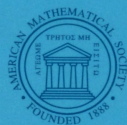


C^* -Algebras and Finite-Dimensional Approximations

Nathanial P. Brown
Narutaka Ozawa

**Graduate Studies
in Mathematics**

Volume 88



American Mathematical Society

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Preface

This is a book about C^* -algebras, various types of approximation, and a few of the surprising applications that have been recently discovered. In short, we will study *approximation theory in the context of operator algebras*.

Approximation is ubiquitous in mathematics; when the object of interest cannot be studied directly, we approximate by tractable relatives and pass to a limit. In our context this is particularly important because C^* -algebras are (almost always) infinite dimensional and we can say precious little without the help of approximation theory. Moreover, most concrete examples enjoy some sort of finite-dimensional localization; hence it is very important to exploit these features to the fullest. Indeed, over the years approximation theory has been at the heart of many of the deepest, most important results: Murray and von Neumann's uniqueness theorem for the hyperfinite II_1 -factor and Connes's remarkable extension to the injective realm; Haagerup's discovery that reduced free group C^* -algebras have the metric approximation property; Higson and Kasparov's resolution of the Baum-Connes conjecture for Haagerup's groups; Popa's work on subfactors and Cartan subalgebras; Voiculescu's whole free entropy industry, which is defined via approximation; Elliott's classification program, which collapses without approximate intertwining arguments; and one can't forget the influential work of Choi, Effros, and Kirchberg on nuclear and exact C^* -algebras.

Approximation is everywhere; it is powerful, important, the backbone of countless breakthroughs. We intend to celebrate it. This subject is a functional analyst's delight, a beautiful mixture of hard and soft analysis, pure joy for the technically inclined. Our wheat may be other texts' chaff, but we see no reason to hide our infatuation with the grace and power which is approximation theory. We don't mean to suggest that mastering technicalities

is the point of operator algebras – it isn't. We simply hope to elevate them from a necessary ally to a revered friend. Also, one shouldn't think these pages are a one-stop shopping place for all aspects of approximation theory – they aren't. The main focus is nuclearity and exactness, with several related concepts and a few applications thrown in for good measure.

From the outset of this project, we were torn between writing user-friendly notes which students would appreciate – many papers in this subject are notoriously difficult to read – or sticking to an expert-oriented, research-monograph level of exposition. In the end, we decided to split the difference. Part 1 of these notes is written with the beginner in mind, someone who just finished a first course in operator algebras (C^* - and W^* -algebras). We wanted the basic theory to be accessible to students working on their own; hence Chapters 2 - 10 have a lot of detail and proceed at a rather slow pace.¹ Chapters 11 - 17 and all of the appendices are written at a higher level, something closer to that found in the literature.

Here is a synopsis of the contents.

Part 1: Basic Theory

The primary objective here is an almost-comprehensive treatment of nuclearity and exactness.² Playing the revisionist-historian role, we define these classes in terms of finite-dimensional approximation properties and later demonstrate the tensor product characterizations. We also study several related ideas which contribute to, and benefit from, nuclearity and exactness.

The first chapter is just a collection of results that we need for later purposes. We often utilize the interplay between C^* -algebras and von Neumann algebras; hence this chapter reviews a number of “basic” facts on both sides. (Some are not so basic and others are so classical that many students never learn them.)

Chapter 2 contains definitions, simple exercises designed to get the reader warmed up, and a few basic examples (AF algebras, C^* -algebras of amenable groups, type I algebras).

¹Except for a few sections in Chapters 4 and 5, where much more is demanded of the reader. This was necessary to keep the book to a reasonable length.

²The most egregious omission is probably Kirchberg's \mathcal{O}_2 -embedding theorem for separable exact C^* -algebras. We felt there were not enough general (i.e., outside of the classification program) applications to warrant including the difficult proof. The paper [107] is readily available and has a self-contained, well-written proof. Rørdam's book [168] has a nearly complete proof and a forthcoming book of Kirchberg and Wassermann will certainly contain all the details. Another significant omission is a discussion of general locally compact groups; we stick to the discrete case. The ideas are adequately exposed in this setting and we don't think beginners benefit from more generality.

In Chapter 3 we give a long introduction to the theory of C^* -tensor products. Most of the chapter is devoted to definitions and a thorough discussion of the subtleties which make C^* -tensor products both interesting and hazardous. However, the last two sections contain important theorems, taking us back to the original definitions of nuclearity and exactness.

In the next two chapters we show that many natural examples of C^* -algebras admit some sort of finite-dimensional approximants. In Chapter 4 we discuss a number of general constructions which one finds in the literature (crossed products by amenable actions, free products, etc.). Chapter 5 is an introduction to exact discrete groups and some related topics which are relevant to noncommutative geometry. Both of these chapters contain redundancies in the sense that we start with special cases and gradually tack on generality. The Bourbakians may protest, but we feel this approach is pedagogically superior.

Someone who works through Chapters 2 - 5 will have a pretty good feel for most aspects of nuclearity and exactness. There is, however, one important permanence property which requires much more work: Both nuclearity and exactness pass to quotients. In some sense, the next four chapters are required to prove these fundamental facts. This doesn't mean we've taken the most direct route, however. On the contrary, we take our sweet time and present a number of related approximation properties which are of independent interest and play crucial roles in the quotient results.

Chapter 6 contains the basics of amenable tracial states. These "invariant means" on C^* -algebras can be characterized in terms of approximation or tensor products. They also yield a simple proof of the deep fact that every finite injective von Neumann algebra is semidiscrete.

In Chapter 7 we study quasidiagonal C^* -algebras. They are also defined via approximation, but the flavor is quite different from nuclearity or exactness. Most of the basic theory is presented, including Voiculescu's homotopy invariance theorem, though much of it isn't necessary for applications to exactness. (For this we only need Dadarlat's approximation theorem for exact quasidiagonal C^* -algebras; see Section 7.5.)

This leads naturally to Chapter 8: AF Embeddability. For applications, the most important fact is that every exact C^* -algebra is a subquotient of an AF C^* -algebra. We give the proof in the beginning of the chapter so those only interested in exactness can quickly proceed forward. For others, we have included the homotopy invariance theorem for AF embeddability and a short survey of related results.

In Chapter 9 we put all the pieces together, completing the basic-theory portion of the book. The main result gives two more tensor product characterizations of exactness, from which corollaries flow: Exact C^* -algebras are

locally reflexive (another important finite-dimensional approximation property), nuclearity and exactness pass to quotients, and a few others.

Finally, we conclude Part 1 with a chapter summarizing permanence properties. This is just for ease of reference, in case one forgets whether or not extensions of exact C^* -algebras are exact.

Part 2: Special Topics

The next four chapters are a disjoint collection of related concepts. They are logically independent and meant to spark the reader's interest – much more could be written about any one of them.

Chapter 11 is primarily about simple quasidiagonal C^* -algebras. Motivated by Elliott's classification program, we spend time discussing the generalized inductive limit approach (of Blackadar and Kirchberg) to nuclear quasidiagonal C^* -algebras. We also prove a theorem of Popa, showing that quasidiagonality is often detectable internally. Finally, we present Connes's amazing uniqueness theorem for the injective II_1 -factor, exploiting Popa's techniques.

Chapter 12 introduces some properties of discrete groups that have been extremely important over the years. First, we discuss Kazhdan's property (T), prove that $SL(3, \mathbb{Z})$ has this property, and demonstrate Connes's result that II_1 -factors with property (T) have few outer automorphisms. Next, we define Haagerup's approximation property – the antithesis of property (T) – and prove that a group which acts properly on a tree (e.g., a free group) enjoys this property. The latter sections of this chapter discuss related approximation properties and their interrelations.

Chapter 13 – on Lance's weak expectation property and the local lifting property for C^* -algebras – gives a streamlined approach to some of Kirchberg's influential work around these ideas. We also reproduce Junge and Pisier's theorem on the tensor product of $\mathbb{B}(\ell^2)$ with itself.

Part 2 concludes with Chapter 14: Weakly Exact von Neumann Algebras. This concept was first suggested by Kirchberg; the theorems and proofs are similar to C^* -results found in Part 1 of the book. It is not yet clear if this theory will bear fruit like its C^* -predecessor, but it seemed like a natural topic to include.

Part 3: Applications

The last three chapters, comprising Part 3, are devoted to applications. We hope to convince you that approximation properties are useful; seemingly unrelated problems will crack wide open when pried with the right technical tool.

Chapter 15 contains solidity and prime factorization results for certain group von Neumann algebras. The solidity results generalize one of the celebrated achievements of free probability theory, while the prime factorization results are natural analogues of some spectacular recent work in dynamical systems. Both depend in a crucial way on some of the C^* -ideas and tensor product techniques contained in Part 1.

Chapter 16 resolves a problem in single operator theory which, at present, appears to require exact quasidiagonal C^* -algebras. We need the fact that exactness implies local reflexivity – one of the deepest, most difficult theorems in C^* -algebras – and it is hard to imagine an operator-theoretic proof which could circumvent this fact.

The final chapter is based on some work of Simon Wassermann. He observed that property (T) groups together with quasidiagonal ideas lead to natural examples for which the Brown-Douglas-Fillmore semigroup is not a group. Approximation properties, or the lack thereof, are at the heart of the argument.

So, that's what you'll find in this book. To the student: We hope these notes are reasonably accessible and a helpful introduction to an area of active research. To the veteran: We hope this will be a useful reference for C^* -approximation theory. As mentioned earlier, Kirchberg and Wassermann are working on an exact C^* -algebra text and there will certainly be overlap between these notes and theirs. However, the emphasis and selection of topics will likely differ; with any luck, the union of our books will satisfy the needs of most. We should also mention that a web page correcting this book's inevitable errors can be found at www.ams.org/bookpages/gsm-88.

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Bibliography

1. C.A. Akemann, J. Anderson and G.K. Pedersen, *Excising states of C^* -algebras*, *Canad. J. Math.* **38** (1986), 1239–1260.
2. C.A. Akemann and P.A. Ostrand, *On a tensor product C^* -algebra associated with the free group on two generators*, *J. Math. Soc. Japan* **27** (1975), 589–599.
3. C. Anantharaman-Delaroche, *Systèmes dynamiques non commutatifs et moyennabilité*, *Math. Ann.* **279** (1987), 297–315.
4. C. Anantharaman-Delaroche, *Amenable correspondences and approximation properties for von Neumann algebras*. *Pacific J. Math.* **171** (1995), 309–341.
5. C. Anantharaman-Delaroche, *Amenability and exactness for dynamical systems and their C^* -algebras*, *Trans. Amer. Math. Soc.* **354** (2002), 4153–4178.
6. C. Anantharaman-Delaroche and J. Renault, *Amenable groupoids*, *Monographies de L'Enseignement Mathématique* 36. L'Enseignement Mathématique, Geneva, 2000.
7. J. Anderson, *A C^* -algebra A for which $\text{Ext}(A)$ is not a group*, *Ann. of Math.* **107** (1978), 455–458.
8. R.J. Archbold and C.J.K. Batty, *C^* -tensor norms and slice maps*, *J. London Math. Soc.* **22** (1980), 127–138.
9. W.B. Arveson, *Subalgebras of C^* -algebras*, *Acta Math.* **123** (1969), 141–224.
10. W.B. Arveson, *An invitation to C^* -algebras*. *Graduate Texts in Mathematics*, No. 39. Springer-Verlag, New York-Heidelberg, 1976.
11. W.B. Arveson, *Notes on extensions of C^* -algebras*, *Duke Math. J.* **44** (1977), 329–355.
12. G.N. Arzhantseva, J. Burillo, M. Lustig, L. Reeves, H. Short and E. Ventura, *Uniform non-amenability*. *Adv. Math.* **197** (2005), 499–522.
13. G. Baumslag, *Topics in combinatorial group theory*. *Lectures in Mathematics ETH Zürich*. Birkhäuser Verlag, Basel, 1993.
14. M.B. Bekka, *On the full C^* -algebras of arithmetic groups and the congruence subgroup problem*, *Forum Math.* **11** (1999), 705–715.
15. B. Bekka, P. de la Harpe and A. Valette, *Kazhdan's property (T)*. *New Mathematical Monographs* 11. Cambridge University Press 2008.

16. G.C. Bell, *Property A for groups acting on metric spaces*, Topology Appl. **130** (2003), 239–251.
17. B. Blackadar, *Nonnuclear subalgebras of C^* -algebras*, J. Operator Theory **14** (1985), 347–350.
18. B. Blackadar and E. Kirchberg, *Generalized inductive limits of finite-dimensional C^* -algebras*, Math. Ann. **307** (1997), 343–380.
19. B. Blackadar and E. Kirchberg, *Inner quasidiagonality and strong NF algebras*, Pacific J. Math. **198** (2001), 307–329.
20. E.F. Blanchard and K.J. Dykema, *Embeddings of reduced free products of operator algebras*, Pacific J. Math. **199** (2001), 1–19.
21. D.P. Blecher, *The standard dual of an operator space*, Pacific J. Math. **153** (1992), 15–30.
22. D.P. Blecher and V.I. Paulsen, *Tensor products of operator spaces*. J. Funct. Anal. **99** (1991), 262–292.
23. F.P. Boca, *A note on full free product C^* -algebras, lifting and quasidiagonality*, Operator theory, operator algebras and related topics (Timisoara, 1996), 51–63, Theta Foundation, Bucharest, 1997.
24. M. Bożejko and G. Fendler, *Herz-Schur multipliers and completely bounded multipliers of the Fourier algebra of a locally compact group*. Boll. Un. Mat. Ital. A (6) **3** (1984), 297–302.
25. M. Bożejko and M.A. Picardello, *Weakly amenable groups and amalgamated products*. Proc. Amer. Math. Soc. **117** (1993), 1039–1046.
26. N.P. Brown, *AF embeddability of crossed products of AF algebras by the integers*, J. Funct. Anal. **160** (1998), 150–175.
27. N.P. Brown, *Herrero’s approximation problem for quasidiagonal operators*, J. Funct. Anal. **186** (2001), 360–365.
28. N.P. Brown, *On quasidiagonal C^* -algebras*, Operator algebras and applications, 19–64, Adv. Stud. Pure Math., 38, Math. Soc. Japan, Tokyo, 2004.
29. N.P. Brown, *Excision and a theorem of Popa*, J. Operator Theory **54** (2005), 3–8.
30. N.P. Brown, *Invariant means and finite representation theory of C^* -algebras*, Mem. Amer. Math. Soc. **184** (2006), no. 865, viii+105 pp.
31. N.P. Brown and M. Dadarlat, *Extensions of quasidiagonal C^* -algebras and K -theory*, Operator algebras and applications, 65–84, Adv. Stud. Pure Math., 38, Math. Soc. Japan, Tokyo, 2004.
32. M. Burger, *Kazhdan constants for $SL(3, \mathbb{Z})$* . J. Reine Angew. Math. **413** (1991), 36–67.
33. P.-A. Cherix, M. Cowling, P. Jolissaint, P. Julg and A. Valette, *Groups with the Haagerup property. Gromov’s a - T -menability*. Progress in Mathematics, **197**. Birkhauser Verlag, Basel, 2001.
34. M. Choda, *Group factors of the Haagerup type*. Proc. Japan Acad. Ser. A Math. Sci. **59** (1983), 174–177.
35. M. Choda, *Reduced free products of completely positive maps and entropy for free product of automorphisms*, Publ. Res. Inst. Math. Sci. **32** (1996), 371–382.
36. M.D. Choi, *A simple C^* -algebra generated by two finite-order unitaries*, Canad. J. Math. **31** (1979), 867–880.
37. M.D. Choi, *The full C^* -algebra of the free group on two generators*, Pacific J. Math. **87** (1980), 41–48.

38. M.D. Choi and E.G. Effros, *The completely positive lifting problem for C^* -algebras*, Ann. of Math. **104** (1976), 585–609.
39. M.D. Choi and E.G. Effros, *Nuclear C^* -algebras and injectivity: the general case*, Indiana Univ. Math. J. **26** (1977), 443–446.
40. M.D. Choi and E.G. Effros, *Nuclear C^* -algebras and the approximation property*, Amer. J. Math. **100** (1978), 61–79.
41. A. Connes, *Classification of injective factors: cases II_1 , II_∞ , III_λ , $\lambda \neq 1$* , Ann. Math. **104** (1976), 73–115.
42. A. Connes, *On the cohomology of operator algebras*, J. Functional Analysis **28** (1978), 248–253.
43. A. Connes, *A factor of type II_1 with countable fundamental group*. J. Operator Theory **4** (1980), 151–153.
44. A. Connes, *Noncommutative geometry*, Academic Press, Inc., San Diego, CA, 1994. xiv+661 pp.
45. A. Connes and V. Jones, *Property T for von Neumann algebras*. Bull. London Math. Soc. **17** (1985), 57–62.
46. M. Cowling, *Harmonic analysis on some nilpotent Lie groups (with application to the representation theory of some semisimple Lie groups)*. Topics in modern harmonic analysis, Vol. I, II (Turin/Milan, 1982), 81–123, Ist. Naz. Alta Mat. Francesco Severi, Rome, 1983.
47. M. Cowling and U. Haagerup, *Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one*. Invent. Math. **96** (1989), 507–549.
48. J. Cuntz, *Simple C^* -algebras generated by isometries*, Comm. Math. Phys. **57** (1977), 173–185.
49. M. Dadarlat, *On the approximation of quasidiagonal C^* -algebras*, J. Funct. Anal. **167** (1999), 69–78.
50. M. Dadarlat, *Nonnuclear subalgebras of AF algebras*, Amer. J. Math. **122** (2000), 581–597.
51. M. Dadarlat, *Residually finite dimensional C^* -algebras and subquotients of the CAR algebra*, Math. Res. Lett. **8** (2001), 545–555.
52. M. Dadarlat, *On the topology of the Kasparov groups and its applications*, J. Funct. Anal. **228** (2005), 394–418.
53. K.R. Davidson, *C^* -algebras by example*, Fields Institute Monographs 6, American Mathematical Society, Providence, RI, 1996.
54. J. De Cannière and U. Haagerup, *Multipliers of the Fourier algebras of some simple Lie groups and their discrete subgroups*. Amer. J. Math. **107** (1985), 455–500.
55. Y. De Cornulier, Y. Stalder and A. Valette, *Proper actions of lamplighter groups associated with free groups*. Preprint, 2007 (arXiv:0707.2039).
56. B. Dorofaeff, *Weak amenability and semidirect products in simple Lie groups*. Math. Ann. **306** (1996), 737–742.
57. K.J. Dykema, *Exactness of reduced amalgamated free product C^* -algebras*, Forum Math. **16** (2004), 161–180.
58. K.J. Dykema and D. Shlyakhtenko, *Exactness of Cuntz-Pimsner C^* -algebras*, Proc. Edinb. Math. Soc. **44** (2001), 425–444.
59. E.G. Effros and U. Haagerup, *Lifting problems and local reflexivity for C^* -algebras*, Duke Math. J. **52** (1985), 103–128.

60. E.G. Effros and E.C. Lance, *Tensor products of operator algebras*, Adv. Math. **25** (1977), no. 1, 1–34.
61. E.G. Effros, N. Ozawa and Z.-J. Ruan, *On injectivity and nuclearity for operator spaces*. Duke Math. J. **110** (2001), 489–521.
62. E.G. Effros and Z.-J. Ruan, *A new approach to operator spaces*, Canad. Math. Bull. **34** (1991), 329–337.
63. E.G. Effros and Z.-J. Ruan, *Operator spaces*, London Mathematical Society Monographs. New Series, 23. The Clarendon Press, Oxford University Press, New York, 2000.
64. G.A. Elliott, *Automorphisms determined by multipliers on ideals of a C^* -algebra*, J. Functional Analysis **23** (1976), 1–10.
65. G.A. Elliott and G. Gong, *On the classification of C^* -algebras of real rank zero II*, Ann. of Math. **144** (1996), 497–610.
66. N.J. Fowler, P.S. Muhly and I. Raeburn, *Representations of Cuntz-Pimsner algebras*, Indiana Univ. Math. J. **52** (2003), 569–605.
67. A. Furman, *Orbit equivalence rigidity*. Ann. of Math. (2) **150** (1999), 1083–1108.
68. L. Ge, *Applications of free entropy to finite von Neumann algebras. II*. Ann. of Math. **147** (1998), 143–157.
69. E. Ghys and P. de la Harpe, *Sur les groupes hyperboliques d’après Mikhael Gromov*. Progress in Math., 83, Birkhäuser, 1990.
70. M. Gromov, *Hyperbolic groups*, Essays in group theory, 75–263, Math. Sci. Res. Inst. Publ. 8, Springer, New York, 1987.
71. M. Gromov, *Random walk in random groups*, Geom. Funct. Anal. **13** (2003), 73–146.
72. E. Guentner, *Exactness of the one relator groups*, Proc. Amer. Math. Soc. **130** (2002), 1087–1093.
73. E. Guentner, N. Higson and S. Weinberger, *The Novikov conjecture for linear groups*, Publ. Math. Inst. Hautes Etudes Sci. No. 101 (2005), 243–268.
74. E. Guentner and J. Kaminker, *Exactness and the Novikov conjecture*, Topology **41** (2002), 411–418.
75. U. Haagerup, *An example of a nonnuclear C^* -algebra, which has the metric approximation property*, Invent. Math. **50** (1978/79), 279–293.
76. U. Haagerup, *All nuclear C^* -algebras are amenable*, Invent. Math. **74** (1983), 305–319.
77. U. Haagerup, *A new proof of the equivalence of injectivity and hyperfiniteness for factors on a separable Hilbert space*, J. Funct. Anal. **62** (1985), 160 – 201.
78. U. Haagerup, *Group C^* -algebras without the completely bounded approximation property*. Preprint, 1986.
79. U. Haagerup, *Connes’ bicentralizer problem and uniqueness of the injective factor of type III₁*, Acta. Math. **158** (1987), 95–148.
80. U. Haagerup and J. Kraus, *Approximation properties for group C^* -algebras and group von Neumann algebras*. Trans. Amer. Math. Soc. **344** (1994), 667–699.
81. U. Haagerup and S. Thorbjørnsen, *A new application of random matrices: $\text{Ext}(C_r^*(\mathbb{F}_2))$ is not a group*, Ann. of Math. **162** (2005), 711–775.
82. P.R. Halmos, *Ten problems in Hilbert space*, Bull. Amer. Math. Soc. **76** (1970), 887–933.

83. P. de la Harpe, A.G. Robertson and A. Valette, *On the spectrum of the sum of generators for a finitely generated group*. Israel J. Math. **81** (1993), 65–96.
84. P. de la Harpe and A. Valette, *La propriété (T) de Kazhdan pour les groupes localement compacts*, Astérisque **175** (1989).
85. N. Higson and E. Guentner, *Group C^* -algebras and K -theory*, Noncommutative geometry, 137–251, Lecture Notes in Math., **1831**, Springer, Berlin, 2004.
86. N. Higson and J. Roe, *Analytic K -homology*, Oxford Mathematical Monographs. Oxford Science Publication. Oxford University Press, Oxford, 2000.
87. T.W. Hungerford, *Algebra*, Reprint of the 1974 original. Graduate Texts in Mathematics, 73. Springer-Verlag, New York-Berlin, 1980.
88. T. Huruya, *On compact completely bounded maps of C^* -algebras*, Michigan Math. J. **30** (1983), 213–220.
89. A. Ioana, J. Peterson and S. Popa, *Amalgamated Free Products of w -Rigid Factors and Calculation of their Symmetry Groups*. Acta. Math., to appear.
90. P. Jolissaint, *Haagerup approximation property for finite von Neumann algebras*. J. Operator Theory **48** (2002), 549–571.
91. P. Jolissaint, *On property (T) for pairs of topological groups*. Enseign. Math. (2) **51** (2005), 31–45.
92. V. Jones and V.S. Sunder, *Introduction to subfactors*, London Mathematical Society Lecture Note Series, 234. Cambridge University Press, Cambridge, 1997. xii+162 pp.
93. M. Junge and G. Pisier, *Bilinear forms on exact operator spaces and $B(H) \otimes B(H)$* , Geom. Funct. Anal. **5** (1995), 329 – 363.
94. R.V. Kadison, *Diagonalizing matrices*. Amer. J. Math. **106** (1984), 1451–1468.
95. R.V. Kadison and J.R. Ringrose, *Fundamentals of the theory of operator algebras*, Vol. II, Graduate Studies in Mathematics, 16. American Mathematical Society, Providence, RI, 1997.
96. V. Kaimanovich, *Boundary amenability of hyperbolic spaces*. Discrete geometric analysis, 83–111, Contemp. Math., **347**, Amer. Math. Soc., Providence, RI, 2004.
97. G.G. Kasparov, *Hilbert C^* -modules: theorems of Stinespring and Voiculescu*, J. Operator Theory **4** (1980), 133–150.
98. T. Katsura, *AF -embeddability of crossed products of Cuntz algebras*, J. Funct. Anal. **196** (2002), 427–442.
99. T. Katsura, *On C^* -algebras associated with C^* -correspondences*, J. Funct. Anal. **217** (2004), 366–401.
100. E. Kirchberg, *C^* -nuclearity implies CPAP*, Math. Nachr. **76** (1977), 203–212.
101. E. Kirchberg, *On the matricial approximation property*. Preprint, 1991.
102. E. Kirchberg, *On nonsemisplit extensions, tensor products and exactness of group C^* -algebras*, Invent. Math. **112** (1993), 449–489.
103. E. Kirchberg, *Commutants of unitaries in UHF algebras and functorial properties of exactness*, J. Reine Angew. Math. **452** (1994), 39–77.
104. E. Kirchberg, *Exact C^* -algebras, tensor products and the classification of purely infinite algebras*, Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zurich, 1994), 943–954.
105. E. Kirchberg, *Discrete groups with Kazhdan’s property T and factorization property are residually finite*, Math. Ann. **299** (1994), 551–563.
106. E. Kirchberg, *On subalgebras of the CAR-algebra*, J. Funct. Anal. **129** (1995), 35–63.

107. E. Kirchberg and N.C. Phillips, *Embedding of exact C^* -algebras in the Cuntz algebra \mathcal{O}_2* , J. Reine Angew. Math. **525** (2000), 17–53.
108. E. Kirchberg and S. Wassermann, *Exact groups and continuous bundles of C^* -algebras*, Math. Ann. **315** (1999), 169–203.
109. E. Kirchberg and S. Wassermann, *Permanence properties of C^* -exact groups*, Doc. Math. **4** (1999), 513–558.
110. E. Kirchberg and W. Winter, *Covering dimension and quasidiagonality*, Internat. J. Math. **15** (2004), 63–85.
111. J. Kraus, *The slice map problem and approximation properties*. J. Funct. Anal. **102** (1991), 116–155.
112. A. Kumjian, D. Pask and I. Raeburn, *Cuntz-Krieger algebras of directed graphs*, Pacific J. Math. **184** (1998), 161–174.
113. E.C. Lance, *On nuclear C^* -algebras*, J. Funct. Anal. **12** (1973), 157–176.
114. E.C. Lance, *Hilbert C^* -modules. A toolkit for operator algebraists*, London Mathematical Society Lecture Note Series, 210. Cambridge University Press, Cambridge, 1995.
115. C. Le Merdy, *On the duality of operator spaces*. Canad. Math. Bull. **38** (1995), 334–346.
116. H. Lin, *Tracially AF C^* -algebras*, Trans. Amer. Math. Soc. **353** (2001), 693–722.
117. H. Lin, *Classification of simple C^* -algebras of tracial topological rank zero*, Duke Math. J. **125** (2004), 91–119.
118. H. Lin, *AF-embedding of crossed products of AH algebras by \mathbb{Z} and asymptotic AF embedding*. Preprint, 2006 (math/0612529).
119. H. Lin, *AF-embedding of the crossed products of AH-algebras by a finitely generated abelian groups*. Preprint, 2007 (arXiv:0706.2229).
120. J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces. I. Sequence spaces*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Vol. 92. Springer-Verlag, Berlin-New York, 1977.
121. R.C. Lyndon and P.E. Schupp, *Combinatorial group theory*. Reprint of the 1977 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.
122. A. Lubotzky, *Discrete groups, expanding graphs and invariant measures*. With an appendix by Jonathan D. Rogawski. Progress in Mathematics, 125. Birkhäuser Verlag, Basel, 1994.
123. A. Lubotzky and Y. Shalom, *Finite representations in the unitary dual and Ramanujan groups*, Discrete geometric analysis, 173–189, Contemp. Math., 347, Amer. Math. Soc., Providence, RI, 2004.
124. H. Matui, *AF embeddability of crossed products of AT algebras by the integers and its application*, J. Funct. Anal. **192** (2002), 562–580.
125. I. Moerdijk and J. Mrčun, *Introduction to foliations and Lie groupoids*. Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge, 2003.
126. P.S. Muhly, J.N. Renault, D.P. Williams, *Equivalence and isomorphism for groupoid C^* -algebras*, J. Operator Theory **17** (1987), 3–22.
127. G.J. Murphy, *C^* -algebras and operator theory*, Academic Press, Inc., Boston, MA, 1990.
128. F.J. Murray and J. von Neumann, *On rings of operators. IV*, Ann. of Math. **44** (1943), 716–808.

129. A. Olshanskii, *SQ-universality of hyperbolic groups*, Sb. Math. **186** (1995), 1199 – 1211.
130. N. Ozawa, *Amenable actions and exactness for discrete groups*, C.R. Acad. Sci. Paris Ser. I Math. **330** (2000), 691–695.
131. N. Ozawa, *Homotopy invariance of AF-embeddability*, Geom. Funct. Anal. **13** (2003), 216–222.
132. N. Ozawa, *Weakly exact von Neumann algebras*. J. Math. Soc. Japan, to appear.
133. N. Ozawa, *Solid von Neumann algebras*, Acta Math. **192** (2004), 111–117.
134. N. Ozawa, *About the QWEP conjecture*, Internat. J. Math. **15** (2004), 501–530.
135. N. Ozawa, *A Kurosh type theorem for type II₁ factors*, Int. Math. Res. Not. (2006), Art. ID 97560, 21 pp.
136. N. Ozawa, *Boundary amenability of relatively hyperbolic groups*, Topology Appl. **153** (2006), 2624–2630.
137. N. Ozawa and S. Popa, *Some prime factorization results for type II₁ factors*, Invent. Math. **156** (2004), 223–234.
138. N. Ozawa and S. Popa, *On a class of II₁ factors with at most one Cartan subalgebra*. Preprint, 2007 (arXiv:0706.3623).
139. A. Paterson, *Amenability*, Mathematical Surveys and Monographs, 29. American Mathematical Society, Providence, RI, 1988.
140. A. Paterson, *Groupoids, inverse semigroups, and their operator algebras*, Progress in Mathematics, 170. Birkhauser Boston, Inc., Boston, MA, 1999.
141. V. Paulsen, *Completely bounded maps and operator algebras*, Cambridge Studies in Advanced Mathematics, 78. Cambridge University Press, Cambridge, 2002.
142. G.K. Pedersen, *C*-algebras and their automorphism groups*, London Mathematical Society Monographs, 14. Academic Press, Inc., London-New York, 1979.
143. J. Peterson, *L²-rigidity in von Neumann algebras*. Preprint, 2006 (math.OA/0605033).
144. J. Peterson and S. Popa, *On the notion of relative property (T) for inclusions of von Neumann algebras*. J. Funct. Anal. **219** (2005), 469–483.
145. M.V. Pimsner, *Embedding some transformation group C*-algebras into AF-algebras*, Ergodic Theory Dynam. Systems **3** (1983), 613–626.
146. M.V. Pimsner, *A class of C*-algebras generalizing both Cuntz-Krieger algebras and crossed products by Z*, Free probability theory (Waterloo, ON, 1995), 189–212, Fields Inst. Commun., 12, Amer. Math. Soc., Providence, RI, 1997.
147. M.V. Pimsner, *Embedding covariance algebras of flows into AF-algebras*, Ergodic Theory Dynam. Systems **19** (1999), 723–740.
148. M. Pimsner and D. Voiculescu, *Imbedding the irrational rotation C*-algebra into an AF-algebra*, J. Operator Theory **4** (1980), 201–210.
149. G. Pisier, *Exact operator spaces*, Recent advances in operator algebras (Orléans, 1992). Astérisque No. 232 (1995), 159–186.
150. G. Pisier, *A simple proof of a theorem of Kirchberg and related results on C*-norms*. J. Operator Theory **35** (1996), 317–335.
151. G. Pisier, *Similarity problems and completely bounded maps*. Second, expanded edition. Includes the solution to “The Halmos problem”. Lecture Notes in Mathematics, 1618. Springer-Verlag, Berlin, 2001.

152. G. Pisier, *Introduction to operator space theory*, London Mathematical Society Lecture Note Series, 294. Cambridge University Press, Cambridge, 2003.
153. G. Pisier, *Remarks on $B(H) \otimes B(H)$* . Preprint, 2005 (math.OA/0509297).
154. S. Popa, *Orthogonal pairs of $*$ -subalgebras in finite von Neumann algebras*. J. Operator Theory **9** (1983), 253–268.
155. S. Popa, *A short proof of “injectivity implies hyperfiniteness” for finite von Neumann algebras*, J. Operator Theory **16** (1986), 261–272.
156. S. Popa, *Correspondences*. INCREST Preprint, 56/1986.
157. S. Popa, *On amenability in type II_1 factors*, Operator algebras and applications, Vol. 2, 173–183, London Math. Soc. Lecture Notes Ser., 136, Cambridge Univ. Press, Cambridge, 1988.
158. S. Popa, *Classification of subfactors and their endomorphisms*. CBMS Regional Conference Series in Mathematics, 86. American Mathematical Society, Providence, RI, 1995.
159. S. Popa, *On local finite-dimensional approximation of C^* -algebras*, Pacific J. Math. **181** (1997), 141–158.
160. S. Popa, *On the fundamental group of type II_1 factors*. Proc. Natl. Acad. Sci. USA **101** (2004), 723–726.
161. S. Popa, *On the Superrigidity of Malleable Actions with Spectral Gap*. J. Amer. Math. Soc., to appear.
162. T. Pytlik and R. Szwarc, *An analytic family of uniformly bounded representations of free groups*. Acta Math. **157** (1986), 287–309.
163. I. Raeburn, *Graph algebras*, CBMS Regional Conference Series in Mathematics, 103. American Mathematical Society, Providence, RI, 2005.
164. J. Renault, *A groupoid approach to C^* -algebras*, Lecture Notes in Mathematics, 793. Springer, Berlin, 1980.
165. E. Ricard and Q. Xu, *Khinchine type inequalities for free product and applications*. J. Reine Angew. Math. **599** (2006), 27–59.
166. A.G. Robertson, *Property (T) for II_1 factors and unitary representations of Kazhdan groups*, Math. Ann. **296** (1993), 547–555.
167. J. Roe, *Lectures on coarse geometry*, University Lecture Series, 31. American Mathematical Society, Providence, RI, 2003.
168. M. Rørdam, *Classification of nuclear, simple C^* -algebras* Classification of nuclear C^* -algebras. Entropy in operator algebras, 1–145, Encyclopaedia Math. Sci., 126, Springer, Berlin, 2002.
169. M. Rørdam, *A purely infinite AH-algebra and an application to AF-embeddability*, Israel J. Math. **141** (2004), 61–82.
170. J. Rosenberg and C. Schochet, *The Kunneth theorem and the universal coefficient theorem for Kasparov’s generalized K -functor*, Duke Math. J. **55** (1987), 431–474.
171. N. Salinas, *Homotopy invariance of $Ext(A)$* , Duke Math. J. **44** (1977), 777–794.
172. J.-P. Serre, *Trees*. Translated from the French original by John Stillwell. Corrected 2nd printing of the 1980 English translation. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003.
173. Y. Shalom, *Bounded generation and Kazhdan’s property (T)*. Inst. Hautes Etudes Sci. Publ. Math. No. **90** (1999), 145–168.
174. Y. Shalom, *Elementary linear groups and Kazhdan’s property (T)*, in preparation.

175. A.M. Sinclair and R.R. Smith, *The Haagerup invariant for tensor products of operator spaces*. Math. Proc. Cambridge Philos. Soc. **120** (1996), 147–153.
176. G. Skandalis. *Une notion de nucléarité en K -théorie (d'après J. Cuntz)*. *K-Theory* **1** (1988), 549–573.
177. G. Skandalis, J.L. Tu and G. Yu, *The coarse Baum-Connes conjecture and groupoids*, *Topology* **41** (2002), 807–834.
178. R.R. Smith, *Completely bounded module maps and the Haagerup tensor product*, *J. Funct. Anal.* **102** (1991), 156–175.
179. J.S. Spielberg, *Embedding C^* -algebra extensions into AF algebras*, *J. Funct. Anal.* **81** (1988), 325–344.
180. A. Szankowski, *$B(\mathcal{H})$ does not have the approximation property*, *Acta Math.* **147** (1981), 89–108.
181. S. Szarek, *An exotic quasidiagonal operator*, *J. Funct. Anal.* **89** (1990), 274–290.
182. M. Takesaki, *On the cross-norm of the direct product of C^* -algebras*, *Tohoku Math. J.* **16** (1964), 111–122.
183. M. Takesaki, *Theory of operator algebras I*. Encyclopedia of Mathematical Sciences, 124. Operator Algebras and Non-commutative Geometry, 5. Springer-Verlag, Berlin, 2002.
184. M. Takesaki, *Theory of operator algebras II*. Encyclopedia of Mathematical Sciences, 125. Operator Algebras and Non-commutative Geometry, 5. Springer-Verlag, Berlin, 2002.
185. M. Takesaki, *Theory of operator algebras III*. Encyclopedia of Mathematical Sciences, 127. Operator Algebras and Non-commutative Geometry, 5. Springer-Verlag, Berlin, 2003.
186. J. Tomiyama, *Applications of Fubini type theorem to the tensor products of C^* -algebras*. *Tôhoku Math. J. (2)* **19** (1967), 213–226.
187. J.L. Tu, *Remarks on Yu's "property A" for discrete metric spaces and groups*, *Bull. Soc. Math. France* **129** (2001), 115–139.
188. D.V. Voiculescu, *Almost inductive limit automorphisms and embeddings into AF-algebras*, *Ergodic Theory Dynam. Systems* **6** (1986), 475–484.
189. D.V. Voiculescu, *A note on quasidiagonal operators*, *Operator Theory: Advances and Applications*, Vol. 32, Birkhauser Verlag, Basel, 1988, 265–274.
190. D.V. Voiculescu, *On the existence of quasicontral approximate units relative to normed ideal. Part I*, *J. Funct. Anal.* **91** (1990), 1–36.
191. D.V. Voiculescu, *A note on quasi-diagonal C^* -algebras and homotopy*, *Duke Math. J.* **62** (1991), 267–271.
192. D.V. Voiculescu, *Around quasidiagonal operators*, *Integr. Equ. and Op. Thy.* **17** (1993), 137–149.
193. D.V. Voiculescu, K.J. Dykema and A. Nica, *Free random variables*, A noncommutative probability approach to free products with applications to random matrices, operator algebras and harmonic analysis on free groups. CRM Monograph Series, 1. American Mathematical Society, Providence, RI, 1992.
194. S. Wassermann, *Injective W^* -algebras*, *Math. Proc. Cambridge Philos. Soc.* **82** (1977), 39–47.
195. S. Wassermann, *C^* -algebras associated with groups with Kazhdan's property T*, *Ann. of Math.* **134** (1991), 423–431.

196. S. Wassermann, *A separable quasidiagonal C^* -algebra with a nonquasidiagonal quotient by the compact operators*, Math. Proc. Cambridge Philos. Soc. **110** (1991), 143–145.
197. S. Wassermann, *Exact C^* -algebras and related topics*, Lecture Notes Series, no.19, GARC, Seoul National University, 1994.
198. N.E. Wegge-Olsen, *K -theory and C^* -algebras. A friendly approach*. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1993.
199. J. Zacharias, *On the invariant translation approximation property for discrete groups*. Proc. Amer. Math. Soc. **134** (2006), 1909–1916.
200. R.J. Zimmer, *Ergodic theory and semisimple groups*, Monographs in Mathematics, 81. Birkhauser Verlag, Basel, 1984.
201. A. Żuk, *Property (T) and Kazhdan constants for discrete groups*, Geom. Funct. Anal. **13** (2003), 643–670.

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