Curves and Surfaces

SECOND EDITION

Sebastián Montiel
Antonio Ros

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This book is dedicated to Eni and Juana, our wives, and to our sons and daughters. We are greatly indebted to them for their encouragement and support, without which this book would have remained a set of teaching notes and exercises.

Contents

Preface to the Second Edition

This second edition follows the plan and the arrangement of topics of the first edition. Some proofs have been revised or simplified. A number of typos and mistakes in the exercises and in the hints for solving them have been corrected. We are very grateful to the publishing staff of the American Mathematical Society and of the Royal Spanish Mathematical Society, especially to Edward Dunne and Guillermo Curbera, for their willingness to undertake a second edition and for their support. We also thank Donald Babbitt for his effort and considerable improvment of both the English and the mathematics in this second edition.

Sebastián Montiel
Antonio Ros
10 February 2009

Preface to the English Edition

This book was begun after a course on the classical differential geometry of surfaces was given by the authors over several years at the University of Granada, Spain. The text benefits largely from comments from students as well as from their reaction to the different topics explained in the lectures.

Since then, different parts of the text have been used as a guide text in several undergraduate and graduate courses. The more that time passes, the more we are persuaded that the study of the geometry of curves and surfaces should be an essential part of the basic training of each mathematician and, at the same time, it could likely be the best way to introduce all the students concerned with differential geometry, both mathematicians and physicists or engineers, to this field.

The book is based on our course, but it has also been completed by including some other alternative subjects, giving on one hand a larger coherence to the text and on the other hand allowing the teacher to focus the course in a variety of ways. Our aim has been to present some of the most relevant global results of classical differential geometry, relative both to the study of curves and that of surfaces.

This text is indeed an improved and updated English version of our earlier Spanish book *Curvas y Superficies* [**12**], published by Proyecto Sur de Ediciones, S. L., Granada, in 1997, and republished in 1998. We are indebted to our colleague Francisco Urbano for many of the improvements and corrections that this English version incorporates. We also recognize a great debt to Joaquín Pérez, another colleague in the Departmento de

Geometría y Topología of our university, for creating the sixty-four figures accompanying the text.

It is also a pleasure to thank Edward Dunne, Editor at the American Mathematical Society, who proposed the possibility of translating our Spanish text, giving us the opportunity of exposing our work to a much wider audience. We also owe a great debt of gratitude to our Production Editor, Arlene O'Sean, for substantially improving the English and the presentation of the text.

The authors gratefully acknowledge that, while this book was being written, they were supported in part by MEC-FEDER Spanish and EU Grants MTM2004-00109 and MTM2004-02746, respectively.

S. Montiel and A. Ros
Granada, 2005

Preface

Over the past decades, there has been an outstanding increase in interest in aspects of global differential geometry, to the neglect of local differential geometry. We have wished, for some years, to present this point of view in a text devoted to classical differential geometry, that is, to the study of curves and surfaces in ordinary Euclidean space. For this, we need to use more sophisticated tools than those usually used in this type of book. Likewise, some topological questions arise that we must unavoidably pay attention to. We intend, at the same time, that our text might serve as an introduction to differential geometry and might be used as a guidebook for a year's study of the differential geometry of curves and surfaces. For this reason, we find additional difficulties of a pedagogical nature and relative to the previous topics that we have to assume a hypothetical reader knows about. These different aspects that we want to consider in our study oppose each other on quite a number of occasions. We have written this text by taking the above problems into account, and we think that it might supply a certain point of equilibrium for all of them.

To read this book, it is necessary to know the basics of linear algebra and to have some ease with the topology of Euclidean space and with the calculus of two and three variables, together with an elementary study of the rudiments of the theory of ordinary differential equations. Besides these standard requirements, the reader should be acquainted with the theory of Lebesgue integration. This novelty is one of the features of our approach. Even though students of our universities usually learn the theories of integration before starting the study of differential geometry, in the past, these theories have been used only in a superficial way in the introductory courses to this subject. Instead, we will use integration as a powerful tool to obtain geometrical results of a global nature.

The second basic difference in our approach, with respect to other books on the same subject, is the particular attention that we will pay, on several occasions throughout the text, to some questions of a topological character. The understanding of such questions will appear sometimes as a goal in itself and often as an essential step to obtaining results of a purely geometrical type.

A third remarkable characteristic of this course on classical differential geometry, this time of a pedagogical nature, although it also has some consequences in the theoretical development of the text, is the authors' determination to use a language free of coordinates whenever possible. We think that, in this way, we get a clearer statement of the results and their proofs.

We would like to thank Manuel Ritoré for his work on drawing the figures accompanying the text. We would also like to thank someone for having typed it and for having decoded the numerous and endless error messages of LaTeX when it was processed, but, in truth, we cannot. Instead, it is easy to recognize our indebtedness to the students of the third year of mathematics at the University of Granada, who, for three years, brought about, with their questions and misgivings, new proofs and points of view relative to diverse problems of classical differential geometry. We are also indebted to earlier textbooks on this subject, mainly to those cited in the bibliography and in a very special way to the book *Differential Geometry of Curves and Surfaces* ([**2**]) of M. P. do Carmo. With it, we passed from ignorance to surprise with respect to the knowledge of differential geometry.

Last, we point out that, for the sake of self-containedness, we have included answers to many of the exercises that we posed within the main text and in the list of exercises that we have included in each chapter. The exercises that we have chosen to answer are not necessarily the most difficult, but those more significant from our point of view. We have marked them with a vertical arrow, like $\upharpoonleft$, at the beginning of the exercise.

S. Montiel and A. Ros
Granada, 1997

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This is a text on the classical differential geometry of curves and surfaces from a modern point of view. In recent years, there has been a marked increase in the interest in the global aspects of this topic, which is the emphasis here. In order to study the global properties of curves and surfaces, more sophisticated tools are necessary than are usually found in texts on the same topic. Also, topological questions arise and must be understood. Thus, for instance, there is a treatment of the Gauss-Bonnet theorem and a discussion of the Euler characteristic. The authors also cover Alexandrov's theorem on compact surfaces in $\mathbb{R}^3$ with constant mean curvature. The last chapter discusses the global geometry of curves, including periodic space curves and the four-vertices theorem in the general case where the curves are not necessarily convex.

Besides being an introduction to the lively subject of curves and surfaces, this book can also be used as an entry to a wider study of differential geometry in general. It is suitable for a first-year graduate course or an advanced undergraduate course.

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