauss and L. Tzafriri, On the complemented subspaces problem, *Israel J. Math.* **9** (1971), 263–269.

52. J. Lindenstrauss and L. Tzafriri, *Classical Banach Spaces I*, Springer–Verlag, Berlin and New York, 1977.

53. V. I. Lomonosov, A counterexample to the Bishop–Phelps theorem in complex spaces, *Israel J. Math.* **115** (2000), 25–28.

54. G. Ya. Lozanovsky, On a theorem of N. Dunford, *Izv. Vyssh. Uchebn. Zaved. Mat.* **8** (1974), 58–59. (Russian)

55. H. P. Lotz, Minimal and reflexive Banach lattices, *Trans. Amer. Math. Soc.* **211** (1975), 85–100.

56. P. Meyer-Nieberg, Quasitriangulierbare Operatoren und invariante Untervekrorräume stetiger linearer Operatoren, *Arch. Math. (Basel)* **22** (1971), 186–199.

57. D. P. Milman, On some criteria for the regularity of spaces of the type (B), *Dokl. Akad. Nauk SSSR, N. S.,* **20** (1938), 243–246. (In Russian.)

58. S-T. C. Moy, Characterization of conditional expectation as a transformation on function spaces, *Pacific J. Math.* **4** (1954), 47–63.

59. J. R. Munkres, *Topology*, Prentice-Hall, Englewood Cliffs, NJ, 1975.

60. H. Nakano, *Modern Spectral Theory*, Maruzen Co., Tokyo, 1950.

61. B. de Pagter, Irreducible compact operators, *Math. Z.* **192** (1986), 149–153.

62. C. Pearcy and N. Salinas, An invariant-subspace theorem, *Michigan Math. J.* **20** (1973), 21–31.

63. A. Pelczynski, On the impossibility of embedding of the space L in certain Banach spaces, *Coll. Math.* **8** (1961), 199–203.

64. B. J. Pettis, A proof that every uniformly convex space is reflexive, *Duke Math. J.* **5** (1939), 369–374.

65. A. M. Plichko and M. M. Popov, *Symmetric function spaces on atomless probability spaces*, Dissertationes Mathematicae, **306**, 1990.

66. C. J. Read, A solution to the invariant subspace problem on the space ℓ_1, *Bulletin London Math. Soc.* **17** (1985), 305–317.

67. C. J. Read, A short proof concerning the invariant subspace problem, *J. London Math. Soc.* (2) **34** (1986), 335–348.

68. C. J. Read, Quasinilpotent operators and the invariant subspace problem, *J. London Math. Soc.* (2) **56** (1997), 595–606.

69. H. H. Schaefer, Spekraleigenschaften positwer linearer Operatoren, *Math. Z.* **82** (1963), 303–313.

70. H. H. Schaefer, *Banach Lattices and Positive Operators*, Springer–Verlag, Berlin and New York, 1974.

71. H. H. Schaefer, On theorems of de Pagter and Andô–Krieger, *Math. Z.* **192** (1986), 155–157.

72. I. Singer, *Bases in Banach Spaces*, Vol. 1, Springer–Verlag, Berlin and New York, 1970.

73. G. G. Sirotkin, Compact-friendly multiplication operators on Banach function spaces, *J. Funct. Analysis*, forthcoming.

74. V. L. Šmulian, Sur la structure de la sphere unitaire dans l'espace de Banach, *Mat. Sb. (N.S.)* **9** (1941), 545–561.

75. A. Sobczyk, Projection of the space m on its subspace c_0, *Bull. Amer. Math. Soc.* **47** (1941), 938–947.

76. A. Spalsbury, Operators not positive with respect to any basis, *Quaestiones Math.* **23** (2000), 489–494.

77. A. Taylor and D. C. Lay, *Introduction to Functional Analysis*, R. E. Krieger, Malabar, Florida, 1986.

78. T. Terzioglu, A characterization of compact linear mappings, *Arch. Math. (Basel)* **22** (1971), 76–78.

79. V. G. Troitsky, On the modulus of C. J. Read's operator, *Positivity* **3** (1998), 257–264.

80. L. Tzafriri, Remarks on contractive projections in L_p-spaces, *Israel J. Math.* **7** (1969), 9–15.

81. L. Tzafriri, An isomorphic characterization of L_p- and c_0-spaces, II, *Michigan Math. J.* **18** (1971), 21–31.

82. L. Weis and D. Werner, The Daugavet equation for operators not fixing a copy of $C(S)$, *J. Operator Theory* **39** (1998), 89–98.

83. A. C. Zaanen, *Riesz Spaces II*, North-Holland, Amsterdam, 1983.

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This is one of the few books available in the literature that contains problems devoted entirely to the theory of operators on Banach spaces and Banach lattices. The book contains complete solutions to the more than 600 exercises in the companion volume, *An Invitation to Operator Theory*, Volume 50 in the AMS series Graduate Studies in Mathematics, also by Abramovich and Aliprantis.

The exercises and solutions contained in this volume serve many purposes. First, they provide an opportunity to the readers to test their understanding of the theory. Second, they are used to demonstrate explicitly technical details in the proofs of many results in operator theory, providing the reader with rigorous and complete accounts of such details. Third, the exercises include many well-known results whose proofs are not readily available elsewhere. Finally, the book contains a considerable amount of additional material and further developments. By adding extra material to many exercises, the authors have managed to keep the presentation as self-contained as possible.

The book can be very useful as a supplementary text to graduate courses in operator theory, real analysis, function theory, integration theory, measure theory, and functional analysis. It will also make a nice reference tool for researchers in physics, engineering, economics, and finance.

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