Topics in Random Matrix Theory

Terence Tao

Graduate Studies in Mathematics

Volume 132

AMERICAN MATHEMATICAL SOCIETY · FOUNDED 1888 · ΑΓΕΩΜΕΤΡΗΤΟΣ ΜΗΔΕΙΣ ΕΙΣΙΤΩ

American Mathematical Society

Topics in Random Matrix Theory

http://dx.doi.org/10.1090/gsm/132

Topics in Random Matrix Theory

Terence Tao

Graduate Studies
in Mathematics

Volume 132

American Mathematical Society
Providence, Rhode Island

2010 *Mathematics Subject Classification.* Primary 60B20, 15B52, 15B35.

For additional information and updates on this book, visit
www.ams.org/bookpages/gsm-132

Library of Congress Cataloging-in-Publication Data

Tao, Terence, 1975–
Topics in random matrix theory / Terence Tao.
p. cm. – (Graduate studies in mathematics ; v. 132)
Includes bibliographical references and index.
ISBN 978-0-8218-7430-1 (alk. paper)
1. Random matrices. I. Title.

QA196.5.T36 2012
512.9′434–dc23 2011045194

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 17 16 15 14 13 12

To Garth Gaudry, who set me on the road;
To my family, for their constant support;
And to the readers of my blog, for their feedback and contributions.

Contents

Preface

In the winter of 2010, I taught a topics graduate course on random matrix theory, the lecture notes of which then formed the basis for this text. This course was inspired by recent developments in the subject, particularly with regard to the rigorous demonstration of universal laws for eigenvalue spacing distributions of Wigner matrices (see the recent survey [**Gu2009b**]). This course does not directly discuss these laws, but instead focuses on more foundational topics in random matrix theory upon which the most recent work has been based. For instance, the first part of the course is devoted to basic probabilistic tools such as concentration of measure and the central limit theorem, which are then used to establish basic results in random matrix theory, such as the Wigner semicircle law on the bulk distribution of eigenvalues of a Wigner random matrix, or the circular law on the distribution of eigenvalues of an iid matrix. Other fundamental methods, such as free probability, the theory of determinantal processes, and the method of resolvents, are also covered in the course.

This text begins in Chapter 1 with a review of the aspects of probability theory and linear algebra needed for the topics of discussion, but assumes some existing familiarity with both topics, as well as a first-year graduate-level understanding of measure theory (as covered for instance in my books [**Ta2011, Ta2010**]). If this text is used to give a graduate course, then Chapter 1 can largely be assigned as reading material (or reviewed as necessary), with the lectures then beginning with Section 2.1.

The core of the book is Chapter 2. While the focus of this chapter is ostensibly on random matrices, the first two sections of this chapter focus more on random *scalar* variables, in particular, discussing extensively the concentration of measure phenomenon and the central limit theorem in this setting. These facts will be used repeatedly when we then turn our attention to random matrices, and also many of the proof techniques used in the scalar setting (such as the moment method) can be adapted to the matrix context. Several of the key results in this chapter are developed through the exercises, and the book is designed for a student who is willing to work through these exercises as an integral part of understanding the topics covered here.

The material in Chapter 3 is related to the main topics of this text, but is optional reading (although the material on Dyson Brownian motion from Section 3.1 is referenced several times in the main text).

This text is *not* intended as a comprehensive introduction to random matrix theory, which is by now a vast subject. For instance, only a small amount of attention is given to the important topic of invariant matrix ensembles, and we do not discuss connections between random matrix theory and number theory, or to physics. For these topics we refer the reader to other texts such as [**AnGuZi2010**], [**DeGi2007**], [**De1999**], [**Fo2010**], [**Me2004**]. We hope, however, that this text can serve as a foundation for the reader to then tackle these more advanced texts.

Acknowledgments

I am greatly indebted to my students of the course on which this text was based, as well as many further commenters on my blog, including Ahmet Arivan, Joshua Batson, Florent Benaych-Georges, Sivaraman Balakrishnan, Alex Bloemendal, Kaihua Cai, Andres Caicedo, Emmanuel Candés, Jérôme Chauvet, Brian Davies, Ben Golub, Stephen Heilman, John Jiang, Li Jing, Rowan Killip, Sungjin Kim, Allen Knutson, Greg Kuperberg, Choongbum Lee, George Lowther, Rafe Mazzeo, Mark Meckes, William Meyerson, Samuel Monnier, Andreas Naive, Srivatsan Narayanan, Giovanni Peccati, Leonid Petrov, Anand Rajagopalan, Brian Simanek, James Smith, Mads Sørensen, David Speyer, Ambuj Tewari, Luca Trevisan, Qiaochu Yuan, and several anonymous contributors, for comments and corrections. These comments, as well as the original lecture notes for this course, can be viewed online at:

`terrytao.wordpress.com/category/teaching/254a-random-matrices`

The author is supported by a grant from the MacArthur Foundation, by NSF grant DMS-0649473, and by the NSF Waterman award.

Last, but not least, I am indebted to my co-authors Emmanuel Candés and Van Vu, for introducing me to the fascinating world of random matrix theory, and to the anonymous referees of this text for valuable feedback and suggestions.

Bibliography

[AhWi2002] R. Ahlswede, A. Winter, *Strong converse for identification via quantum channels*, IEEE Trans. Information Theory 48 (2002), 568–579.

[AlKrVu2002] N. Alon, M. Krivelevich, V. Vu, *On the concentration of eigenvalues of random symmetric matrices*, Israel J. Math. **131** (2002), 259–267.

[AnGuZi2010] G. Anderson, A. Guionnet, O. Zeitouni, An introduction to Random Matrices, Cambridge Studies in Advanced Mathematics, 118. Cambridge University Press, Cambridge, 2010.

[At1982] M. F. Atiyah, *Convexity and commuting Hamiltonians*, Bull. London Math. Soc. **14** (1982), no. 1, 1–5.

[Ba1993] Z. D. Bai, *Convergence rate of expected spectral distributions of large random matrices. I. Wigner matrices*, Ann. Probab. **21** (1993), no. 2, 625–648.

[Ba1993b] Z. D. Bai, *Convergence rate of expected spectral distributions of large random matrices. II. Sample covariance matrices.* Ann. Probab. **21** (1993), no. 2, 649–672.

[Ba1997] Z. D. Bai, *Circular law*, Ann. Probab. **25** (1997), no. 1, 494–529.

[BaSi2010] Z. Bai, J. Silverstein, Spectral analysis of large dimensional random matrices. Second edition. Springer Series in Statistics. Springer, New York, 2010. xvi+551 pp.

[BaYi1988] Z.D. Bai, Y.Q. Yin, *Necessary and sufficient conditions for almost sure convergence of the largest eigenvalue of a Wigner matrix*, Ann. Probab. **16** (1988), no. 4, 1729–1741.

[BaHa1984] A. D. Barbour, P. Hall, *Stein's method and the Berry-Esseen theorem*, Austral. J. Statist. **26** (1984), no. 1, 8–15.

[BeBo2010] G. Ben Arous, P. Bourgade, *Extreme gaps between eigenvalues of random matrices*, `arXiv:1010.1294`.

[BeCoDyLiTi2010] H. Bercovici, B. Collins, K. Dykema, W. Li, D. Timotin, *Intersections of Schubert varieties and eigenvalue inequalities in an arbitrary finite factor*, J. Funct. Anal. **258** (2010), no. 5, 1579–1627.

[Bh1997] R. Bhatia, Matrix analysis, Graduate Texts in Mathematics, 169. Springer-Verlag, New York, 1997.

[Bi2003] P. Biane, *Free probability for probabilists*, Quantum Probability Communications, Volume XI, 55–72, World Scientific (2003).

[BiLe2001] P. Biane, F. Lehner, *Computation of some examples of Brown's spectral measure in free probability*, Colloq. Math. **90** (2001), no. 2, 181–211.

[BoCaCh2008] C. Bordenave, P. Caputo, D. Chafai, *Circular law theorem for random Markov matrices*, `arXiv:0808.1502`.

[BoVuWo2010] J. Bourgain, V. Vu, P. Wood, *On the singularity probability of discrete random matrices*, J. Funct. Anal. **258** (2010), no. 2, 559–603.

[BrHi1996] E. Brézin, S. Hikami, *Correlations of nearby levels induced by a random potential*, Nuclear Phys. B **479** (1996), no. 3, 697–706.

[Br1986] L. G. Brown, *Lidskii's theorem in the type II case*, Geometric methods in operator algebras (Kyoto, 1983), 1–35, Pitman Res. Notes Math. Ser., 123, Longman Sci. Tech., Harlow, 1986.

[De1999] P. Deift, Orthogonal polynomials and random matrices: a Riemann-Hilbert approach. Courant Lecture Notes in Mathematics, 3. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 1999.

[DeGi2007] P. Deift, D. Gioev, *Universality at the edge of the spectrum for unitary, orthogonal, and symplectic ensembles of random matrices*, Comm. Pure Appl. Math. **60** (2007), no. 6, 867–910.

[Dy1962] F. Dyson, *A Brownian-motion model for the eigenvalues of a random matrix*, J. Mathematical Phys. **3** (1962) 1191–1198.

[Ed1988] A. Edelman, *Eigenvalues and condition numbers of random matrices*, SIAM J. Matrix Anal. Appl. **9** (1988), 543–560.

[Ed1996] A. Edelman, *The probability that a random real Gaussian matrix has k real eigenvalues, related distributions, and the circular law*, J. Multivariate Anal. **60** (1997), no. 2, 203–232.

[Er1945] P. Erdős, *On a lemma of Littlewood and Offord*, Bull. Amer. Math. Soc. **51**, (1945) 898–902.

[ErScYa2008] L. Erdős, B. Schlein, H.-T. Yau, *Local semicircle law and complete delocalization for Wigner random matrices*, Comm. Math. Phys. **287** (2009), 641–655.

[ErScYa2009] L. Erdős, B. Schlein, H.-T. Yau, *Universality of Random Matrices and Local Relaxation Flow*, Invent. Math. **185** (2011), 75-119.

[Fe1969] P. Federbush, *Partially alternative derivation of a result of Nelson*, J. Math. Phys. **10** (1969) 50–52.

[Fo2010] Forrester, P. J. Log-gases and random matrices. London Mathematical Society Monographs Series, 34. Princeton University Press, Princeton, NJ, 2010.

[Ga2007] J. Garnett, Bounded analytic functions. Revised first edition. Graduate Texts in Mathematics, 236. Springer, New York, 2007.

[Ge1980] S. Geman, *A limit theorem for the norm of random matrices*, Ann. Probab. **8** (1980), no. 2, 252–261.

[Gi1965] J. Ginibre, *Statistical ensembles of complex, quaternion, and real matrices*, J. Mathematical Phys. **6** (1965), 440–449.

[Gi1984] V. L. Girko, *The circular law*, Teor. Veroyatnost. i Primenen. **29** (1984), no. 4, 669–679.

[Gi2004] V. L. Girko, *The strong circular law. Twenty years later. II*, Random Oper. Stochastic Equations 12 (2004), no. 3, 255–312.

[Go1965] S. Golden, *Lower bounds for the Helmholtz function*, Phys. Rev. **137** (1965), B1127–B1128.

[GoTi2007] F. Götze, A. Tikhomirov, *The circular law for random matrices*, Ann. Probab. **38** (2010), 1444–1491.

[Go2008] W. T. Gowers, *Quasirandom groups*, Combin. Probab. Comput. **17** (2008), no. 3, 363–387.

[Gr1975] L. Gross, *Logarithmic Sobolev inequalities*, Amer. J. Math. **97** (1975), no. 4, 1061–1083.

[Gr2009] D. Gross, *Recovering low-rank matrices from few coefficients in any basis*, `arXiv:0910.1879`.

[Gr1999] D. Grabiner, *Brownian motion in a Weyl chamber, non-colliding particles, and random matrices*, Ann. Inst. H. Poincaré Probab. Statist. **35** (1999), no. 2, 177–204.

[GuSt1982] V. Guillemin, S. Sternberg, *Convexity properties of the moment mapping*, Invent. Math. **67** (1982), no. 3, 491–513.

[Gu2009] A. Guionnet, Large random matrices: lectures on macroscopic asymptotics. Lectures from the 36th Probability Summer School held in Saint-Flour, 2006. Lecture Notes in Mathematics, 1957. Springer-Verlag, Berlin, 2009.

[Gu2009b] A. Guionnet, Grandes matrices aléatoires et théorèmes d'universalité (d'aprés Erdős, Schlein, Tao, Vu, et Yau), Séminaire Bourbaki, 62éme anée, 2009–2010, no. 1019.

[GuKrZe2009] A. Guionnet, M. Krishnapur, O. Zeitouni, *The single ring theorem*, `arXiv:0909.2214`.

[GuZe2000] A. Guionnet, O. Zeitouni, *Concentration of the spectral measure for large matrices*, Electron. Comm. Probab. **5** (2000), 119–136.

[Gu2005] J. Gustavsson, *Gaussian fluctuations of eigenvalues in the GUE*, Ann. Inst. H. Poincaré Probab. Statist. **41** (2005), no. 2, 151–178.

[Ha1977] G. Halász,*Estimates for the concentration function of combinatorial number theory and probability*, Period. Math. Hungar. **8** (1977), no. 3–4, 197–211.

[HeRo1995] U. Helmke, J. Rosenthal, *Eigenvalue inequalities and Schubert calculus*, Math. Nachr. **171** (1995), 207–225.

[Ho1954] A. Horn, *Doubly stochastic matrices and the diagonal of a rotation matrix*, Amer. J. Math. **76** (1954), 620–630.

[Jo2001] K. Johansson, *Universality of the local spacing distribution in certain ensembles of Hermitian Wigner matrices*, Comm. Math. Phys. 215 (2001), no. 3, 683–705.

[Jo1982] D. Jonsson, *Some limit theorems for the eigenvalues of a sample covariance matrix*, J. Multivariate Anal. 12 (1982), no. 1, 1–38.

[Ka1985] J. P. Kahane. Some random series of functions. Second edition, Cambridge Studies in Advanced Math. 5 (1985), Cambridge University Press.

[KaKoSz1995] J. Kahn, J. Komlós, E. Szemerédi, *On the probability that a random* ± 1*-matrix is singular*, J. Amer. Math. Soc. **8** (1995), no. 1, 223–240.

[Ka2002] O. Kallenberg, Foundations of modern probability. Second edition. Probability and its Applications (New York). Springer-Verlag, New York, 2002.

[KeVa2007] J. Keller, J. Vanden-Broeck, *Stirling's formula derived simply*, `arXiv:0711.4412`

[Kh2009] O. Khorunzhiy, *High moments of large Wigner random matrices and asymptotic properties of the spectral norm*, `arXiv:0907.3743`.

[Klyachko] A. Klyachko, *Stable bundles, representation theory and Hermitian operators*, Selecta Math. (N.S.) **4** (1998), no. 3, 419–445.

[KnTa2001] A. Knutson, T. Tao, *Honeycombs and sums of Hermitian matrices*, Notices Amer. Math. Soc. **48** (2001), no. 2, 175–186.

[KnTaWo2004] A. Knutson, T. Tao, C. Woodward, *The honeycomb model of $GL_n(\mathbf{C})$ tensor products. II. Puzzles determine facets of the Littlewood-Richardson cone*, J. Amer. Math. Soc. **17** (2004), no. 1, 19–48.

[Ko1967] J. Komlós, *On the determinant of (0,1) matrices*, Studia Sci. Math. Hungar **2** (1967), 7–21.

[KuNi2006] L. Kuipers; H. Niederreiter, Uniform Distribution of Sequences, Dover Publishing, 2006.

[La2005] R. Latała, *Some estimates of norms of random matrices*, Proc. Amer. Math. Soc. 133 (2005), no. 5, 1273–1282.

[Le2001] M. Ledoux, The concentration of measure phenomenon, Mathematical Surveys and Monographs, 89. American Mathematical Society, Providence, RI, 2001.

[Le1995] M. Ledoux, *On Talagrand's deviation inequalities for product measures*, ESAIM Probab. Statist. **1** (1995/97), 63–87.

[Li1922] J.W. Lindeberg, *Eine neue Herleitung des Exponentialgesetzes in der Wahscheinlichkeitsrechnung*, Math. Zeit. **15** (1922), pp. 211–225.

[LiPaRuTo2005] A. E. Litvak, A. Pajor, M. Rudelson, N. Tomczak-Jaegermann, *Smallest singular value of random matrices and geometry of random polytopes*, Adv. Math. **195** (2005), no. 2, 491–523.

[Ly2009] A. Lytova, L. Pastur, *Central limit theorem for linear eigenvalue statistics of random matrices with independent entries*, Ann. Probab. **37** (2009), no. 5, 1778–1840.

[Me2004] M. Meckes, *Concentration of norms and eigenvalues of random matrices*, J. Funct. Anal. **211** (2004), no. 2, 508–524.

[Me2004] M. Mehta, Random matrices. Third edition. Pure and Applied Mathematics (Amsterdam), 142. Elsevier/Academic Press, Amsterdam, 2004.

[MeGa1960] M. L. Mehta, M. Gaudin, *On the density of eigenvalues of a random matrix*, Nuclear Phys. **18** (1960) 420–427.

[Ol2008] G. Olshanski, *Difference operators and determinantal point processes*, `arXiv:0810.3751`.

[PaZh2010] G. Pan, W. Zhou, *Circular law, extreme singular values and potential theory*, J. Multivariate Anal. **101** (2010), no. 3, 645–656.

[Pa1973] L. Pastur, *On the spectrum of random matrices*, Teoret. Mat.Fiz. **10**, 102–112 (1973).

[Pe2006] S. Péché, *The largest eigenvalue of small rank perturbations of Hermitian random matrices*, Probab. Theory Related Fields **134** (2006), no. 1, 127–173.

[Pe2009] S. Péché, *Universality results for the largest eigenvalues of some sample covariance matrix ensembles*, Probab. Theory Related Fields **143** (2009), no. 3–4, 481–516.

[PeSo2007] S. Péché, A. Soshnikov, *On the lower bound of the spectral norm of symmetric random matrices with independent entries*, Electron. Commun. Probab. **13** (2008), 280–290.

[Ru2008] M. Rudelson, *Invertibility of random matrices: norm of the inverse*, Ann. of Math. (2) **168** (2008), no. 2, 575–600.

[RuVe2008] M. Rudelson, R. Vershynin, *The Littlewood-Offord problem and invertibility of random matrices*, Adv. Math. **218** (2008), no. 2, 600–633.

[RuVe2009] M. Rudelson, R. Vershynin, *The least singular value of a random square matrix is $O(n^{-1/2})$*, C. R. Math. Acad. Sci. Paris **346** (2008), no. 15–16, 893–896.

[Ru2007] A. Ruzmaikina, *Universality of the edge distribution of eigenvalues of Wigner random matrices with polynomially decaying distributions of entries*, Comm. Math. Phys. **261** (2006), no. 2, 277–296.

[Sn2002] P. Sniady, *Random regularization of Brown spectral measure*, J. Funct. Anal. **193** (2002), no. 2, 291–313.

[So1999] A. Soshnikov, *Universality at the edge of the spectrum in Wigner random matrices*, Comm. Math. Phys. **207** (1999), no. 3, 697–733.

[So2004] A. Soshnikov, *Poisson statistics for the largest eigenvalues of Wigner random matrices with heavy tails*, Electron. Comm. Probab. **9** (2004), 82–91.

[SoFy2005] A. Soshnikov, Y. Fyodorov, *On the largest singular values of random matrices with independent Cauchy entries*, J. Math. Phys. **46** (2005), no. 3, 033302, 15 pp.

[Sp] R. Speicher, `www.mast.queensu.ca/∼speicher/survey.html`

[St1970] C. Stein, *A bound for the error in the normal approximation to the distribution of a sum of dependent random variables*, Proc. Wh. Berk. Symp. Math. Star. Hob. 11 (1970), 583–602.

[Ta1995] M. Talagrand, *Concentration of measure and isoperimetric inequalities in product spaces*, Inst. Hautes Études Sci. Publ. Math. No. **81** (1995), 73–205.

[Ta2005] M. Talagrand, The generic chaining. Upper and lower bounds of stochastic processes. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2005.

[Ta2008] T. Tao, Structure and Randomness, American Mathematical Society, 2008.

[Ta2009] T. Tao, Poincaré's legacies, Vol. I., American Mathematical Society, 2009.

[Ta2009b] T. Tao, Poincaré's legacies, Vol. II., American Mathematical Society, 2009.

[Ta2010] T. Tao, An epsilon of room, Vol. I., American Mathematical Society, 2010.

[Ta2010b] T. Tao, An epsilon of room, Vol. II., American Mathematical Society, 2010.

[Ta2011] T. Tao, An introduction to measure theory, American Mathematical Society, 2011.

[TaVu2007] T. Tao, V. Vu, *On the singularity probability of random Bernoulli matrices*, J. Amer. Math. Soc. **20** (2007), no. 3, 603–628.

[TaVu2008] T. Tao, V. Vu, *Random matrices: the circular law*, Commun. Contemp. Math. **10** (2008), no. 2, 261–307.

[TaVu2009] T. Tao, V. Vu, *Inverse Littlewood-Offord theorems and the condition number of random discrete matrices*, Ann. of Math. (2) **169** (2009), no. 2, 595–632.

[TaVu2009b] T. Tao, V. Vu, *Random matrices: Universality of local eigenvalue statistics*, Acta Math. **206** (2011), 127–204.

[TaVu2009c] T. Tao, V. Vu, *Random matrices: Universality of local eigenvalue statistics up to the edge*, Comm. Math. Phys. **298** (2010), 549–572.

[TaVu2010] T. Tao, V. Vu, *Random matrices: the distribution of the smallest singular values*, Geom. Funct. Anal. **20** (2010), no. 1, 260–297.

[TaVuKr2010] T. Tao, V. Vu, M. Krishnapur, *Random matrices: Universality of ESDs and the circular law*, Annals of Probability **38** (2010), no. 5, 2023–206.

[Th1965] C. Thompson, *Inequality with applications in statistical mechanics*, Journal of Mathematical Physics **6** (1965), 1812–1813.

[To1994] B. Totaro, *Tensor products of semistables are semistable*, Geometry and analysis on complex manifolds, 242–250, World Sci. Publ., River Edge, NJ, 1994.

[TrWi2002] C. Tracy, H. Widom, *Distribution functions for largest eigenvalues and their applications*, Proceedings of the International Congress of Mathematicians, Vol. I (Beijing, 2002), 587–596, Higher Ed. Press, Beijing, 2002.

[Tr1991] L. N. Trefethen, *Pseudospectra of matrices*, Numerical analysis 1991 (Dundee, 1991), 234–266, Pitman Res. Notes Math. Ser., 260, Longman Sci. Tech., Harlow, 1992.

[Tr2010] J. Tropp, *User-friendly tail bounds for sums of random matrices*, `arXiv:1004.4389`.

[Tr1984] H. Trotter, *Eigenvalue distributions of large Hermitian matrices; Wigner's semi-circle law and a theorem of Kac, Murdock, and Szegö*, Adv. in Math. **54**(1):67–82, 1984.

[Ve2008] R. Vershynin, *Golden-Thompson inequality*, `www-personal.umich.edu/~romanv/teaching/reading-group/golden-thompson.pdf`

[Vo1991] D. Voiculescu, *Limit laws for random matrices and free products*, Invent. Math. **104** (1991), no. 1, 201–220.

[Vu2007] V. Vu, *Spectral norm of random matrices*, Combinatorica **27** (2007), no. 6, 721–736.

[Wo2003] T. H. Wolff, Lectures on Harmonic Analysis, American Mathematical Society, University Lecture Series vol. 29, 2003.

[YiBaKr1988] Y. Q. Yin, Z. D. Bai, P. R. Krishnaiah, *On the limit of the largest eigenvalue of the large-dimensional sample covariance matrix*, Probab. Theory Related Fields **78** (1988), no. 4, 509–521.

Index

Titles in This Series

132 **Terence Tao,** Topics in Random Matrix Theory, 2012

131 **Ian M. Musson,** Lie Superalgebras and Enveloping Algebras, 2012

130 **Viviana Ene and Jürgen Herzog,** Gröbner Bases in Commutative Algebra, 2011

129 **Stuart P. Hastings, J. Bryce McLeod, and J. Bryce McLeod,** Classical Methods in Ordinary Differential Equations, 2012

128 **J. M. Landsberg,** Tensors: Geometry and Applications, 2012

127 **Jeffrey Strom,** Modern Classical Homotopy Theory, 2011

126 **Terence Tao,** An Introduction to Measure Theory, 2011

125 **Dror Varolin,** Riemann Surfaces by Way of Complex Analytic Geometry, 2011

124 **David A. Cox, John B. Little, and Henry K. Schenck,** Toric Varieties, 2011

123 **Gregory Eskin,** Lectures on Linear Partial Differential Equations, 2011

122 **Teresa Crespo and Zbigniew Hajto,** Algebraic Groups and Differential Galois Theory, 2011

121 **Tobias Holck Colding and William P. Minicozzi, II,** A Course in Minimal Surfaces, 2011

120 **Qing Han,** A Basic Course in Partial Differential Equations, 2011

119 **Alexander Korostelev and Olga Korosteleva,** Mathematical Statistics, 2011

118 **Hal L. Smith and Horst R. Thieme,** Dynamical Systems and Population Persistence, 2011

117 **Terence Tao,** An Epsilon of Room, I: Real Analysis, 2010

116 **Joan Cerdà,** Linear Functional Analysis, 2010

115 **Julio González-Díaz, Ignacio García-Jurado, and M. Gloria Fiestras-Janeiro,** An Introductory Course on Mathematical Game Theory, 2010

114 **Joseph J. Rotman,** Advanced Modern Algebra, 2010

113 **Thomas M. Liggett,** Continuous Time Markov Processes, 2010

112 **Fredi Tröltzsch,** Optimal Control of Partial Differential Equations, 2010

111 **Simon Brendle,** Ricci Flow and the Sphere Theorem, 2010

110 **Matthias Kreck,** Differential Algebraic Topology, 2010

109 **John C. Neu,** Training Manual on Transport and Fluids, 2010

108 **Enrique Outerelo and Jesús M. Ruiz,** Mapping Degree Theory, 2009

107 **Jeffrey M. Lee,** Manifolds and Differential Geometry, 2009

106 **Robert J. Daverman and Gerard A. Venema,** Embeddings in Manifolds, 2009

105 **Giovanni Leoni,** A First Course in Sobolev Spaces, 2009

104 **Paolo Aluffi,** Algebra: Chapter 0, 2009

103 **Branko Grünbaum,** Configurations of Points and Lines, 2009

102 **Mark A. Pinsky,** Introduction to Fourier Analysis and Wavelets, 2002

101 **Ward Cheney and Will Light,** A Course in Approximation Theory, 2000

100 **I. Martin Isaacs,** Algebra, 1994

99 **Gerald Teschl,** Mathematical Methods in Quantum Mechanics, 2009

98 **Alexander I. Bobenko and Yuri B. Suris,** Discrete Differential Geometry, 2008

97 **David C. Ullrich,** Complex Made Simple, 2008

96 **N. V. Krylov,** Lectures on Elliptic and Parabolic Equations in Sobolev Spaces, 2008

95 **Leon A. Takhtajan,** Quantum Mechanics for Mathematicians, 2008

94 **James E. Humphreys,** Representations of Semisimple Lie Algebras in the BGG Category $\mathcal{O}$, 2008

93 **Peter W. Michor and Peter W. Michor,** Topics in Differential Geometry, 2008

92 **I. Martin Isaacs,** Finite Group Theory, 2008

91 **Louis Halle Rowen,** Graduate Algebra: Noncommutative View, 2008

90 **Larry J. Gerstein,** Basic Quadratic Forms, 2008

89 **Anthony Bonato,** A Course on the Web Graph, 2008

Courtesy of Reed Hutchinson/UCLA

The field of random matrix theory has seen an explosion of activity in recent years, with connections to many areas of mathematics and physics. However, this makes the current state of the field almost too large to survey in a single book. In this graduate text, we focus on one specific sector of the field, namely the spectral distribution of random Wigner matrix ensembles (such as the Gaussian Unitary Ensemble), as well as iid matrix ensembles. The text is largely self-contained and starts with a review of relevant aspects of probability theory and linear algebra. With over 200 exercises, the book is suitable as an introductory text for beginning graduate students seeking to enter the field.

ISBN 978-0-8218-7430-1

9 780821 874301

GSM/132

For additional information and updates on this book, visit

www.ams.org/bookpages/gsm-132

AMS *on the* Web
www.ams.org