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Training Manual on Transport and Fluids

John C. Neu

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Preface

This text presents models of transport in continuous media and a corresponding body of mathematical techniques. Within this text, I have embedded a subtext of problems. Topics and problems are listed together in the table of contents. Each problem is followed by a detailed solution emphasizing process and craftsmanship. These problems and solutions express the *practice* of applied mathematics as the examination and re-examination of essential ideas in many interrelated examples.

Since the science that falls under the headings "transport" or "fluids" is so broad, this introductory text for a one-term advanced undergraduate or beginning graduate course must select a highly specific path. The main requirement is that topics and exercises be logically interconnected and form a self-contained whole.

Briefly, the physical topics are: convection and diffusion as the simplest models of transport, local conservation laws with sources as a general "frame" of continuum mechanics, ideal fluid as the simplest example of an actual physical medium with mass, momentum and energy transport, and finally, free surface waves and shallow water theory. The idea behind this lineup is the progression from purely geometric and kinematic to genuinely physical.

The mathematical prerequisites for engaging the practice of this text are: fluency in advanced calculus and vector analysis, and acquaintance with PDEs from an introductory undergraduate course.

The mathematical skills *developed* in this text have two tracks: First, classical constructions of solutions to linear PDEs and related tools, such as the Dirac δ-function, are presented with a relentless sense of connection to the geometric-physical situations they articulate. Second, and more essential, is the emphasis on dimensional analysis and scaling. Some topics, such as physical similarity and similarity solutions, are traditional. In addition, there are asymptotic reductions based on scaling, such as incompressible flow as a limit of compressible flow, shallow water theory from the full equations of free surface waves, and further reduction of shallow water theory to an asymptotic model of a tsunami approaching the shore. The art of scaling reduction is introduced in the very first chapter, and the drum beat of examples and problems persists throughout the body of the text.

I am grateful to several people who assisted me with the development of this text. Craig Evans asked me to write this book in the first place, and Sergei Gelfand, the editor, reinforced and upheld this request. I thank Tom Beale for discussions and suggestions regarding the title. Katherine Schwarz, a student in my Fall 2004 graduate course on which this book is based, developed the LaTeX version of my lecture notes and figures, many of which are included here. Yu Tanouchi typed the revised and extended version, bringing the book into its present form. Rahel Wachs and Yossi Farjoun made the computerized versions of the figures. Bianca Cerchiai reviewed the book close to its final form and offered many thoughtful suggestions. Finally, I acknowledge my wife, Wanda Krassowska Neu, for her help and support during this work.

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I have learned a lot from John Neu over the past years, and his book reflects very well his sense of style and purpose.

—Walter Craig, McMaster University, Hamilton, Ontario, Canada and Fields Institute for Research in Mathematical Sciences, Toronto, Ontario, Canada

This book presents elementary models of transport in continuous media and a corresponding body of mathematical technique. Physical topics include convection and diffusion as the simplest models of transport; local conservation laws with sources as the general framework of continuum mechanics; ideal fluid as the simplest model of a medium with mass; momentum and energy transport; and finally, free surface waves, in particular, shallow water theory.

There is a strong emphasis on dimensional analysis and scaling. Some topics, such as physical similarity and similarity solutions, are traditional. In addition, there are reductions based on scaling, such as incompressible flow as a limit of compressible flow, and shallow water theory derived asymptotically from the full equations of free surface waves. More and deeper examples are presented as problems, including a series of problems that model a tsunami approaching the shore.

The problems form an embedded subtext to the book. Each problem is followed by a detailed solution emphasizing process and craftsmanship. The problems express the practice of applied mathematics as the examination and re-examination of simple but essential ideas in many interrelated examples.

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