

Ordinary Differential Equations

Qualitative Theory

Luis Barreira
Claudia Valls

**Graduate Studies
in Mathematics**

Volume 137



American Mathematical Society

Ordinary Differential Equations

Qualitative Theory

Ordinary Differential Equations

Qualitative Theory

Luis Barreira

Claudia Valls

Translated by the authors

Graduate Studies
in Mathematics

Volume 137



American Mathematical Society
Providence, Rhode Island

EDITORIAL COMMITTEE

David Cox (Chair)
Daniel S. Freed
Rafe Mazzeo
Gigliola Staffilani

This work was originally published in Portuguese by IST Press under the title “Equações Diferenciais: Teoria Qualitativa” by Luis Barreira and Clàudia Valls, © IST Press 2010, Instituto Superior Técnico. All Rights Reserved.

The present translation was created under license for the American Mathematical Society and published by permission.

Translated by the authors.

2010 *Mathematics Subject Classification*. Primary 34-01, 34Cxx, 34Dxx, 37Gxx, 37Jxx.

For additional information and updates on this book, visit
www.ams.org/bookpages/gsm-137

Library of Congress Cataloging-in-Publication Data

Barreira, Luis, 1968–

[Equações diferenciais. English]

Ordinary differential equations : qualitative theory / Luis Barreira, Claudia Valls ; translated by the authors.

p. cm. – (Graduate studies in mathematics ; v. 137)

Includes bibliographical references and index.

ISBN 978-0-8218-8749-3 (alk. paper)

1. Differential equations—Qualitative theory. I. Valls, Claudia, 1973– II. Title.

QA372.B31513 2010
515'.352–dc23

2012010848

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294 USA. Requests can also be made by e-mail to reprint-permission@ams.org.

© 2012 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 17 16 15 14 13 12

Contents

Preface	ix
Part 1. Basic Concepts and Linear Equations	
Chapter 1. Ordinary Differential Equations	3
§1.1. Basic notions	3
§1.2. Existence and uniqueness of solutions	9
§1.3. Additional properties	21
§1.4. Existence of solutions for continuous fields	32
§1.5. Phase portraits	35
§1.6. Equations on manifolds	48
§1.7. Exercises	53
Chapter 2. Linear Equations and Conjugacies	57
§2.1. Nonautonomous linear equations	57
§2.2. Equations with constant coefficients	63
§2.3. Variation of parameters formula	75
§2.4. Equations with periodic coefficients	78
§2.5. Conjugacies between linear equations	85
§2.6. Exercises	97
Part 2. Stability and Hyperbolicity	
Chapter 3. Stability and Lyapunov Functions	105
§3.1. Notions of stability	105

§3.2.	Stability of linear equations	108
§3.3.	Stability under nonlinear perturbations	113
§3.4.	Lyapunov functions	116
§3.5.	Exercises	123
Chapter 4.	Hyperbolicity and Topological Conjugacies	127
§4.1.	Hyperbolic critical points	127
§4.2.	The Grobman–Hartman theorem	129
§4.3.	Hölder conjugacies	139
§4.4.	Structural stability	141
§4.5.	Exercises	143
Chapter 5.	Existence of Invariant Manifolds	147
§5.1.	Basic notions	147
§5.2.	The Hadamard–Perron theorem	149
§5.3.	Existence of Lipschitz invariant manifolds	150
§5.4.	Regularity of the invariant manifolds	157
§5.5.	Exercises	167
Part 3. Equations in the Plane		
Chapter 6.	Index Theory	171
§6.1.	Index for vector fields in the plane	171
§6.2.	Applications of the notion of index	176
§6.3.	Index of an isolated critical point	179
§6.4.	Exercises	181
Chapter 7.	Poincaré–Bendixson Theory	185
§7.1.	Limit sets	185
§7.2.	The Poincaré–Bendixson theorem	190
§7.3.	Exercises	196
Part 4. Further Topics		
Chapter 8.	Bifurcations and Center Manifolds	201
§8.1.	Introduction to bifurcation theory	201
§8.2.	Center manifolds and applications	206
§8.3.	Theory of normal forms	215
§8.4.	Exercises	222

Chapter 9. Hamiltonian Systems	225
§9.1. Basic notions	225
§9.2. Linear Hamiltonian systems	229
§9.3. Stability of equilibria	231
§9.4. Integrability and action-angle coordinates	235
§9.5. The KAM theorem	239
§9.6. Exercises	240
Bibliography	243
Index	245

Preface

The main objective of this book is to give a comprehensive introduction to the qualitative theory of ordinary differential equations. In particular, among other topics, we study the existence and uniqueness of solutions, phase portraits, linear equations and their perturbations, stability and Lyapunov functions, hyperbolicity, and equations in the plane.

The book is also intended to serve as a bridge to important topics that are often left out of a second course of ordinary differential equations. Examples include the smooth dependence of solutions on the initial conditions, the existence of topological and differentiable conjugacies between linear systems, and the Hölder continuity of the conjugacies in the Grobman–Hartman theorem. We also give a brief introduction to bifurcation theory, center manifolds, normal forms, and Hamiltonian systems.

We describe mainly notions, results and methods that allow one to discuss the qualitative properties of the solutions of an equation without solving it explicitly. This can be considered the main aim of the qualitative theory of ordinary differential equations.

The book can be used as a basis for a second course of ordinary differential equations. Nevertheless, it has more material than the standard courses, and so, in fact, it can be used in several different ways and at various levels. Among other possibilities, we suggest the following courses:

- a) advanced undergraduate/beginning graduate second course: Chapters 1–5 and 7–8 (without Sections 1.4, 2.5 and 8.3, and without the proofs of the Grobman–Hartman and Hadamard–Perron theorems);
- b) advanced undergraduate/beginning graduate course on equations in the plane: Chapters 1–3 and 6–7;

- c) advanced graduate course on stability: Chapters 1–3 and 8–9;
- d) advanced graduate course on hyperbolicity: Chapters 1–5.

Other selections are also possible, depending on the audience and on the time available for the course. In addition, some sections can be used for short expositions, such as Sections 1.3.2, 1.4, 2.5, 3.3, 6.2 and 8.3.

Other than some basic pre-requisites of linear algebra and differential and integral calculus, all concepts and results used in the book are recalled along the way. Moreover, (almost) everything is proven, with the exception of some results in Chapters 8 and 9 concerning more advanced topics of bifurcation theory, center manifolds, normal forms and Hamiltonian systems. Being self-contained, the book can also serve as a reference or for independent study.

Now we give a more detailed description of the contents of the book. Part 1 is dedicated to basic concepts and linear equations.

- In Chapter 1 we introduce the basic notions and results of the theory of ordinary differential equations, in particular, concerning the existence and uniqueness of solutions (Picard–Lindelöf theorem) and the dependence of solutions on the initial conditions. We also establish the existence of solutions of equations with a continuous vector field (Peano’s theorem). Finally, we give an introduction to the description of the qualitative behavior of the solutions in the phase space.
- In Chapter 2 we consider the particular case of (nonautonomous) linear equations and we study their fundamental solutions. It is often useful to see an equation as a perturbation of a linear equation, and to obtain the solutions (even if implicitly) using the variation of parameters formula. This point of view is often used in the book. We then consider the particular cases of equations with constant coefficients and equations with periodic coefficients. More advanced topics include the C^1 dependence of solutions on the initial conditions and the existence of topological conjugacies between linear equations with hyperbolic matrices of coefficients.

Part 2 is dedicated to the study of stability and hyperbolicity.

- In Chapter 3, after introducing the notions of stability and asymptotic stability, we consider the particular case of linear equations, for which it is possible to give a complete characterization of these notions in terms of fundamental solutions. We also consider the particular cases of equations with constant coefficients and equations with periodic coefficients. We then discuss the persistence of asymptotic stability under sufficiently small perturbations of an asymptotically

stable linear equation. We also give an introduction to the theory of Lyapunov functions, which sometimes yields the stability of a given solution in a more or less automatic manner.

- In Chapters 4–5 we introduce the notion of hyperbolicity and we study some of its consequences. Namely, we establish two key results on the behavior of the solutions in a neighborhood of a hyperbolic critical point: the Grobman–Hartman and Hadamard–Perron theorems. The first shows that the solutions of a sufficiently small perturbation of a linear equation with a hyperbolic critical point are topologically conjugate to the solutions of the linear equation. The second shows that there are invariant manifolds tangent to the stable and unstable spaces of a hyperbolic critical point. As a more advanced topic, we show that all conjugacies in the Grobman–Hartman theorem are Hölder continuous. We note that Chapter 5 is more technical: the exposition is dedicated almost entirely to the proof of the Hadamard–Perron theorem. In contrast to what happens in other texts, our proof does not require a discretization of the problem or additional techniques that would only be used here. We note that the material in Sections 5.3 and 5.4 is used nowhere else in the book.

In Part 3 we describe results and methods that are particularly useful in the study of equations in the plane.

- In Chapter 6 we give an introduction to index theory and its applications to differential equations in the plane. In particular, we describe how the index of a closed path with respect to a vector field varies with the path and with the vector field. We then present several applications, including a proof of the existence of a critical point inside any periodic orbit, in the sense of Jordan’s curve theorem.
- In Chapter 7 we give an introduction to the Poincaré–Bendixson theory. After introducing the notions of α -limit and ω -limit sets, we show that bounded semiorbits have nonempty, compact and connected α -limit and ω -limit sets. Then we establish one of the important results of the qualitative theory of ordinary differential equations in the plane, the Poincaré–Bendixson theorem. In particular, it yields a criterion for the existence of periodic orbits.

Part 4 is of a somewhat different nature and it is only here that not everything is proved. Our main aim is to make the bridge to important topics that are often left out of a second course of ordinary differential equations.

- In Chapter 8 we give an introduction to bifurcation theory, with emphasis on examples. We then give an introduction to the theory of center manifolds, which often allows us to reduce the order of an

equation in the study of stability or the existence of bifurcations. We also give an introduction to the theory of normal forms that aims to eliminate through a change of variables all possible terms in the original equation.

- Finally, in Chapter 9 we give an introduction to the theory of Hamiltonian systems. After introducing some basic notions, we describe several results concerning the stability of linear and nonlinear Hamiltonian systems. We also consider the notion of integrability and the Liouville–Arnold theorem on the structure of the level sets of independent integrals in involution. In addition, we describe the basic ideas of the KAM theory.

The book also includes numerous examples that illustrate in detail the new concepts and results as well as exercises at the end of each chapter.

Luis Barreira and Claudia Valls
Lisbon, February 2012

Bibliography

- [1] R. Abraham and J. Marsden, *Foundations of Mechanics*, Benjamin/Cummings, 1978.
- [2] H. Amann, *Ordinary Differential Equations*, Walter de Gruyter, 1990.
- [3] V. Arnold, *Mathematical Methods of Classical Mechanics*, Springer, 1989.
- [4] V. Arnold, *Ordinary Differential Equations*, Springer, 1992.
- [5] R. Bhatia, *Matrix Analysis*, Springer, 1997.
- [6] J. Carr, *Applications of Centre Manifold Theory*, Springer, 1981.
- [7] C. Chicone, *Ordinary Differential Equations with Applications*, Springer, 1999.
- [8] S.-N. Chow and J. Hale, *Methods of Bifurcation Theory*, Springer, 1982.
- [9] E. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, McGraw-Hill, 1955.
- [10] T. Dieck, *Algebraic Topology*, European Mathematical Society, 2008.
- [11] B. Dubrovin, A. Fomenko and S. Novikov, *Modern Geometry - Methods and Applications: The Geometry and Topology of Manifolds*, Springer, 1985.
- [12] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, 1983.
- [13] J. Hale, *Ordinary Differential Equations*, Robert E. Krieger, 1980.
- [14] J. Hale and H. Koçak, *Dynamics and Bifurcations*, Springer, 1991.
- [15] P. Hartman, *Ordinary Differential Equations*, SIAM, 2002.
- [16] N. Higham, *Functions of Matrices: Theory and Computation*, SIAM, 2008.
- [17] M. Hirsch and S. Smale, *Differential Equations, Dynamical Systems, and Linear Algebra*, Academic Press, 1974.
- [18] M. Irwin, *Smooth Dynamical Systems*, Academic Press, 1980.
- [19] A. Katok and B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems*, Cambridge University Press, 1995.
- [20] J. La Salle and S. Lefschetz, *Stability by Liapunov's Direct Method, With Applications*, Academic Press, 1961.
- [21] J. Milnor, *Morse Theory*, Princeton University Press, 1963.

- [22] J. Moser, *Stable and Random Motions in Dynamical Systems*, Princeton University Press, 1973.
- [23] J. Palis and W. de Melo, *Geometric Theory of Dynamical Systems*, Springer, 1982.
- [24] D. Sánchez, *Ordinary Differential Equations and Stability Theory*, Dover, 1968.
- [25] F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems*, Springer, 1996.

Index

- α -limit set, 186
- ω -limit set, 186

- action-angle coordinates, 235
- almost-integrable Hamiltonian, 239
- Arzelà–Ascoli theorem, 32
- asymptotic stability, 107
- asymptotically stable
 - equation, 110
 - solution, 107
- attracting point, 17
- autonomous equation, 8

- ball
 - closed –, 13
 - open –, 13
- bifurcation, 203
 - Hopf –, 206
 - pitchfork –, 205
 - saddle-node –, 204
 - theory, 201
 - transcritical –, 204
- Brouwer’s fixed point theorem, 178

- canonical
 - matrix, 229
 - transformation, 227
- Cauchy sequence, 14
- center, 74
 - manifold, 201, 206, 208
 - manifold theorem, 208
 - space, 207
- characteristic
 - exponent, 80, 232
 - multiplier, 80
- chart, 48
- closed
 - ball, 13
 - path, 171
- compact set, 10
- complete metric space, 14
- completely integrable Hamiltonian, 236
- conjugacy
 - differentiable –, 87
 - linear –, 87, 88
 - topological –, 87, 90, 127
- connected
 - component, 176
 - set, 176
- conservative equation, 42
- contraction, 12, 16
 - fiber –, 18
- convergent sequence, 14
- coordinate system, 48
- critical point, 35
 - hyperbolic –, 127, 136
 - isolated –, 179
- critical value, 50
- curve, 50, 171

- degree of freedom, 225
- differentiable
 - conjugacy, 87
 - map, 50
 - structure, 48
- differential equation, 3
 - on manifold, 48

- disconnected set, 176
- distance, 12
- equation
 - asymptotically stable –, 110
 - autonomous –, 8
 - conservative –, 42
 - homogeneous –, 76
 - homological –, 217
 - linear –, 57
 - linear variational –, 60
 - nonautonomous –, 57
 - nonhomogeneous –, 76
 - stable –, 110
 - unstable –, 110
- equilibrium, 231
 - nondegenerate –, 231
- existence of solutions, 9, 33
- exponential, 63
- fiber, 18
 - contraction, 18
 - contraction theorem, 18
- first integral, 42
- fixed point, 16
- Floquet's theorem, 79
- flow, 8
 - box theorem, 36
- focus
 - stable –, 74
 - unstable –, 74
- formula
 - Liouville's –, 62
 - variation of parameters –, 75, 76
- frequency vector, 239
- function
 - Lipschitz –, 15
 - locally Lipschitz –, 10, 116
 - Lyapunov –, 105, 116, 117
 - periodic –, 78
 - strict Lyapunov –, 117
- functions
 - in involution, 236
 - independent –, 236
- fundamental
 - solution, 59
 - theorem of algebra, 179
- global solution, 37
- Grobman–Hartman theorem, 129, 136
- Gronwall's lemma, 21
- Hadamard–Perron theorem, 149
- Hamiltonian, 225
 - almost-integrable –, 239
 - completely integrable –, 236
 - matrix, 229
 - system, 225
- heteroclinic orbit, 37
- homoclinic orbit, 37
- homogeneous equation, 76
- homological equation, 217
- homotopic paths, 174
- homotopy, 174
- Hopf bifurcation, 206
- hyperbolic
 - critical point, 127, 136
 - matrix, 90
- hyperbolicity, 127
- independent functions, 236
- index, 171, 172, 180
 - theory, 171
- initial
 - condition, 6
 - value problem, 3, 6
- instability criterion, 121
- integrability, 235
- integral, 42
- invariance of domain theorem, 136
- invariant
 - manifold, 147, 150
 - set, 185
- isolated critical point, 179
- Jordan canonical form, 64
- Jordan's curve theorem, 176
- KAM theorem, 239, 240
- lemma
 - Gronwall's –, 21
 - Morse's –, 231
- line integral, 172
- linear
 - conjugacy, 87, 88
 - equation, 57
 - Hamiltonian system, 229
 - variational equation, 60
- Liouville's formula, 62
- Liouville–Arnold theorem, 237
- Lipschitz function, 15
- locally Lipschitz function, 10, 116
- Lyapunov function, 105, 116, 117
- manifold, 48

- center –, 208
- invariant –, 147, 150
- stable –, 150, 208
- unstable –, 150, 208
- matrix
 - canonical –, 229
 - Hamiltonian –, 229
 - hyperbolic –, 90
 - monodromy –, 80
- maximal interval, 29, 30
- metric space, 12
- monodromy matrix, 80
- Morse's lemma, 231
- negative semiorbit, 186
- node
 - stable –, 68, 69, 71
 - unstable –, 68, 71
- nonautonomous equation, 57
- nondegenerate equilibrium, 231
- nonhomogeneous equation, 76
- nonresonant vector, 239
- norm, 13
- normal form, 215
- open ball, 13
- orbit, 35, 186
 - heteroclinic –, 37
 - homoclinic –, 37
 - periodic –, 36
- path
 - closed –, 171
 - regular –, 171
- Peano's theorem, 33
- periodic
 - function, 78
 - orbit, 36
- phase
 - portrait, 35, 38, 67
 - space, 35
- Picard–Lindelöf theorem, 10
- pitchfork bifurcation, 205
- Poincaré–Bendixson
 - theorem, 190, 192
 - theory, 185
- point
 - attracting –, 17
 - critical –, 35
 - fixed –, 16
- Poisson bracket, 229
- positive semiorbit, 186
- quasi-periodic trajectory, 239
- regular path, 171
- resonant vector, 219
- saddle point, 68
- saddle-node bifurcation, 204
- semiorbit
 - negative –, 186
 - positive –, 186
- sequence
 - Cauchy –, 14
 - convergent –, 14
- set
 - α -limit –, 186
 - ω -limit –, 186
 - compact –, 10
 - connected –, 176
 - disconnected –, 176
 - invariant –, 185
- solution, 3, 4, 51
 - asymptotically stable –, 107
 - fundamental –, 59
 - global –, 37
 - stable –, 106
 - unstable –, 106
- solutions
 - differentially conjugate –, 87
 - linearly conjugate –, 87
 - topologically conjugate –, 87
- space
 - center –, 207
 - complete metric –, 14
 - metric –, 12
 - of solutions, 57
 - stable –, 128, 207
 - unstable –, 128, 207
- stability, 105, 113
 - asymptotic –, 107
 - criterion, 117
 - theory, 105
- stable
 - equation, 110
 - focus, 74
 - manifold, 150, 208
 - node, 68, 69, 71
 - solution, 106
 - space, 128, 207
- strict Lyapunov function, 117
- tangent
 - bundle, 50

- space, 50
- vector, 50
- theorem
 - Arzelà–Ascoli –, 32
 - Brouwer’s fixed point –, 178
 - center manifold –, 208
 - fiber contraction –, 18
 - Floquet’s –, 79
 - flow box –, 36
 - Grobman–Hartman –, 129, 136
 - Hadamard–Perron –, 149
 - invariance of domain –, 136
 - Jordan’s curve –, 176
 - KAM –, 239, 240
 - Liouville–Arnold –, 237
 - Peano’s –, 33
 - Picard–Lindelöf –, 10
 - Poincaré–Bendixson –, 190, 192
- theory
 - of normal forms, 215
 - Poincaré–Bendixson –, 185
- topological conjugacy, 87, 90, 127
- transcritical bifurcation, 204
- transversal, 191
- uniqueness of solutions, 9
- unstable
 - equation, 110
 - focus, 74
 - manifold, 150, 208
 - node, 68, 71
 - solution, 106
 - space, 128, 207
- variation of parameters formula, 75, 76
- vector
 - field, 51
 - frequency –, 239
 - nonresonant –, 239
 - resonant –, 219

Selected Titles in This Series

- 137 **Luis Barreira and Claudia Valls**, Ordinary Differential Equations, 2012
- 136 **Arshak Petrosyan, Henrik Shahgholian, and Nina Uraltseva**, Regularity of Free Boundaries in Obstacle-Type Problems, 2012
- 135 **Pascal Cherrier and Albert Milani**, Linear and Quasi-linear Evolution Equations in Hilbert Spaces, 2012
- 134 **Jean-Marie De Koninck and Florian Luca**, Analytic Number Theory, 2012
- 133 **Jeffrey Rauch**, Hyperbolic Partial Differential Equations and Geometric Optics, 2012
- 132 **Terence Tao**, Topics in Random Matrix Theory, 2012
- 131 **Ian M. Musson**, Lie Superalgebras and Enveloping Algebras, 2012
- 130 **Viviana Ene and Jürgen Herzog**, Gröbner Bases in Commutative Algebra, 2011
- 129 **Stuart P. Hastings and J. Bryce McLeod**, Classical Methods in Ordinary Differential Equations, 2012
- 128 **J. M. Landsberg**, Tensors: Geometry and Applications, 2012
- 127 **Jeffrey Strom**, Modern Classical Homotopy Theory, 2011
- 126 **Terence Tao**, An Introduction to Measure Theory, 2011
- 125 **Dror Varolin**, Riemann Surfaces by Way of Complex Analytic Geometry, 2011
- 124 **David A. Cox, John B. Little, and Henry K. Schenck**, Toric Varieties, 2011
- 123 **Gregory Eskin**, Lectures on Linear Partial Differential Equations, 2011
- 122 **Teresa Crespo and Zbigniew Hajto**, Algebraic Groups and Differential Galois Theory, 2011
- 121 **Tobias Holck Colding and William P. Minicozzi, II**, A Course in Minimal Surfaces, 2011
- 120 **Qing Han**, A Basic Course in Partial Differential Equations, 2011
- 119 **Alexander Korostelev and Olga Korosteleva**, Mathematical Statistics, 2011
- 118 **Hal L. Smith and Horst R. Thieme**, Dynamical Systems and Population Persistence, 2011
- 117 **Terence Tao**, An Epsilon of Room, I: Real Analysis, 2010
- 116 **Joan Cerdà**, Linear Functional Analysis, 2010
- 115 **Julio González-Díaz, Ignacio García-Jurado, and M. Gloria Fiestras-Janeiro**, An Introductory Course on Mathematical Game Theory, 2010
- 114 **Joseph J. Rotman**, Advanced Modern Algebra, 2010
- 113 **Thomas M. Liggett**, Continuous Time Markov Processes, 2010
- 112 **Fredi Tröltzsch**, Optimal Control of Partial Differential Equations, 2010
- 111 **Simon Brendle**, Ricci Flow and the Sphere Theorem, 2010
- 110 **Matthias Kreck**, Differential Algebraic Topology, 2010
- 109 **John C. Neu**, Training Manual on Transport and Fluids, 2010
- 108 **Enrique Outerelo and Jesús M. Ruiz**, Mapping Degree Theory, 2009
- 107 **Jeffrey M. Lee**, Manifolds and Differential Geometry, 2009
- 106 **Robert J. Daverman and Gerard A. Venema**, Embeddings in Manifolds, 2009
- 105 **Giovanni Leoni**, A First Course in Sobolev Spaces, 2009
- 104 **Paolo Aluffi**, Algebra: Chapter 0, 2009
- 103 **Branko Grünbaum**, Configurations of Points and Lines, 2009
- 102 **Mark A. Pinsky**, Introduction to Fourier Analysis and Wavelets, 2002
- 101 **Ward Cheney and Will Light**, A Course in Approximation Theory, 2000
- 100 **I. Martin Isaacs**, Algebra, 1994
- 99 **Gerald Teschl**, Mathematical Methods in Quantum Mechanics, 2009
- 98 **Alexander I. Bobenko and Yuri B. Suris**, Discrete Differential Geometry, 2008
- 97 **David C. Ullrich**, Complex Made Simple, 2008
- 96 **N. V. Krylov**, Lectures on Elliptic and Parabolic Equations in Sobolev Spaces, 2008
- 95 **Leon A. Takhtajan**, Quantum Mechanics for Mathematicians, 2008

SELECTED TITLES IN THIS SERIES

- 94 **James E. Humphreys**, Representations of Semisimple Lie Algebras in the BGG Category \mathcal{O} , 2008
- 93 **Peter W. Michor**, Topics in Differential Geometry, 2008
- 92 **I. Martin Isaacs**, Finite Group Theory, 2008
- 91 **Louis Halle Rowen**, Graduate Algebra: Noncommutative View, 2008
- 90 **Larry J. Gerstein**, Basic Quadratic Forms, 2008
- 89 **Anthony Bonato**, A Course on the Web Graph, 2008
- 88 **Nathanial P. Brown and Narutaka Ozawa**, C^* -Algebras and Finite-Dimensional Approximations, 2008
- 87 **Srikanth B. Iyengar, Graham J. Leuschke, Anton Leykin, Claudia Miller, Ezra Miller, Anurag K. Singh, and Uli Walther**, Twenty-Four Hours of Local Cohomology, 2007
- 86 **Yulij Ilyashenko and Sergei Yakovenko**, Lectures on Analytic Differential Equations, 2008
- 85 **John M. Alongi and Gail S. Nelson**, Recurrence and Topology, 2007
- 84 **Charalambos D. Aliprantis and Rabee Tourky**, Cones and Duality, 2007
- 83 **Wolfgang Ebeling**, Functions of Several Complex Variables and Their Singularities, 2007
- 82 **Serge Alinhac and Patrick Gérard**, Pseudo-differential Operators and the Nash–Moser Theorem, 2007
- 81 **V. V. Prasolov**, Elements of Homology Theory, 2007
- 80 **Davar Khoshnevisan**, Probability, 2007
- 79 **William Stein**, Modular Forms, a Computational Approach, 2007
- 78 **Harry Dym**, Linear Algebra in Action, 2007
- 77 **Bennett Chow, Peng Lu, and Lei Ni**, Hamilton’s Ricci Flow, 2006
- 76 **Michael E. Taylor**, Measure Theory and Integration, 2006
- 75 **Peter D. Miller**, Applied Asymptotic Analysis, 2006
- 74 **V. V. Prasolov**, Elements of Combinatorial and Differential Topology, 2006
- 73 **Louis Halle Rowen**, Graduate Algebra: Commutative View, 2006
- 72 **R. J. Williams**, Introduction to the Mathematics of Finance, 2006
- 71 **S. P. Novikov and I. A. Taimanov**, Modern Geometric Structures and Fields, 2006
- 70 **Seán Dineen**, Probability Theory in Finance, 2005
- 69 **Sebastián Montiel and Antonio Ros**, Curves and Surfaces, 2005
- 68 **Luis Caffarelli and Sandro Salsa**, A Geometric Approach to Free Boundary Problems, 2005
- 67 **T.Y. Lam**, Introduction to Quadratic Forms over Fields, 2005
- 66 **Yuli Eidelman, Vitali Milman, and Antonis Tsolomitis**, Functional Analysis, 2004
- 65 **S. Ramanan**, Global Calculus, 2005
- 64 **A. A. Kirillov**, Lectures on the Orbit Method, 2004
- 63 **Steven Dale Cutkosky**, Resolution of Singularities, 2004
- 62 **T. W. Körner**, A Companion to Analysis, 2004
- 61 **Thomas A. Ivey and J. M. Landsberg**, Cartan for Beginners, 2003
- 60 **Alberto Candel and Lawrence Conlon**, Foliations II, 2003
- 59 **Steven H. Weintraub**, Representation Theory of Finite Groups: Algebra and Arithmetic, 2003
- 58 **Cédric Villani**, Topics in Optimal Transportation, 2003
- 57 **Robert Plato**, Concise Numerical Mathematics, 2003

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/.

This textbook provides a comprehensive introduction to the qualitative theory of ordinary differential equations. It includes a discussion of the existence and uniqueness of solutions, phase portraits, linear equations, stability theory, hyperbolicity and equations in the plane. The emphasis is primarily on results and methods that allow one to analyze qualitative properties of the solutions without solving the equations explicitly. The text includes numerous examples that illustrate in detail the new concepts and results as well as exercises at the end of each chapter. The book is also intended to serve as a bridge to important topics that are often left out of a course on ordinary differential equations. In particular, it provides brief introductions to bifurcation theory, center manifolds, normal forms and Hamiltonian systems.

ISBN 978-0-8218-8749-3



9 780821 887493

GSM/137

For additional information
and updates on this book, visitwww.ams.org/bookpages/gsm-137AMS on the Web
www.ams.org