Resolution of Singularities

Steven Dale Cutkosky

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To Hema, Ashok and Maya

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Contents

Preface

The notion of singularity is basic to mathematics. In elementary algebra singularity appears as a multiple root of a polynomial. In geometry a point in a space is non-singular if it has a tangent space whose dimension is the same as that of the space. Both notions of singularity can be detected through the vanishing of derivitives.

Over an algebraically closed field, a variety is non-singular at a point if there exists a tangent space at the point which has the same dimension as the variety. More generally, a variety is non-singular at a point if its local ring is a regular local ring. A fundamental problem is to remove a singularity by simple algebraic mappings. That is, can a given variety be desingularized by a proper, birational morphism from a non-singular variety? This is always possible in all dimensions, over fields of characteristic zero. We give a complete proof of this in Chapter 6.

We also treat positive characteristic, developing the basic tools needed for this study, and giving a proof of resolution of surface singularities in positive characteristic in Chapter 7.

In Section 2.5 we discuss important open problems, such as resolution of singularities in positive characteristic and local monomialization of morphisms.

Chapter 8 gives a classification of valuations in algebraic function fields of surfaces, and a modernization of Zariski's original proof of local uniformization for surfaces in characteristic zero.

This book has evolved out of lectures given at the University of Missouri and at the Chennai Mathematics Institute, in Chennai, (also known as Madras), India. It can be used as part of a one year introductory sequence in algebraic geometry, and would provide an exciting direction after the basic notions of schemes and sheaves have been covered. A core course on resolution is covered in Chapters 2 through 6. The major ideas of resolution have been introduced by the end of Section 6.2, and after reading this far, a student will find the resolution theorems of Section 6.8 quite believable, and have a good feel for what goes into their proofs.

Chapters 7 and 8 cover additional topics. These two chapters are independent, and can be chosen as possible followups to the basic material in the first 5 chapters. Chapter 7 gives a proof of resolution of singularities for surfaces in positive characteristic, and Chapter 8 gives a proof of local uniformization and resolution of singularities for algebraic surfaces. This chapter provides an introduction to valuation theory in algebraic geometry, and to the problem of local uniformization.

The appendix proves foundational results on the singular locus that we need. On a first reading, I recommend that the reader simply look up the statements as needed in reading the main body of the book. Versions of almost all of these statements are much easier over algebraically closed fields of characteristic zero, and most of the results can be found in this case in standard textbooks in algebraic geometry.

I assume that the reader has some familiarity with algebraic geometry and commutative algebra, such as can be obtained from an introductory course on these subjects. This material is covered in books such as Atiyah and MacDonald [**13**] or the basic sections of Eisenbud's book [**37**], and the first two chapters of Hartshorne's book on algebraic geometry [**47**], or Eisenbud and Harris's book on schemes [**38**].

I thank Professors Seshadri and Ed Dunne for their encouragement to write this book, and Laura Ghezzi, Tài Hà, Krishna Hanamanthu, Olga Kashcheyeva and Emanoil Theodorescu for their helpful comments on preliminary versions of the manuscript.

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Steven Dale Cutkosky

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The notion of singularity is basic to mathematics. In algebraic geometry, the resolution of singularities by simple algebraic mappings is truly a fundamental problem. It has a complete solution in characteristic zero and partial solutions in arbitrary characteristic.

The resolution of singularities in characteristic zero is a key result used in many subjects besides algebraic geometry, such as differential equations, dynamical systems, number theory, the theory of $\mathcal{D}$-modules, topology, and mathematical physics.

This book is a rigorous, but instructional, look at resolutions. A simplified proof, based on canonical resolutions, is given for characteristic zero. There are several proofs given for resolution of curves and surfaces in characteristic zero and arbitrary characteristic.

Besides explaining the tools needed for understanding resolutions, Cutkosky explains the history and ideas, providing valuable insight and intuition for the novice (or expert). There are many examples and exercises throughout the text.

The book is suitable for a second course on an exciting topic in algebraic geometry. A core course on resolutions is contained in Chapters 2 through 6. Additional topics are covered in the final chapters. The prerequisite is a course covering the basic notions of schemes and sheaves.

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