Pseudo-differential Operators and the Nash–Moser Theorem
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Translated by Stephen S. Wilson

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Preface to the English edition

We are happy to welcome the English translation of our book, which originally appeared under the title ‘Opérateurs pseudo-différentiels et théorème de Nash–Moser’ in 1991 (InterEditions/Editions du CNRS, Paris).

Though the world of partial differential equations has changed a lot during these years, we think that the elementary presentation of the subjects touched upon in our book is still up to date and can be useful; thus, we made no changes, except for correcting some misprints. On the other hand, several remarkable books on partial differential equations have appeared since: though their scopes largely exceed that of our book, we thought it relevant to mention them in our bibliography.

Finally, we wish to thank the translator, Dr. Stephen S. Wilson, and the editorial board of the AMS, who worked to produce this new edition of our work.

Orsay, November 2006

Serge Alinhac and Patrick Gérard

Translator’s note

The numbering system I have used in my translation is essentially that employed by the authors in the original French edition so that the actual equation numbers etc. are the same in both versions. I did, however, make certain changes to the cross-referencing system: for example, to remove ambiguity, outside of Chapter II Exercise A.1 of that chapter may be referred to here as Exercise II.A.1 although within Chapter II it is referred to as Exercise A.1.

Cheltenham, February 2007

Stephen S. Wilson
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\(C^k(\Omega), C^\infty(\Omega)\), 5

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\(\langle \cdot, \cdot \rangle\), 6

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\(\text{supp } u, \text{sing supp } u\), 7

\(u \ast v\), 8

\(D'(\Omega), \mathcal{E}'(\Omega)\), 6, 8

\(S, S', \hat{u}, \mathcal{F}\), 10, 11

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This book presents two essential and apparently unrelated subjects. The first, microlocal analysis and the theory of pseudo-differential operators, is a basic tool in the study of partial differential equations and in analysis on manifolds. The second, the Nash–Moser theorem, continues to be fundamentally important in geometry, dynamical systems, and nonlinear PDE.

Each of the subjects, which are of interest in their own right as well as for applications, can be learned separately. But the book shows the deep connections between the two themes, particularly in the middle part, which is devoted to Littlewood–Paley theory, dyadic analysis, and the paradifferential calculus and its application to interpolation inequalities.

An important feature is the elementary and self-contained character of the text, to which many exercises and an introductory Chapter 0 with basic material have been added. This makes the book readable by graduate students or researchers from one subject who are interested in becoming familiar with the other. It can also be used as a textbook for a graduate course on nonlinear PDE or geometry.