

Representations of Semisimple Lie Algebras in the BGG Category \mathcal{O}

James E. Humphreys

**Graduate Studies
in Mathematics**

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To my future readers Zoë Humphreys, Asher Gerlis,
Emily Hunter, and Miranda Hunter

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Preface

Representation theory plays a central role in Lie theory and has developed in numerous specialized directions over recent decades. Motivation comes from many areas of mathematics and physics, notably the Langlands program. The methods involved are also diverse, including fruitful interactions with “modern” algebraic geometry. Here we focus primarily on algebraic methods in the case of a semisimple Lie algebra \mathfrak{g} over \mathbb{C} with universal enveloping algebra $U(\mathfrak{g})$, where the prerequisites are relatively modest.

The category $\text{Mod } U(\mathfrak{g})$ of all $U(\mathfrak{g})$ -modules is much too large to be understood algebraically. Fortunately, many interesting Lie group representations can be studied effectively in terms of a more limited subcategory where modules are subjected to appropriate finiteness conditions: the *BGG category* \mathcal{O} introduced in the early 1970s by Joseph Bernstein, Israel Gelfand, and Sergei Gelfand. Their papers, stimulated in part by Verma’s 1966 thesis [251], have led to far-reaching work involving a growing list of researchers. In this book we discuss systematically the early work leading to the Kazhdan–Lusztig Conjecture and its proof around 1980. This is at the core of more recent developments, some of which we go on to introduce in the later chapters. Taken on its own, the study of category \mathcal{O} offers a rewarding tour of the beautiful terrain that lies just beyond the classical Cartan–Weyl theory of finite dimensional representations of \mathfrak{g} .

Part I (comprising Chapters 1–8) is written in textbook style, at the level of a second year graduate course in a U.S. university. The emphasis here is on highest weight modules, starting with Verma modules and culminating in the determination of formal characters of simple highest weight modules in the setting of the Kazhdan–Lusztig Conjecture (1979). The proof of this conjecture requires sophisticated ideas from algebraic geometry which go

well beyond the algebraic framework of earlier chapters. Thus Chapter 8 marks a shift toward the survey style used in the remainder of the book.

The chapters in Part II can to a large extent be read independently. They supplement the more unified theme of Part I in a variety of ways, often motivated by problems arising in Lie group representations. The book ends with an introduction to the influential work of Beilinson, Ginzburg, and Soergel on Koszul duality.

I have tried to keep prerequisites to a minimum. The reader needs to be comfortable with the basic structure theory of semisimple Lie algebras over \mathbb{C} (summarized in Chapter 0) as well as with standard algebraic methods including elementary homological algebra.

Exercises are scattered throughout the text (mainly in Part I) where I thought they would do the most good. Some of the more straightforward ones are used later in the development. At any rate, the most important exercise for the reader is to engage actively with the ideas presented. Examples are also interspersed, though unfortunately it is difficult to gain much direct insight from low rank cases of the sort which can be done by hand. The deeper parts of the theory have required some imaginative leaps not based on examples alone.

The substantial reference list includes all source material cited, together with related books and survey articles. I have added a somewhat arbitrary sample of other research papers to point the reader in directions such as those sketched in Chapter 13. There is also a list of frequently used symbols, most of which are introduced early in the book. Anyone who consults the literature will encounter a wide array of notational choices; here I have tried to keep things simple and consistent to the extent possible.

The mathematics presented here is not original, though parts of the treatment may be. Many people have provided helpful feedback on earlier versions of the chapters, including Troels Agerholm, Henning Andersen, Brian Boe, Tom Braden, Jon Brundan, Walter Mazorchuk, Wolfgang Soergel, Catharina Stropple, and Geordie Williamson. I am especially indebted to Jens Carsten Jantzen for his detailed suggestions at many stages of the writing. His ideas have left a lasting imprint on the study of category \mathcal{O} . Naturally, the final choices made are my own responsibility. Corrections and suggestions from readers are welcome.

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Bibliography

1. N. Abe, *On the existence of homomorphisms between principal series of complex semisimple Lie groups*, arXiv:0712.2122 [math.RT]
2. K. Akin, *On complexes relating the Jacobi–Trudy identity with the Bernstein–Gelfand–Gelfand resolution*, *J. Algebra* **117** (1988), 494–503.
3. H. H. Andersen, *Schubert varieties and Demazure’s character formula*, *Invent. Math.* **79** (1985), 611–618.
4. ———, *Twisted Verma modules and their quantized analogues*, pp. 1–10, *Combinatorial and Geometric Representation Theory (Seoul, 2001)*, *Contemp. Math.*, 325, Amer. Math. Soc., Providence, RI, 2003.
5. H. H. Andersen and N. Lauritzen, *Twisted Verma modules*, pp. 1–26, *Studies in Memory of Issai Schur*, *Progr. Math.*, 210, Birkhäuser, Boston, 2003.
6. H. H. Andersen and J. Paradowski, *Fusion categories arising from semisimple Lie algebras*, *Comm. Math. Phys.* **169** (1995), 563–588.
7. H. H. Andersen and C. Stroppel, *Twisting functors on \mathcal{O}_0* , *Represent. Theory* **7** (2003), 681–699.
8. S. Arkhipov, *A new construction of the semi-infinite BGG resolution*, unpublished preprint, arXiv:q-alg/9605043.
9. ———, *Algebraic construction of contragredient quasi-Verma modules in positive characteristic*, pp. 27–68, *Representation Theory of Algebraic Groups and Quantum Groups*, *Adv. Stud. Pure Math.*, **40**, Math. Soc. Japan, Tokyo, 2004.
10. E. Backelin, *Representation of the category \mathcal{O} in Whittaker categories*, *Internat. Math. Res. Notices* **1997**, no. 4, 153–172.
11. ———, *Koszul duality for parabolic and singular category \mathcal{O}* , *Represent. Theory* **3** (1999), 139–152.
12. ———, *The Hom-spaces between projective functors*, *Represent. Theory* **5** (2001), 267–283.
13. D. Barbasch, *Filtrations on Verma modules*, *Ann. Sci. École Norm. Sup. (4)* **16** (1983), 489–494.
14. H. Bass, *Algebraic K-Theory*, W. A. Benjamin, New York, 1968.

15. A. Beilinson, *Localization of representations of reductive Lie algebras*, pp. 699–710, Proc. Intern. Congr. Math. (Warsaw, 1983), PWN, Warsaw, 1984.
16. A. Beilinson and J. Bernstein, *Localisation de \mathfrak{g} -modules*, C.R. Math. Acad. Sci. Paris **292** (1981), 15–18.
17. A. Beilinson, J. Bernstein, and P. Deligne, *Faisceaux pervers*, Analyse et topologie sur les espaces singuliers, I (Luminy, 1981), 5–171, *Astérisque*, **100**, Soc. Math. France, Paris, 1982.
18. ———, *A proof of Jantzen conjectures*, I. M. Gelfand Seminar, 1–50, Adv. Soviet Math., 16, Part 1, Amer. Math. Soc., Providence, RI, 1993.
19. A. Beilinson, R. Bezrukavnikov, and I. Mirković, *Tilting exercises*, Mosc. Math. J. **4** (2004), 547–557.
20. A. Beilinson and V. Ginzburg, *Mixed categories, Ext-duality and representations (results and conjectures)*, preprint, 1986.
21. ———, *Wall-crossing functors and \mathcal{D} -modules*, Represent. Theory **3** (1999), 1–31.
22. A. Beilinson, V. Ginzburg, and W. Soergel, *Koszul duality patterns in representation theory*, J. Amer. Math. Soc. **9** (1996), 473–527.
23. J. Bernstein, I. Frenkel, and M. Khovanov, *A categorification of the Temperley–Lieb algebra and Schur quotients of $U(\mathfrak{sl}_2)$ via projective and Zuckerman functors*, Selecta Math. (N.S.) **5** (1999), 199–241.
24. J. Bernstein and S. Gelfand, *Tensor products of finite and infinite dimensional representations of semisimple Lie algebras*, Compositio Math. **41** (1980), 245–285.
25. I. N. Bernstein, I. M. Gelfand, and S. I. Gelfand, *Structure of representations generated by highest weights*, Funktsional. Anal. i Prilozhen. **5** (1971), no. 1. 1–9; English transl., Funct. Anal. Appl. **5** (1971), 1–8.
26. ———, *Differential operators on the base affine space and a study of \mathfrak{g} -modules*, pp. 21–64, Lie Groups and their Representations (Proc. Summer School, Bolyai János Math. Soc., Budapest, 1971), Halsted, New York, 1975.
27. ———, *On a category of \mathfrak{g} -modules*, Funktsional. Anal. i Prilozhen. **10** (1976), no. 2, 1–8; English transl., Funct. Anal. Appl. **10** (1976), 87–92.
28. R. Biagioli, *Closed product formulas for extensions of generalized Verma modules*, Trans. Amer. Math. Soc. **356** (2004), 159–184.
29. A. Björner and F. Brenti, *Combinatorics of Coxeter groups*, Springer-Verlag, New York, 2005.
30. B. D. Boe, *Homomorphisms between generalized Verma modules*, Trans. Amer. Math. Soc. **288** (1985), 791–799.
31. ———, *A counterexample to the Gabber–Joseph conjecture*, pp. 1–3, *Kazhdan–Lusztig Theory and Related Topics (Chicago, IL, 1989)*, Contemp. Math., 139, Amer. Math. Soc., Providence, RI, 1992.
32. B. D. Boe and D. H. Collingwood, *A comparison theory for the structure of induced representations*, J. Algebra **94** (1985), 511–545.
33. ———, *A comparison theory for the structure of induced representations II*, Math. Z. **190** (1985), 1–11.
34. ———, *A multiplicity one theorem for holomorphically induced representations*, Math. Z. **192** (1986), 265–282.
35. ———, *Multiplicity free categories of highest weight representations*, Comm. Algebra **18** (1990), 947–1032; part II, 1033–1070.

36. B. D. Boe and M. Hunziker, *Kostant modules in blocks of category \mathcal{O}_S* , arXiv:math.RT/0604336.
37. B. D. Boe and D. K. Nakano, *Representation type of the blocks of category \mathcal{O}_S* , Adv. Math. **196** (2005), 193–256.
38. B. D. Boe, D. K. Nakano, and Emilie Wiesner, *Category \mathcal{O} for the Virasoro algebra: cohomology and Koszulity*, Pacific J. Math. **234** (2007), 1–22.
39. A. Borel and N. Wallach, *Continuous cohomology, discrete subgroups, and representations of reductive groups*, 2nd ed., Math. Surveys Monogr., 67, Amer. Math. Soc., Providence, RI, 2000.
40. W. Borho, *A survey on enveloping algebras of semisimple Lie algebras. I*, pp. 19–50, *Lie algebras and related topics (Windsor, Ont., 1984)*, CMS Conf. Proc., 5, Amer. Math. Soc., Providence, RI, 1986.
41. ———, *Nilpotent orbits, primitive ideals, and characteristic classes (a survey)*, pp. 350–359, Proc. Intern. Congr. Math. (Berkeley, Calif., 1986), Amer. Math. Soc., Providence, RI, 1987.
42. W. Borho and J. C. Jantzen, *Über primitive Ideale in der Einhüllenden einer halbeinfachen Lie-Algebra*, Invent. Math. **39** (1977), 1–53.
43. R. Bott, *Homogeneous vector bundles*, Ann. of Math. **66** (1957), 203–248.
44. A. Bouaziz, *Sur les représentations des algèbres de Lie semi-simples construites par T. Enright*, pp. 57–68, *Noncommutative harmonic analysis and Lie groups (Marseille, 1980)*, Lecture Notes in Math., 880, Springer-Verlag, Berlin, 1981.
45. N. Bourbaki, *Groupes et algèbres de Lie*, Chapters 4–6, Hermann, Paris, 1968; 2nd ed., Masson, Paris, 1981; English transl., Springer-Verlag, Berlin, 2002.
46. ———, *Groupes et algèbres de Lie*, Chapters 7–8, Hermann, Paris, 1975; English transl. (with Chapter 9), Springer-Verlag, Berlin, 2005.
47. F. Brenti, *Kazhdan–Lusztig polynomials: history, problems, and combinatorial invariance*, Sémin. Lothar. Combin., **49** (2002/04), Art. B49b, 30 pp.
48. K. S. Brown, *Extensions of “thickened” modules of the Virasoro algebra*, J. Algebra **269** (2003), 160–188.
49. J. Brundan, *Kazhdan–Lusztig polynomials and character formulae for the Lie superalgebra $\mathfrak{gl}(m | n)$* , J. Amer. Math. Soc. **16** (2003), 185–231.
50. ———, *Tilting modules for Lie superalgebras*, Comm. Algebra **32** (2004), 2251–2268.
51. ———, *Centers of degenerate cyclotomic Hecke algebras and parabolic category \mathcal{O}* , arXiv:math.RT/0607717.
52. ———, *Symmetric functions, parabolic category \mathcal{O} and the Springer fiber*, arXiv:math/0608235 [math.RT], to appear in Duke Math. J.
53. J. Brundan, S. M. Goodwin, and A. Kleshchev, *Highest weight theory for finite W -algebras*, arXiv:0801.1337 [math.RT].
54. Th. Brüstle, S. König, and V. Mazorchuk, *The coinvariant algebra and representation types of blocks of category \mathcal{O}* , Bull. London Math. Soc. **33** (2001), 669–681.
55. J. L. Brylinski and M. Kashiwara, *Kazhdan–Lusztig conjecture and holonomic systems*, Invent. Math. **64** (1981), 387–410.
56. K. Carlin, *Extensions of Verma modules*, Trans. Amer. Math. Soc. **294** (1986), 29–43.
57. ———, *Completion and translation in \mathcal{O}* , Comm. Algebra **16** (1988), 1921–1932.
58. ———, *Local systems of Shapovalov elements*, Comm. Algebra **23** (1995), 3039–3049.

59. H. Cartan and S. Eilenberg, *Homological algebra*, Princeton Univ. Press, Princeton, 1956.
60. R. W. Carter, *Lie algebras of finite and affine type*, Cambridge Univ. Press, Cambridge, 2005.
61. L. Casian and D. H. Collingwood, *The Kazhdan–Lusztig conjecture for generalized Verma modules*, *Math. Z.* **195** (1987), 581–600.
62. ———, *Weight filtrations for induced representations of real reductive Lie groups*, *Adv. in Math.* **73** (1989), 79–146.
63. E. Cline, B. Parshall, and L. Scott, *Finite dimensional algebras and highest weight modules*, *J. Reine Angew. Math.* **391** (1988), 85–99.
64. ———, *Abstract Kazhdan–Lusztig theories*, *Tohoku Math. J.* **45** (1993), 511–534.
65. D. H. Collingwood, *Category \mathcal{O}' , n -homology and the reducibility of generalized principal series representations*, *Duke Math. J.* **50** (1983), 1201–1224.
66. ———, *Representations of rank one Lie groups*, *Research Notes in Mathematics*, 137, Pitman (Advanced Publishing Program), Boston, MA, 1985.
67. ———, *The n -homology of Harish-Chandra modules: generalizing a theorem of Kostant*, *Math. Ann.* **272** (1985), 161–187.
68. ———, *Jacquet modules for semisimple Lie groups having Verma module filtrations*, *J. Algebra* **136** (1991), 353–375.
69. D. H. Collingwood and R. S. Irving, *A decomposition theorem for certain self-dual modules in the category \mathcal{O}* , *Duke Math. J.* **58** (1989), 89–102.
70. D. H. Collingwood, R. S. Irving, and B. Shelton, *Filtrations on generalized Verma modules for Hermitian symmetric pairs*, *J. Reine Angew. Math.* **383** (1988), 54–86.
71. D. H. Collingwood and B. Shelton, *A duality theorem for extensions of induced highest weight modules*, *Pacific J. Math.* **146** (1990), 227–237.
72. N. Conze and J. Dixmier, *Idéaux primitifs de l’algèbre enveloppante d’une algèbre de Lie semi-simple*, *Bull. Sci. Math.* **96** (1972), 339–351.
73. N. Conze-Berline and M. Duflo, *Sur les représentations induites des groupes semi-simples complexes*, *Compositio Math.* **34** (1977), 307–336.
74. C. W. Curtis and I. Reiner, *Methods of representation theory, I*, Wiley, New York, 1981.
75. P. Delorme, *Extensions dans la catégorie \mathcal{O} de Bernstein–Gelfand–Gelfand. Applications*, preprint, 1978.
76. ———, *Extensions in the Bernstein–Gelfand–Gelfand category \mathcal{O} . Applications*, *Funktional. Anal. i Prilozhen.* **14** (1980), no 3, 77–78; English transl., *Funct. Anal. Appl.* **14** (1980), 228–229.
77. V. V. Deodhar, *On a construction of representations and a problem of Enright*, *Invent. Math.* **57** (1980), 101–118.
78. ———, *On the Kazhdan–Lusztig conjectures*, *Indag. Math.* **44** (1982), 1–17.
79. ———, *On some geometric aspects of Bruhat orderings, II. The parabolic analogue of Kazhdan–Lusztig polynomials*, *J. Algebra* **111** (1987), 483–506.
80. ———, *Duality in parabolic set up for questions in Kazhdan–Lusztig theory*, *J. Algebra* **142** (1991), 201–209.
81. ———, ed., *Kazhdan–Lusztig Theory and Related Topics (Chicago, IL, 1989)*, *Contemp. Math.*, 139, Amer. Math. Soc., Providence, RI, 1992.

82. V. V. Deodhar, O. Gabber, and V. Kac, *Structure of some categories of representations of infinite-dimensional Lie algebra*, Adv. in Math. **45** (1982), 92–116.
83. V. V. Deodhar and J. Lepowsky, *On multiplicity in the Jordan–Hölder series of Verma modules*, J. Algebra **49** (1977), 512–524.
84. J. Dixmier, *Algèbres enveloppantes*, Gauthier–Villars, Paris, 1974; reprint of English translation, *Enveloping algebras*, Amer. Math. Soc., Providence, RI, 1996.
85. S. Donkin, *The q -Schur algebra*, London Math. Soc. Lecture Note Series, 253, Cambridge Univ. Press, Cambridge, 1998.
86. M. Duflo, *Construction of primitive ideals in an enveloping algebra*, pp. 77–93, Lie Groups and their Representations (Proc. Summer School, Bolyai János Math. Soc., Budapest, 1971), Halsted, New York, 1975.
87. ———, *Représentations irréductibles des groupes semi-simples complexes*, pp. 26–88, *Analyse harmonique sur les groupes de Lie*, Lecture Notes in Math., 497, Springer-Verlag, Berlin, 1975.
88. ———, *Sur la classification des idéaux primitifs dans l’algèbre enveloppante d’une algèbre de Lie semi-simple*, Ann. of Math. **105** (1977), 107–120.
89. T. J. Enright, *On the fundamental series of a real semisimple Lie algebra: their irreducibility, resolutions and multiplicity formulae*, Ann. of Math. **110** (1979), 1–82.
90. ———, *Lectures on representations of complex semisimple Lie groups*, Tata Inst. Lectures on Mathematics and Physics, 66, Springer-Verlag, Berlin, 1981.
91. T. J. Enright and B. Shelton, *Decompositions in categories of highest weight modules*, J. Algebra **100** (1986), 380–402.
92. ———, *Categories of highest weight modules: Applications to Hermitian symmetric spaces*, Mem. Amer. Math. Soc. **67** (1987), no. 367.
93. T. J. Enright and N. R. Wallach, *Notes on homological algebra and representations of Lie algebras*, Duke Math. J. **47** (1980), 1–15.
94. P. Fiebig, *Centers and translation functors for the category \mathcal{O} over Kac–Moody algebras*, Math. Z. **243** (2003), 689–717.
95. ———, *The combinatorics of category \mathcal{O} over symmetrizable Kac–Moody algebras*, Transform. Groups **11** (2006), 29–49.
96. J. Franklin, *Homomorphisms between Verma modules in characteristic p* , J. Algebra **112** (1988), 58–85.
97. W. Fulton and J. Harris, *Representation theory*, Springer-Verlag, New York, 1991.
98. V. Futorny, S. König, and V. Mazorchuk, *S -subcategories in \mathcal{O}* , Manuscripta Math. **102** (2000), 487–503.
99. ———, *A combinatorial description of blocks in $\mathcal{O}(\mathcal{P}, \Lambda)$ associated with $\mathfrak{sl}(2)$ -induction*, J. Algebra **231** (2000), 86–103.
100. ———, *Categories of induced modules for Lie algebras with triangular decomposition*, Forum Math. **13** (2001), 641–661.
101. V. Futorny and V. Mazorchuk, *Structure of α -stratified modules for finite-dimensional Lie algebras. I*, J. Algebra **183** (1996), 456–482.
102. ———, *BGG-resolution for α -stratified modules over simply-laced finite-dimensional Lie algebra*, J. Math. Kyoto Univ. **38** (1998), 229–240.
103. ———, *Highest weight categories of Lie algebra modules*, J. Pure Appl. Math. **138** (1999), 107–118.
104. V. Futorny, D. K. Nakano, and R. D. Pollack, *Representation type of the blocks of category \mathcal{O}* , Q. J. Math. **52** (2001), 285–305.

105. O. Gabber and A. Joseph, *On the Bernstein–Gelfand–Gelfand resolution and the Duflo sum formula*, *Compositio Math.* **43** (1981), 107–131.
106. ———, *Towards the Kazhdan–Lusztig conjecture*, *Ann. Sci. École Norm. Sup.* (4) **14** (1981), 261–302.
107. D. Gaitsgory, *Geometric representation theory*, lecture notes, Harvard Univ., 2005.
108. H. Garland and J. Lepowsky, *Lie algebra homology and the Macdonald–Kac formulas*, *Invent. Math.* **34** (1976), 37–76.
109. S. Gelfand and R. MacPherson, *Verma modules and Schubert cells: a dictionary*, pp. 1–50, *P. Dubreil and M.-P. Malliavin Algebra Seminar (Paris, 1981)*, Lecture Notes in Math., 924, Springer-Verlag, Berlin, 1982.
110. S. I. Gelfand and Yu. I. Manin, *Methods of homological algebra*, 2nd ed., Springer-Verlag, Berlin, 2003.
111. V. Ginzburg, N. Guay, E. Opdam, and R. Rouquier, *On the category \mathcal{O} for rational Cherednik algebras*, *Invent. Math.* **154** (2003), 617–651.
112. X. Gomez and V. Mazorchuk, *On an analogue of BGG-reciprocity*, *Comm. Algebra* **29** (2001), 5329–5334.
113. I. G. Gordon, *Symplectic reflection algebras*, arXiv:0712.1568 [math.RT].
114. M. Goresky, *Tables of Kazhdan–Lusztig polynomials*, currently available online at <http://www.math.ias.edu/~goresky/tables.html>.
115. W. A. de Graaf, *Constructing homomorphisms between Verma modules*, *J. Lie Theory* **15** (2005), 415–428.
116. N. Guay, *Projective modules in the category \mathcal{O} for the Cherednik algebra*, *J. Pure Appl. Algebra* **182** (2003), 209–221.
117. K. Günzl, *The fine structure of translation functors*, *Represent. Theory* **3** (1999), 223–249.
118. A. Gyoja, *Further generalization of generalized Verma modules*, *Publ. Res. Inst. Math. Sci.* **29** (1993), 349–395.
119. ———, *A remark on homomorphisms between generalized Verma modules*, *J. Math. Kyoto Univ.* **34** (1994), 695–697.
120. ———, *A duality theorem for homomorphisms between generalized Verma modules*, *J. Math. Kyoto Univ.* **40** (2000), 437–450.
121. Harish-Chandra, *On some applications of the universal enveloping algebra of a semisimple Lie algebra*, *Trans. Amer. Math. Soc.* **70** (1951), 28–96.
122. P. J. Hilton and U. Stambach, *A course in homological algebra*, Springer-Verlag, New York, 1971.
123. A. van den Hombergh, *Note on a paper by Bernstein, Gelfand and Gelfand on Verma modules*, *Indag. Math.* **36** (1974), 352–356.
124. R. Hotta, K. Takeuchi, and T. Tanisaki, *D -modules, perverse sheaves, and representation theory*, *Progr. Math.*, 236, Birkhäuser, Boston, 2008.
125. J. E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer-Verlag, New York, 1972.
126. ———, *A construction of projective modules in the category \mathcal{O} of Bernstein–Gelfand–Gelfand*, *Indag. Math.* **39** (1977), 301–303.
127. ———, *Finite and infinite dimensional modules for semisimple Lie algebras*, pp. 1–64, *Lie Theories and their Applications*, Queen’s Papers in Pure and Appl. Math. No. 48, Kingston, Ont., 1978.

128. ———, *Highest weight modules for semisimple Lie algebras*, pp. 72–103, *Representation Theory I*, Lecture Notes in Math., 831, Springer-Verlag, Berlin, 1980.
129. ———, *Reflection groups and Coxeter groups* (Cambridge Studies in Advanced Mathematics, 29), Cambridge Univ. Press, Cambridge, 1990.
130. ———, *Conjugacy classes in semisimple algebraic groups*, Math. Surveys Monographs, 43, Amer. Math. Soc., Providence, RI, 1995.
131. ———, *Modular representations of finite groups of Lie type*, London Math. Soc. Lecture Note Series, 326, Cambridge Univ. Press, Cambridge, 2006.
132. R. S. Irving, *Projective modules in the category \mathcal{O}* , unpublished manuscript, 1982.
133. ———, *Projective modules in the category \mathcal{O}_S : self-duality*, Trans. Amer. Math. Soc. **291** (1985), 701–732.
134. ———, *Projective modules in the category \mathcal{O}_S : Loewy series*, Trans. Amer. Math. Soc. **291** (1985), 733–754.
135. ———, *The socle filtration of a Verma module*, Ann. Sci. École Norm. Sup. (4) **21** (1988), 47–65.
136. ———, *Singular blocks of the category \mathcal{O}* , Math. Z. **204** (1990), 209–224.
137. ———, *BGG algebras and the BGG reciprocity principle*, J. Algebra **135** (1990), 363–380.
138. ———, *A filtered category \mathcal{O}_S and applications*, Mem. Amer. Math. Soc. **83** (1990), no. 419.
139. ———, *Graded BGG algebras*, pp. 181–200, *Abelian Groups and Nonabelian Rings*, Contemp. Math., 130, Amer. Math. Soc., Providence, RI, 1992.
140. ———, *Singular blocks of the category \mathcal{O} , II*, pp. 237–248, *Kazhdan–Lusztig Theory and Related Topics (Chicago, IL, 1989)*, Contemp. Math., 139, Amer. Math. Soc., Providence, RI, 1992.
141. ———, *Shuffled Verma modules and principal series modules over complex semisimple Lie algebras*, J. London Math. Soc. **48** (1993), 263–277.
142. R. S. Irving and B. Shelton, *Loewy series and simple projective modules in the category \mathcal{O}_S* , Pacific J. Math. **132** (1988), 319–342; correction, *ibid.* **135** (1988), 395–396.
143. N. Jacobson, *Lie algebras*, Wiley Interscience, New York, 1962; Dover reprint, 1979.
144. J. C. Jantzen, *Darstellungen halbeinfacher algebraischer Gruppen und zugeordnete kontravariante Formen*, Bonner Math. Schriften, No. 67, 1973.
145. ———, *Zur Charakterformel gewisser Darstellungen halbeinfacher Gruppen und Lie-Algebren*, Math. Z. **140** (1974), 127–149.
146. ———, *Kontravariante Formen auf induzierten Darstellungen halbeinfacher Lie-Algebren*, Math. Ann. **226** (1977), 53–65.
147. ———, *Moduln mit einem höchsten Gewicht*, Lecture Notes in Math., 750, Springer-Verlag, Berlin, 1979.
148. ———, *Einhüllende Algebren halbeinfacher Lie-Algebren*, Springer-Verlag, Berlin, 1983 (reviewed by D. A. Vogan, Jr. in Bull. Amer. Math. Soc. **12** (1985), 279–283).
149. ———, *Einhüllende Algebren halbeinfacher Lie-Algebren*, pp. 393–401, *Proc. Intern. Congr. Math. (Warsaw, 1983)*, PWN, Warsaw, 1984.
150. ———, *Primitive ideals in the enveloping algebra of a semisimple Lie algebra*, pp. 29–36, *Noetherian Rings and their Applications*, Math. Surv. Monogr., 24, Amer. Math. Soc., Providence, RI, 1987.
151. ———, *Lectures on quantum groups*, Amer. Math. Soc., Providence, RI, 1996.

152. ———, *Representations of algebraic groups*, Academic Press, Orlando, 1987; 2nd ed., Amer. Math. Soc., Providence, RI, 2003.
153. ———, *Character formulae from Hermann Weyl to the present*, to appear.
154. A. Joseph, *Gelfand–Kirillov dimension for the annihilators of simple quotients of Verma modules*, J. London Math. Soc. **18** (1978), 50–60.
155. ———, *Dixmier’s problem for Verma and principal series submodules*, J. London Math. Soc. **20** (1979), 193–204.
156. ———, *Kostant’s problem, Goldie rank and the Gelfand–Kirillov conjecture*, Invent. Math. **56** (1980), 191–213.
157. ———, *Towards the Jantzen conjecture*, Compositio Math. **40** (1980), 35–67; II **40**, 69–78; III **41** (1981), 23–30.
158. ———, *The Enright functor and the Bernstein–Gelfand–Gelfand category \mathcal{O}* , Invent. Math. **67** (1982), 423–445.
159. ———, *Completion functors in the \mathcal{O} category*, pp. 80–105, *Noncommutative harmonic analysis and Lie groups (Marseille, 1982)*, Lecture Notes in Math., 1020, Springer-Verlag, Berlin, 1983.
160. ———, *Primitive ideals in enveloping algebras*, pp. 403–414, *Proc. Intern. Congr. Math. (Warsaw, 1983)*, PWN, Warsaw, 1984.
161. ———, *Quantum groups and their primitive ideals*, Springer-Verlag, Berlin, 1995.
162. ———, *Sur l’annulateur d’un module de Verma*, pp. 237–300, *Representation Theories and Algebraic Geometry (Montreal, PQ, 1997)*, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 514, Kluwer Acad. Publ., Dordrecht, 1998.
163. A. Joseph, G. S. Perets, and P. Polo, *Sur l’équivalence de catégories de Beilinson et Bernstein*, C.R. Math. Acad. Sci. Paris **313** (1991), 705–709.
164. V. G. Kac, *Infinite-dimensional Lie algebras and Dedekind’s η -function*, Functional Anal. Appl. **8** (1974), 68–70.
165. ———, *Infinite dimensional Lie algebras*, 3rd ed., Cambridge Univ. Press, Cambridge, 1990.
166. V. G. Kac and D. A. Kazhdan, *Structure of representations with highest weight of infinite-dimensional Lie algebras*, Adv. Math. **34** (1979), 97–108.
167. J. Kåhrström, *Tensoring with infinite-dimensional modules in \mathcal{O}_0* , arXiv:0708/2218 [math.RT].
168. J. Kåhrström and V. Mazorchuk, *A new approach to Kostant’s problem*, arXiv:0712.3117 [math.RT].
169. M. Kashiwara and T. Tanisaki, *Characters of irreducible modules with non-critical highest weights over affine Lie algebras*, pp. 275–296, *Representations and Quantizations (Shanghai, 1998)*, China High. Educ. Press, Shanghai, 2000.
170. D. Kazhdan and G. Lusztig, *Representations of Coxeter groups and Hecke algebras*, Invent. Math. **53** (1979), 165–184.
171. ———, *Schubert varieties and Poincaré duality*, pp. 185–203, *Geometry of the Laplace operator*, Proc. Sympos. Pure Math., XXXVI, Amer. Math. Soc., Providence, R.I., 1980.
172. A. Khare, *Category \mathcal{O} over a deformation of the symplectic oscillator algebra*, J. Pure Appl. Algebra **195** (2005), 131–166; erratum **199** (2005), 319–320.
173. ———, *Axiomatic framework for the BGG category \mathcal{O}* , arXiv:math.RT/0502227.
174. O. Khomenko, *Categories with projective functors*, Proc. London Math. Soc. **90** (2005), 711–737.

175. O. Khomenko and V. Mazorchuk, *The Shapovalov form for generalized Verma modules*, Dopov. Nats. Akad. Nauk Ukr. Mat. Prirodozn. Tekh. Nauki **1999**, no. 6, 28–32.
176. ———, *A note on simplicity of generalized Verma modules*, Comment. Math. Univ. St. Paul. **48** (1999), 145–148.
177. ———, *On the determinant of Shapovalov form for generalized Verma modules*, J. Algebra **215** (1999), 318–329.
178. ———, *Generalized Verma modules over the Lie algebra of type G_2* , Comm. Algebra **27** (1999), 777–783.
179. ———, *Schubert filtration for simple quotients of generalized Verma modules*, Ark. Mat. **38** (2000), 319–326.
180. ———, *An irreducibility criterion for generalized Verma modules*, Dopov. Nats. Akad. Nauk Ukr. Mat. Prirodozn. Tekh. Nauki **2001**, no. 5, 22–24.
181. ———, *Generalized Verma modules induced from $\mathfrak{sl}(2, \mathbb{C})$ and associated Verma modules*, J. Algebra **242** (2001), 561–576.
182. ———, *On multiplicities of simple subquotients in generalized Verma modules*, Czechoslovak Math. J. **52** (127) (2002), 337–343.
183. ———, *Rigidity of generalized Verma modules*, Colloq. Math. **92** (2002), 45–57.
184. ———, *Structure of modules induced from simple modules with minimal annihilator*, Canad. J. Math. **56** (2004), 293–309.
185. ———, *On Arkhipov’s and Enright’s functors*, Math. Z. **249** (2005), 357–386.
186. S. Khoroshkin, *Projective functors and the restriction of a Verma module to a subalgebra of Levi type*, Ann. Global Anal. Geom. **10** (1992), 81–86.
187. S. L. Kleiman, *The development of intersection homology theory*, Pure Appl. Math. Q. **3** (2007), 225–282.
188. A. W. Knap, *Lie groups, Lie algebras, and cohomology*, Math. Notes **34**, Princeton Univ. Press, Princeton, NJ, 1988.
189. S. König, *The global dimension of the regular blocks of the BGG-category \mathcal{O} of a semisimple complex Lie algebra*, unpublished preprint.
190. ———, *Cartan decompositions and BGG-resolutions*, Manuscripta Math. **86** (1995), 103–111.
191. ———, *Blocks of category \mathcal{O} , double centralizer properties, and Enright’s completions*, pp. 113–134 in *Algebra—Representation Theory*, K. W. Roggenkamp and M. Ştefănescu, eds., Kluwer, Dordrecht, 2001.
192. ———, *Ringel duality and Kazhdan–Lusztig theory*, Pacific J. Math. **203** (2002), 415–428.
193. S. König and V. Mazorchuk, *Enright’s completions and injectively copresented modules*, Trans. Amer. Math. Soc. **354** (2002), 2725–2743.
194. ———, *An equivalence of two categories of $\mathfrak{sl}(n, \mathbb{C})$ -modules*, Algebr. Represent. Theory **5** (2002), 319–329.
195. B. Kostant, *A formula for the multiplicity of a weight*, Trans. Amer. Math. Soc. **93** (1959), 53–73.
196. ———, *Lie algebra cohomology and the generalized Borel–Weil–Bott theorem*, Ann. of Math. **74** (1961), 329–387.
197. S. Kumar, *Bernstein–Gelfand–Gelfand resolution for arbitrary Kac–Moody algebras*, Math. Ann. **286** (1990), 709–729.

198. J. Lepowsky, *Conical vectors in induced modules*, Trans. Amer. Math. Soc. **208** (1975), 219–272.
199. ———, *Existence of conical vectors in induced modules*, Ann. of Math. **102** (1975), 17–40.
200. ———, *Uniqueness of embeddings of certain induced modules*, Proc. Amer. Math. Soc. **56** (1976), 55–58.
201. ———, *Generalized Verma modules, the Cartan–Helgason theorem, and the Harish-Chandra homomorphism*, J. Algebra **49** (1977), 470–495.
202. ———, *A generalization of the Bernstein–Gelfand–Gelfand resolution*, J. Algebra **49** (1977), 496–511.
203. G. Lusztig and D. A. Vogan, Jr., *Singularities of closures of K -orbits on flag manifolds*, Invent. Math. **71** (1983), 365–379.
204. A. V. Lutsyuk, *Homomorphisms of the modules M_χ* , Funktsional. Anal. i Prilozhen. **8** (1974), no. 4, 91–92; English transl., Funct. Anal. Appl. **8** (1975), 351–352.
205. C. Marastoni, *Generalized Verma modules, b -functions of semi-invariants and duality for twisted \mathcal{D} -modules on generalized flag manifolds*, C.R. Math. Acad. Sci. Paris **335** (2002), 111–116.
206. O. Mathieu, *Classification of irreducible weight modules*, Ann. Inst. Fourier (Grenoble) **50** (2000), 537–592.
207. H. Matumoto, *On the existence of homomorphisms between scalar generalized Verma modules*, pp. 259–274, *Representation Theory of Groups and Algebras*, Contemp. Math., 145, Amer. Math. Soc., Providence, RI, 1993.
208. ———, *The homomorphisms between scalar generalized Verma modules associated to maximal parabolic subalgebras*, Duke Math. J. **131** (2006), 75–118.
209. V. Mazorchuk, *α -stratified modules over the Lie algebra $\mathfrak{sl}(n, \mathbb{C})$* , Ukrain. Mat. Zh. **45** (1993), no. 9, 1215–1224; English transl., Ukrainian Math. J. **45** (1993), no. 9, 1360–1371.
210. ———, *On the structure of an α -stratified generalized Verma module over Lie algebra $\mathfrak{sl}(n, \mathbb{C})$* , Manuscripta Math. **88** (1995), 59–72.
211. ———, *Generalized Verma modules*, Mathematical Studies Monograph Series, 8. VNTL Publishers, L’viv, 2000.
212. ———, *Twisted and shuffled filtrations on tilting modules*, C.R. Math. Acad. Sci. Soc. R. Can. **25** (2003), no. 1, 26–32.
213. ———, *A twisted approach to Kostant’s problem*, Glasg. Math. J. **47** (2005), 549–561.
214. ———, *Some homological properties of the category \mathcal{O}* , Pacific J. Math. **232** (2007), 313–341.
215. ———, *Applications of the category of linear complexes of tilting modules associated with the category \mathcal{O}* , to appear in Algebr. Represent. Theory.
216. V. Mazorchuk, S. Ovsienko, and C. Stroppel, *Quadratic duals, Koszul dual functors, and applications*, arXiv:math.RT/0603475, to appear in Trans. Amer. Math. Soc.
217. V. Mazorchuk and C. Stroppel, *Translation and shuffling of projectively presentable modules and a categorification of a parabolic Hecke module*, Trans. Amer. Math. Soc. **357** (2005), 2939–2973.
218. ———, *Projective-injective modules, Serre functors and symmetric algebras*, arXiv:math.RT/0508119, to appear in J. Reine Angew. Math.

219. ———, *On functors associated to a simple root*, J. Algebra **314** (2007), 97–128.
220. ———, *Categorification of (induced) cell modules and the rough structure of generalized Verma modules*, arXiv:math.RT/070281.
221. ———, *Categorification of Wedderburn’s basis for $\mathbb{C}[S_n]$* , arXiv:0708.3949 [math.RT], to appear in Arch. Math. (Basel).
222. R. Mirollo and K. Vilonen, *Bernstein–Gelfand–Gelfand reciprocity on perverse sheaves*, Ann. Sci. École Norm. Sup. (4) **20** (1987), 311–324.
223. R. V. Moody and A. Pianzola, *Lie algebras with triangular decompositions*, Wiley Interscience, New York–Toronto, 1995.
224. H. Oda and T. Oshima, *Minimal polynomials and annihilators of generalized Verma modules of the scalar type*, J. Lie Theory **16** (2006), 155–219.
225. P. Ostapenko, *Inverting the Shapovalov form*, J. Algebra **147** (1992), 90–95.
226. A. Rocha-Caridi, *Splitting criteria for \mathfrak{g} -modules induced from a parabolic and the Bernstein–Gelfand–Gelfand resolution of a finite-dimensional, irreducible \mathfrak{g} -module*, Trans. Amer. Math. Soc. **262** (1980), 335–366.
227. A. Rocha-Caridi and N. R. Wallach, *Projective modules over graded Lie algebras. I*, Math. Z. **180** (1982), 151–177.
228. ———, *Highest weight modules over graded Lie algebras: resolutions, filtrations and character formulas*, Trans. Amer. Math. Soc. **277** (1983), 133–162.
229. ———, *Characters of irreducible representations of the Virasoro algebra*, Math. Z. **185** (1984), 1–21.
230. S. Ryom-Hansen, *Koszul duality of translation and Zuckerman functors*, J. Lie Theory **14** (2004), 151–163.
231. N. N. Shapovalov, *On a bilinear form on the universal enveloping algebra of a complex semisimple Lie algebra*, Funktsional. Anal. i Prilozhen. **6** (1972), no. 4, 65–70; English transl., Funct. Anal. Appl. **6** (1972), 307–312.
232. B. Shelton, *Extensions between generalized Verma modules: the Hermitian symmetric cases*, Math. Z. **197** (1988), 305–318.
233. W. Soergel, *\mathcal{D} -modules et équivalence de Enright–Shelton*, C.R. Math. Acad. Sci. Paris **307** (1988), 19–22.
234. ———, *Universelle versus relative Einhüllende: Eine geometrische Untersuchung von Quotienten von universellen Einhüllenden halbeinfacher Lie-Algebren*, Math. Ann. **284** (1989), 177–198.
235. ———, *\mathfrak{n} -cohomology of simple highest weight modules on walls and purity*, Invent. Math. **98** (1989), 565–680.
236. ———, *Parabolisch-singuläre Dualität für Kategorie \mathcal{O}* , Max-Planck-Institut, Bonn, MPI/89–68.
237. ———, *Kategorie \mathcal{O} , perverse Garben und Moduln über den Koinvarianten zur Weylgruppe*, J. Amer. Math. Soc. **3** (1990), 421–445.
238. ———, *Gradings on representation categories*, pp. 800–806, Proc. Intern. Congr. Math. (Zürich, 1994), Birkhäuser Verlag, Basel, 1995.
239. ———, *Kazhdan–Lusztig polynomials and a combinatoric for tilting modules*, Represent. Theory **1** (1997), 83–114.
240. ———, *Character formulas for tilting modules over Kac–Moody algebras*, Represent. Theory **2** (1998), 432–448.
241. ———, *Andersen filtration and Hard Lefschetz*, Geom. Funct. Anal. **17** (2008), 2066–2089.

242. T. A. Springer, *Quelques applications de la cohomologie d'intersection*, Bourbaki Seminar, Vol. 1981/1982, Exp. 589, *Astérisque*, **92–93**, Soc. Math. France, Paris, 1982.
243. R. Steinberg, *Lectures on Chevalley groups*, Yale Univ. Math. Dept., 1967–68.
244. C. Stroppel, *Homomorphisms and extensions of principal series*, *J. Lie Theory* **13** (2003), 193–212.
245. ———, *Category \mathcal{O} : gradings and translation functors*, *J. Algebra* **268** (2003), 301–326.
246. ———, *Category \mathcal{O} : quivers and endomorphism rings of projectives*, *Represent. Theory* **7** (2003), 322–345.
247. ———, *Composition factors of quotients of the universal enveloping algebra by primitive ideals*, *J. London Math. Soc.* **70** (2004), 643–658.
248. ———, *Categorification of the Temperley–Lieb category, tangles, and cobordisms via projective functors*, *Duke Math. J.* **126** (2005), 547–596.
249. T. Tanisaki, *Character formulas of Kazhdan–Lusztig type*, pp. 261–276, *Representations of finite dimensional algebras and related topics in Lie theory and geometry*, Fields Inst. Commun., 40, Amer. Math. Soc., Providence, RI, 2004.
250. V. S. Varadarajan, *Lie groups, Lie algebras, and their representations*, Prentice–Hall, Englewood Cliffs, NJ, 1974; reprinted by Springer-Verlag, New York, 1984.
251. D. N. Verma, *Structure of certain induced representations of complex semisimple Lie algebras*, Ph.D. thesis, Yale Univ., 1966.
252. ———, *Structure of certain induced representations of complex semisimple Lie algebras*, *Bull. Amer. Math. Soc.* **74** (1968), 160–166; errata, 628.
253. D. A. Vogan, Jr., *Irreducible characters of semisimple Lie groups I*, *Duke Math. J.* **46** (1979), 61–108.
254. ———, *Irreducible characters of semisimple Lie groups II. The Kazhdan–Lusztig conjectures*, *Duke Math. J.* **46** (1979), 805–859.
255. ———, *Irreducible characters of semisimple Lie groups III. Proof of the Kazhdan–Lusztig conjecture in the integral case*, *Invent. Math.* **71** (1983), 381–417.
256. ———, *The character table for E_8* , *Notices Amer. Math. Soc.* **54** (2007), 1122–1134.
257. M. Vybornov, *Perverse sheaves, Koszul IC-modules, and the quiver for the category \mathcal{O}* , *Invent. Math.* **167** (2007), 19–46.
258. N. R. Wallach, *On the Enright–Varadarajan modules: a construction of the discrete series*, *Ann. Sci. École Norm. Sup. (4)* **9** (1976), 81–101.
259. C. A. Weibel, *An introduction to homological algebra*, Cambridge Univ. Press, Cambridge, 1994.
260. Wai Ling Yee, *The signature of the Shapovalov form on irreducible Verma modules*, *Represent. Theory* **9** (2005), 638–677.
261. A. V. Zelevinsky, *Resolutions, dual pairs, and character formulas*, *Funktsional. Anal. i Prilozhen.* **21** (1987), no. 2, 74–75; English transl., *Funct. Anal. Appl.* **21** (1987), 152–154.
262. G. Zuckerman, *Tensor products of finite and infinite dimensional representations of semisimple Lie groups*, *Ann. of Math.* **106** (1977), 295–308.

Frequently Used Symbols

Symbol	Description	Section
\mathfrak{g}	semisimple Lie algebra	0.1
\mathfrak{h}	Cartan subalgebra	0.1
ℓ	rank of \mathfrak{g} (= $\dim \mathfrak{h}$)	0.1
Φ	root system of \mathfrak{g} relative to \mathfrak{h}	0.1
Δ	simple system in Φ	0.1
Φ^+	positive roots relative to Δ	0.1
\mathfrak{g}_α	root space	0.1
\mathfrak{n}	sum of positive root spaces	0.1
\mathfrak{n}^-	sum of negative root spaces	0.1
\mathfrak{b}	standard Borel subalgebra $\mathfrak{h} \oplus \mathfrak{n}$	0.1
$\mathfrak{p} = \mathfrak{p}_I$	parabolic subalgebra, $I \subset \Delta$	0.1
$\mathfrak{l} = \mathfrak{l}_I$	Levi subalgebra	0.1
$\mathfrak{u} = \mathfrak{u}_I$	nilradical of \mathfrak{p}	0.1
$(h_\alpha, x_\alpha, y_\alpha)$	standard basis elements, $\alpha \in \Phi^+$	0.1
E	euclidean space spanned by Φ	0.2
α^\vee	coroot $2\alpha/(\alpha, \alpha)$	0.2
s_α	reflection relative to $\alpha \in \Phi$	0.2
Λ_r	root lattice in E	0.2

Symbol	Description	Section
W	Weyl group	0.3
$\ell(w)$	length of $w \in W$	0.3
w_\circ	longest element of W	0.3
W_I	parabolic subgroup of W ($I \subset \Delta$)	0.3
$w' \leq w$	Bruhat ordering of W	0.4
$U(\mathfrak{g})$	universal enveloping algebra of \mathfrak{g}	0.5
$Z(\mathfrak{g})$	center of $U(\mathfrak{g})$	0.5
τ	transpose map on $U(\mathfrak{g})$	0.5
Λ	integral weight lattice in E	0.6
Λ^+	dominant integral weights	0.6
$\mu \leq \lambda$	partial ordering of weights	0.6
Γ	\mathbb{Z}^+ -linear combinations of Δ	0.6
ϖ_α	fundamental weight, $\alpha \in \Delta$	0.6
ρ	sum of fundamental weights	0.6
C	Weyl chamber in E	0.6
$\text{Mod } U(\mathfrak{g})$	category of all $U(\mathfrak{g})$ -modules	0.7
M_λ	weight space in M for $\lambda \in \mathfrak{h}^*$	0.7
$\Pi(M)$	set of weights of M	0.7
\mathcal{O}	subcategory of $\text{Mod } U(\mathfrak{g})$	1.1
\mathcal{O}_{int}	modules in \mathcal{O} with integral weights	1.1
$M(\lambda)$	Verma module	1.3
$L(\lambda)$	simple quotient of $M(\lambda)$	1.3
$N(\lambda)$	maximal submodule of $M(\lambda)$	1.3
χ_λ	central character	1.7
$w \cdot \lambda$	$w(\lambda + \rho) - \rho$ ($w \in W, \lambda \in \mathfrak{h}^*$)	1.8
$[M : L(\lambda)]$	composition factor multiplicity	1.11
$K(\mathcal{O})$	Grothendieck group of \mathcal{O}	1.11
$\text{Rad } M$	radical of module M	1.11
$\text{Soc } M$	socle of module M	1.11
\mathcal{O}_χ	subcategory of \mathcal{O}	1.12
M^χ	summand of M in \mathcal{O}_χ	1.12
\mathcal{O}_0	principal block of \mathcal{O}	1.13
\mathcal{X}_0	additive group of formal characters	1.14
$\text{ch } M$	formal character of M	1.14
p	Kostant function in \mathcal{X}_0	1.16
\mathcal{P}	original Kostant partition function	1.16

Symbol	Description	Section
M^\vee	dual of module $M \in \mathcal{O}$	3.2
$\Phi_{[\lambda]}$	integral root system of $\lambda \in \mathfrak{h}^*$	3.4
$\Delta_{[\lambda]}$	simple system in $\Phi_{[\lambda]}$	3.4
$W_{[\lambda]}$	integral Weyl group of $\lambda \in \mathfrak{h}^*$	3.4
\mathcal{O}_λ	block in \mathcal{O}	3.5
$(M : M(\lambda))$	standard filtration multiplicity	3.7
$P(\lambda)$	projective cover of $L(\lambda)$	3.9
$Q(\lambda)$	injective hull of $L(\lambda)$	3.9
$\mu \uparrow \lambda$	strong linkage of weights	5.1
$M(\lambda)^i$	i th submodule in Jantzen filtration	5.3
$M(\lambda)_i$	i th layer in Jantzen filtration	5.3
Φ_λ^+	index set in Jantzen Sum Formula	5.3
T_λ^μ	translation functor	7.1
H_α	α -hyperplane shifted by $-\rho$ in E	7.3
\widehat{F}	upper closure of facet F	7.3
\widehat{C}	upper closure of Weyl chamber C	7.3
$E(\lambda)$	span of $\Phi_{[\lambda]}$ in E	7.4
λ^\natural	integral part of $\lambda \in \mathfrak{h}^*$	7.4
W_λ°	stabilizer of λ	7.4
w_λ	longest element in $W_{[\lambda]}$	7.13
Θ_s	wall-crossing functor	7.14
$\ell(x, w)$	$\ell(w) - \ell(x)$ when $x \leq w$	8.0
$P_{x,w}(q)$	Kazhdan–Lusztig polynomial, $x \leq w$	8.2
G/B	flag variety	8.5
X_w	Bruhat cell in G/B	8.5
\mathcal{O}^p	parabolic subcategory of \mathcal{O}	9.3
$M_{\Gamma}(\lambda)$	parabolic Verma module	9.4
W^{Γ}	minimal coset representatives in $W_{\Gamma} \backslash W$	9.4

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