Introduction to 3-Manifolds

Jennifer Schultens

Graduate Studies in Mathematics

Volume 151

American Mathematical Society

https://doi.org/10.1090//gsm/151

Introduction to 3-Manifolds

Jennifer Schultens

Graduate Studies in Mathematics

Volume 151

American Mathematical Society
Providence, Rhode Island

2010 *Mathematics Subject Classification.* Primary 57N05, 57N10, 57N16, 57N40, 57N50, 57N75, 57Q15, 57Q25, 57Q40, 57Q45.

For additional information and updates on this book, visit
www.ams.org/bookpages/gsm-151

Library of Congress Cataloging-in-Publication Data

Schultens, Jennifer, 1965–
Introduction to 3-manifolds / Jennifer Schultens.
pages cm — (Graduate studies in mathematics ; v. 151)
Includes bibliographical references and index.
ISBN 978-1-4704-1020-9 (alk. paper)
1. Topological manifolds. 2. Manifolds (Mathematics) I. Title. II. Title: Introduction to three-manifolds.

QA613.2.S35 2014
514′.34—dc23

2013046541

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 19 18 17 16 15 14

Dedicated to Misha and Esther and our extended family

Contents

Preface

This book grew out of a graduate course on 3-manifolds taught at Emory University in the spring of 2003. It aims to introduce the beginning graduate student to central topics in the study of 3-manifolds. Prerequisites are kept to a minimum but do include some point set topology (see [**109**]) and some knowledge of general position (see [**128**]). In a few places, it is worth our while to mention results or proofs involving concepts from algebraic topology or differential geometry. This should not stop the interested reader with no background in algebraic topology or differential geometry from enjoying the material presented here. The sections and exercises involving algebraic topology are marked with a *, those involving differential geometry with a **.

This book conveys my personal path through the subject of 3-manifolds during a certain period of time (roughly 1990 to 2007). Marty Scharlemann deserves credit for setting me on this path. He remains a much appreciated guide. Other guides include Misha Kapovich, Andrew Casson, Rob Kirby, and my collaborators.

In Chapter 1 we introduce the notion of a manifold of arbitrary dimension and discuss several structures on manifolds. These structures may or may not exist on a given manifold. In addition, if a particular structure exists on a given manifold, it may or may not be presented as part of the information given. In Chapter 2 we consider manifolds of a particular dimension, namely 2-manifolds, also known as surfaces. Here we provide an overview of the classification of surfaces and discuss the mapping class group. Chapter 3 gives examples of 3-manifolds and standard techniques used to study 3-manifolds. In Chapter 4 we catch a glimpse of the interaction of pairs of manifolds, specifically pairs of the form (3-manifold, 1-manifold). Of particular interest here is the consideration of knots from the point of view of the complement ("Not Knot"). For other perspectives, we refer the reader to the many books, both new and old, mentioned in Chapter 4, that provide a more in-depth study. In Chapter 5 we consider triangulated 3-manifolds, normal surfaces, almost normal surfaces, and how these set the stage for algorithms pertaining to 3-manifolds. In Chapter 6 we cover a subject near and dear to the author's heart: Heegaard splittings. Heegaard splittings are decompositions of 3-manifolds into symmetric pieces. They can be thought of in many different ways. We discuss key examples, classical problems, and recent advances in the subject of Heegaard splittings. In Chapter 7 we introduce hyperbolic structures on manifolds and complexes and provide a glimpse of how they affect our understanding of 3-manifolds. We include two appendices: one on general position and one on Morse functions. Exercises appear at the end of most sections.

I wish to thank the many colleagues and students who have given me the opportunity to learn and teach. I also wish to thank the institutions that have supported me through the years: University of California, Emory University, Max-Planck-Institut für Mathematik Bonn, Max-Planck-Institut für Mathematik Leipzig, and the National Science Foundation.

Bibliography

1. Colin C. Adams, *The knot book. An elementary introduction to the mathematical theory of knots*, W. H. Freeman and Company, New York, 1994. Reprinted with corrections, Amer. Math. Soc., Providence, RI, 2004.
2. Ian Agol, *The Virtual Haken Conjecture*, arXiv:1204.2810 [math.GT].
3. Ian Agol and Yi Liu, *Presentation length and Simon's conjecture*, J. Amer. Math. Soc. **25** (2012), no. 1, 151–187. MR2833481
4. Lars Ahlfors and Leo Sario, *Riemann surfaces*, Princeton Mathematical Series, vol. 26, Princeton University Press, Princeton, NJ, 1960.
5. Selman Akbulut, *Scharlemann's manifold is standard*, Ann. of Math. (2) **149** (1999), no. 2, 497–510. MR1689337 (2000d:57033)
6. James Waddell Alexander, *Note on Riemann spaces*, Bull. Amer. Math. Soc. **26** (1920), no. 8, 370–372.
7. ______, *A lemma on systems of knotted curves*, Proc. Nat. Acad. Sci. U.S.A. **9** (1923), no. 3, 93–95.
8. Riccardo Benedetti and Carlo Petronio, *Lectures on hyperbolic geometry*, Universitext, Springer-Verlag, Berlin, 1992. MR1219310 (94e:57015)
9. Laurent Bessières, Gérard Besson, Sylvain Maillot, Michel Boileau, and Joan Porti, *Geometrisation of* 3*-manifolds*, EMS Tracts in Mathematics, vol. 13, European Mathematical Society (EMS), Zürich, 2010. MR2683385 (2012d:57027)
10. Joan S. Birman, *Braids, links, and mapping class groups*, Princeton University Press, Princeton, NJ, 1974, Annals of Mathematics Studies, No. 82. MR0375281 (51 #11477)
11. Steven Bleiler and Martin Scharlemann, *A projective plane in* $\mathbf{R}^4$ *with three critical points is standard. Strongly invertible knots have property* P, Topology **27** (1988), no. 4, 519–540.
12. Michel Boileau, Donald Collins, and Heiner Zieschang, *Scindements de Heegaard des petites variétes de Seifert*, C. R. Acad. Sci. Paris Sér. I Math. **305** (1989), no. 12, 557–560.
13. Michel Boileau and Jean-Pierre Otal, *Sur les scindements de Heegaard du tore* T^3, J. Differential Geom. **32** (1990), no. 1, 209–233.

14. Francis Bonahon and Jean-Pierre Otal, *Scindements de Heegaard des espaces lenticulaires*, C. R. Acad. Sci. Paris Sér. I Math. **294** (1982), no. 17, 585–587.
15. Brian H. Bowditch, *Intersection numbers and the hyperbolicity of the curve complex*, J. Reine Angew. Math. **598** (2006), 105–129.
16. Mark Brittenham and Ying-Qing Wu, *The classification of exceptional Dehn surgeries on 2-bridge knots*, Comm. Anal. Geom. **9** (2001), no. 1, 97–113. MR1807953 (2001m:57008)
17. Jeffrey F. Brock, Kenneth W. Bromberg, Richard D. Canary, and Yair N. Minsky, *Local topology in deformation spaces of hyperbolic 3-manifolds*, Geom. Topol. **15** (2011), no. 2, 1169–1224. MR2831259
18. M. Brown, *A proof of the generalized Schoenflies Theorem*, Bull. Amer. Math. Soc. **66** (1960), 74–76.
19. Gerhard Burde, Heiner Zieschang, and Michael Heusener, *Knots*, third ed., de Gruyter Studies in Mathematics, vol. 5, Walter de Gruyter & Co., Berlin, 2013.
20. Danny Calegari and David Gabai, *Shrinkwrapping and the taming of hyperbolic 3-manifolds*, J. Amer. Math. Soc. **19** (2006), no. 2, 385–446.
21. Alberto Candel and Lawrence Conlon, *Foliations. I*, Graduate Studies in Mathematics, vol. 23, American Mathematical Society, Providence, RI, 2000. MR1732868 (2002f:57058)
22. ______, *Foliations. II*, Graduate Studies in Mathematics, vol. 60, American Mathematical Society, Providence, RI, 2003. MR1994394 (2004e:57034)
23. Huai-Dong Cao and Xi-Ping Zhu, *A complete proof of the Poincaré and Geometrization Conjectures—application of the Hamilton-Perelman Theory of the Ricci flow*, Asian Journal of Mathematics **10** (2006), no. 2, 165–492.
24. A. J. Casson and C. McA. Gordon, *A loop theorem for duality spaces and fibred ribbon knots*, Invent. Math. **74** (1983), no. 1, 119–137.
25. Andrew J. Casson and Steven A. Bleiler, *Automorphisms of surfaces after Nielsen and Thurston*, London Mathematical Society Student Texts, vol. 9, Cambridge University Press, Cambridge, 1988. MR964685 (89k:57025)
26. Jean Cerf, *La stratification naturelle des espaces de fonction différentiables réelles et le théorème de la pseudo-isotopie*, Inst. Hautes Études Sci. Publ. Math. **39** (1970), 5–173.
27. Éric Charpentier, Étienne Ghys, and Annick Lesne (eds.), *The scientific legacy of Poincaré*, History of Mathematics, vol. 36, American Mathematical Society, Providence, RI, 2010, Translated from the 2006 French original by Joshua Bowman. MR2605614 (2011b:00005)
28. J. H. Conway, *An enumeration of knots and links, and some of their algebraic properties*, Computational Problems in Abstract Algebra (Proc. Conf., Oxford, 1967), Pergamon, Oxford, 1970, pp. 329–358. MR0258014 (41 #2661)
29. Daryl Cooper and Martin Scharlemann, *The structure of a solvmanifold's Heegaard splittings*, Proceedings of 6th Gökova Geometry-Topology Conference, vol. 23, 1999, pp. 1–18. MR1701636 (2000h:57034)
30. Marc Culler, C. McA. Gordon, J. Luecke, and Peter B. Shalen, *Dehn surgery on knots*, Ann. of Math. (2) **125** (1987), no. 2, 237–300.
31. Oliver T. Dasbach and Tao Li, *Property P for knots admitting certain Gabai disks*, Topology Appl. **142** (2004), no. 1-3, 113–129.
32. M. Dehn, *Über die Topologie des dreidimensionalen Raumes*, Math. Ann. **69** (1910), 137–168.

33. ______, *Die Gruppe der Abbildungsklassen. (German) Das arithmetische Feld auf Flächen*, Acta Math. **69** (1938), no. 1, 135–206.

34. Charles Delman and Rachel Roberts, *Alternating knots satisfy strong Property P*, Comment. Math. Helv. **74** (1999), no. 3, 376–397.

35. Y. Diao, J. C. Nardo, and Y. Sun, *Global knotting in equilateral random polygons*, JKTR **10** (2001), no. 4, 597–697.

36. Yuanan Diao, Nicholas Pippenger, and De Witt Sumners, *On random knots. Random knotting and linking*, JKTR **3** (1994), no. 3, 419–429.

37. Manfredo Perdigão do Carmo, *Riemannian geometry*, Mathematics: Theory & Applications, Birkhäuser Boston Inc., Boston, MA, 1992, Translated from the second Portuguese edition by Francis Flaherty. MR1138207 (92i:53001)

38. Cornelia Drutu and Michael Kapovich, *Lectures on geometric group theory*, preprint.

39. A. Fathi, F. Laudenbach, and V. Poenaru, *Travaux de Thurston sur les surfaces*, Astérisque **66-67** (1979), 1–284.

40. M. Feighn, *Branched covers according to J. W. Alexander*, Collect. Math. **37** (1986), no. 1, 55–60.

41. David Gabai, *Foliations and the topology of* 3*-manifolds*, J. Differential Geom. **18** (1983), no. 3, 445–503. MR723813 (86a:57009)

42. ______, *Genera of the arborescent links*, Mem. Amer. Math. Soc., vol. 59, American Mathematical Society, Providence, RI, 1986.

43. ______, *Foliations and the topology of* 3*-manifolds. III*, J. Differential Geom. **26** (1987), no. 3, 479–536. MR910018 (89a:57014b)

44. ______, 3 *lectures on foliations and laminations on* 3*-manifolds*, Laminations and foliations in dynamics, geometry and topology (Stony Brook, NY, 1998), Contemp. Math., vol. 269, Amer. Math. Soc., Providence, RI, 2001, pp. 87–109. MR1810537 (2002g:57032)

45. David Gabai and Ulrich Oertel, *Essential laminations in* 3*-manifolds*, Ann. of Math. (2) **130** (1989), no. 1, 41–73. MR1005607 (90h:57012)

46. C. McA. Gordon and J. Luecke, *Knots are determined by their complements*, Bull. Amer. Math. Soc. (N.S.) **20** (1989), no. 1.

47. V.K.A.M. Gugenheim, *Piecewise linear isotopy and embedding of elements and spheres. I, II*, Proc. LMS **3** (1953), 29–53.

48. V. Guillemin and A Pollack, *Differential topology*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1974.

49. André Haefliger, *Variétés feuilletées*, Ann. Scuola Norm. Sup. Pisa (3) **16** (1962), 367–397. MR0189060 (32 #6487)

50. ______, *Feuilletages riemanniens*, Astérisque (1989), no. 177-178, Exp. No. 707, 183–197, Séminaire Bourbaki, Vol. 1988/89. MR1040573 (91e:57047)

51. ______, *Travaux de Novikov sur les feuilletages*, Séminaire Bourbaki, Vol. 10, Soc. Math. France, Paris, 1995, Exp. No. 339, 433–444. MR1610457

52. Wolfgang Haken, *Ein Verfahren zur Aufspaltung einer* 3*-Mannigfaltigkeit in irreduzible* 3*-Mannigfaltigkeiten*, Math. Z. **76** (1961), 427–467. MR0141108 (25 #4519c)

53. ______, *Theorie der Normalflächen*, Acta Math. **105** (1961), 245–375. MR0141106 (25 #4519a)

54. Richard S. Hamilton, *A compactness property for solutions of the Ricci flow*, Amer. J. Math. **117** (1995), no. 3, 545–572. MR1333936 (96c:53056)

55. ———, *The formation of singularities in the Ricci flow*, Surveys in differential geometry, Vol. II (Cambridge, MA, 1993), Int. Press, Cambridge, MA, 1995, pp. 7–136. MR1375255 (97e:53075)

56. Kevin Hartshorn, *Heegaard splittings of Haken manifolds have bounded distance*, Pacific J. Math. **204** (2002), no. 1, 61–75. MR1905192 (2003a:57037)

57. William James Harvey, *Geometric structure of surface mapping class groups*, Homological group theory (Proc. Sympos., Durham, 1977), London Math. Soc. Lecture Note Ser., vol. 36, Cambridge Univ. Press, Cambridge-New York, 1979, pp. 255–269.

58. Joel Hass, Paul Norbury, and J. Hyam Rubinstein, *Minimal spheres of arbitrarily high Morse index*, Comm. Anal. Geom. **11** (2003), no. 3, 425–439. MR2015753 (2004i:53082)

59. Allen Hatcher, *Notes on basic* 3*-manifold topology*, preprint.

60. P. Heegaard, *Sur l'"Analysis situs"*, Bull. Soc. Math. France **44** (1916), 161–242. MR1504754

61. Geoffrey Hemion, *The classification of knots and* 3*-dimensional spaces*, Oxford Science Publications, The Clarendon Press Oxford University Press, New York, 1992. MR1211184 (94g:57015)

62. John Hempel, 3*-manifolds as viewed from the curve complex*, Topology **40** (2001), no. 3, 631–657.

63. ———, 3*-manifolds*, AMS Chelsea Publishing, Providence, RI, 2004, reprint of the 1976 original.

64. H. Hilden, M. T. Lozano, and J. M. Montesinos, *Universal knots*, Bull. Amer. Math. Soc. (N.S.) **8** (1983), no. 3, 449–450.

65. Dale Husemoller, *Fibre bundles*, third ed., Graduate Texts in Mathematics, vol. 20, Springer-Verlag, New York, 1994. MR1249482 (94k:55001)

66. Michael Hutchings, *Floer homology of families. I*, Algebr. Geom. Topol. **8** (2008), no. 1, 435–492. MR2443235 (2009h:57046)

67. William Jaco, *Lectures on three-manifold topology*, CBMS Regional Conference Series in Mathematics, vol. 43, American Mathematical Society, Providence, RI, 1980.

68. William Jaco and Ulrich Oertel, *An algorithm to decide if a* 3*-manifold is a Haken manifold*, Topology **23** (1984), no. 2, 195–209.

69. William Jaco and J. Hyam Rubinstein, *PL minimal surfaces in* 3*-manifolds*, J. Differential Geom. **27** (1988), no. 3, 493–524.

70. ———, *PL equivariant surgery and invariant decompositions of* 3*-manifolds*, Adv. in Math. **73** (1989), no. 2, 149–191.

71. William Jaco and Peter B. Shalen, *Seifert fibered spaces in irreducible, sufficiently-large* 3*-manifolds*, Bull. Amer. Math. Soc. **82** (1976), no. 5, 765–767. MR0415623 (54 #3706)

72. Klaus Johannson, *On exotic homotopy equivalences of* 3*-manifolds*, Geometric topology (Proc. Georgia Topology Conf., Athens, GA, 1977), Academic Press, New York, 1979, pp. 101–111. MR537729 (80m:57002)

73. ———, *Topology and combinatorics of* 3*-manifolds*, Lecture Notes in Mathematics, vol. 1599, Springer-Verlag, Berlin, 1995. MR1439249 (98c:57014)

74. Michael Kapovich, *Hyperbolic manifolds and discrete groups*, Modern Birkhäuser Classics, Birkhäuser Boston Inc., Boston, MA, 2009, reprint of the 2001 edition. MR2553578 (2010k:57039)

75. Svetlana Katok, *Fuchsian groups*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1992. MR1177168 (93d:20088)
76. Mikhail Khovanov, *Link homology and categorification*, International Congress of Mathematicians. Vol. II, Eur. Math. Soc., Zürich, 2006, pp. 989–999. MR2275632 (2008f:57009)
77. Robion Kirby, *A calculus for framed links in* S^3, Invent. Math. **45** (1978), no. 1, 35–56. MR0467753 (57 #7605)
78. Robion Kirby and Paul Melvin, *The* E_8*-manifold, singular fibers and handlebody decompositions*, Proceedings of the Kirbyfest (Berkeley, CA, 1998), Geom. Topol. Monogr., vol. 2, Geom. Topol. Publ., Coventry, 1999, pp. 233–258. MR1734411 (2000j:57051)
79. Bruce Kleiner and John Lott, *Notes on Perelman's papers*, Geom. Topol. **12** (2008), no. 5, 2587–2855. MR2460872 (2010h:53098)
80. Helmut Kneser, *Geschlossene Flächen in dreidimensionalen Mannigfaltigkeiten*, Jber. d. D. Math. Verein. **38** (1929), 248–260.
81. Tsuyoshi Kobayashi, *A construction of arbitrarily high degeneration of tunnel numbers of knots under connected sum*, J. Knot Theory Ramifications **3** (1994), no. 2, 179–186. MR1279920 (95g:57011)
82. P. B. Kronheimer and T. S. Mrowka, *Witten's conjecture and property P*, Geom. Topol. **8** (2004), 295–310 (electronic). MR2023280 (2004m:57023)
83. Marc Lackenby, *Heegaard splittings, the virtually Haken conjecture and property* (τ), Invent. Math. **164** (2006), no. 2, 317–359. MR2218779 (2007c:57030)
84. Tao Li, *Heegaard surfaces and measured laminations. I. The Waldhausen conjecture*, Invent. Math. **167** (2007), no. 1, 135–177. MR2264807 (2008h:57033)
85. ———, *An algorithm to determine the Heegaard genus of a* 3*-manifold*, Geom. Topol. **15** (2011), no. 2, 1029–1106. MR2821570 (2012m:57033)
86. ———, *Rank and genus of* 3*-manifolds*, J. Amer. Math. Soc. **26** (2013), no. 3, 777–829. MR3037787
87. W. B. Raymond Lickorish, *An introduction to knot theory*, Graduate Texts in Mathematics, vol. 175, Springer-Verlag, New York, 1997.
88. W. B. R. Lickorish, *A representation of orientable combinatorial* 3*-manifolds*, Ann. of Math. **76** (1962), no. 2, 531–540.
89. Charles Livingston, *Knot theory*, Carus Mathematical Monographs, vol. 24, Mathematical Association of America, Washington, DC, 1993. MR1253070 (94m:57021)
90. D. D. Long, A. Lubotzky, and A. W. Reid, *Heegaard genus and property* τ *for hyperbolic* 3*-manifolds*, J. Topol. **1** (2008), no. 1, 152–158. MR2365655 (2008j:57036)
91. Feng Luo, *Automorphisms of the complex of curves*, Topology **39** (2000), no. 2, 283–298. MR1722024 (2000j:57045)
92. Jason Manning, *Algorithmic detection and description of hyperbolic structures on closed* 3*-manifolds with solvable word problem*, Geom. Topol. **6** (2002), 1–26.
93. William S. Massey, *Algebraic topology: An introduction*, Springer-Verlag, New York, 1977, reprint of the 1967 edition, Graduate Texts in Mathematics, Vol. 56. MR0448331 (56 #6638)
94. H. A. Masur and Y. N. Minsky, *Geometry of the complex of curves. I. Hyperbolicity*, Invent. Math. **138** (1999), no. 1, 103–149.
95. ———, *Geometry of the complex of curves. II. Hierarchical structure*, Geom. Funct. Anal. **10** (2000), no. 4, 902–974.

96. Sergei Matveev, *Algorithmic topology and classification of* 3*-manifolds*, Algorithms and Computation in Mathematics, vol. 9, Springer-Verlag, Berlin, 2003. MR1997069 (2004i:57026)

97. Curtis T. McMullen, *The evolution of geometric structures on* 3*-manifolds*, Bull. Amer. Math. Soc. (N.S.) **48** (2011), no. 2, 259–274. MR2774092 (2012a:57024)

98. John Milnor, *On manifolds homeomorphic to the* 7*-sphere*, Ann. of Math. (2) **64** (1956), 399–405. MR0082103 (18,498d)

99. ______, *Morse theory. Based on lecture notes by M. Spivak and R. Wells*, Annals of Mathematics Studies, vol. 51, Princeton University Press, Princeton, NJ, 1963.

100. John W. Milnor, *Topology from the differentiable viewpoint*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, 1997, based on notes by David W. Weaver, revised reprint of the 1965 original. MR1487640 (98h:57051)

101. Edwin E. Moise, *Affine structures in* 3*-manifolds. V. The triangulation theorem and Hauptvermutung*, Ann. of Math. **56** (1952), no. 2, 96–114.

102. ______, *Geometric topology in dimensions* 2 *and* 3, Springer-Verlag, New York, 1977, Graduate Texts in Mathematics, Vol. 47. MR0488059 (58 #7631)

103. Frank Morgan, *Manifolds with density and Perelman's proof of the Poincaré conjecture*, Amer. Math. Monthly **116** (2009), no. 2, 134–142. MR2478057 (2010a:53164)

104. John Morgan and Gang Tian, *Ricci flow and the Poincaré conjecture*, Clay Mathematics Monographs, vol. 3, American Mathematical Society, Providence, RI, 2007. MR2334563 (2008d:57020)

105. John W. Morgan and Hyman Bass (eds.), *The Smith conjecture*, Pure and Applied Mathematics, vol. 112, Academic Press Inc., Orlando, FL, 1984, Papers presented at the symposium held at Columbia University, New York, 1979. MR758459 (86i:57002)

106. Yoav Moriah and Jennifer Schultens, *Irreducible Heegaard splittings of Seifert fibered spaces are either vertical or horizontal*, Topology **37** (1998), no. 5, 1089–1112. MR1650355 (99g:57021)

107. Kanji Morimoto, *There are knots whose tunnel numbers go down under connected sum*, Proc. Amer. Math. Soc. **123** (1995), no. 11, 3527–3532. MR1317043 (96a:57022)

108. Kanji Morimoto, Makoto Sakuma, and Yoshiyuki Yokota, *Examples of tunnel number one knots which have the property "$1 + 1 = 3$"*, Math. Proc. Cambridge Philos. Soc. **119** (1996), no. 1, 113–118. MR1356163 (96i:57007)

109. James R. Munkres, *Topology: A first course*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1975. MR0464128 (57 #4063)

110. Kunio Murasugi, *On the braid index of alternating links*, Trans. Amer. Math. Soc. **326** (1991), no. 1, 237–260.

111. Hossein Namazi, *Heegaard splittings and hyperbolic geometry*, ProQuest LLC, Ann Arbor, MI, 2005, Thesis (Ph.D.)–State University of New York at Stony Brook. MR2707466

112. Hossein Namazi and Juan Souto, *Heegaard splittings and pseudo-Anosov maps*, Geom. Funct. Anal. **19** (2009), no. 4, 1195–1228. MR2570321 (2011a:57035)

113. Y. Ni, *Uniqueness of PL minimal surfaces*, Acta Math. Sin. (Engl. Ser.) **23** (2007), no. 6, 961–964.

114. S. P. Novikov, *The topology of foliations*, Trudy Moskov. Mat. Obšč. **14** (1965), 248–278. MR0200938 (34 #824)

115. Peter Ozsváth, András I. Stipsicz, and Zoltán Szabó, *Combinatorial Heegaard Floer homology and nice Heegaard diagrams*, Adv. Math. **231** (2012), no. 1, 102–171. MR2935385

116. Peter Ozsváth and Zoltán Szabó, *Knot Floer homology and the four-ball genus*, Geom. Topol. **7** (2003), 615–639. MR2026543 (2004i:57036)

117. ———, *Knot Floer homology, genus bounds, and mutation*, Topology Appl. **141** (2004), no. 1-3, 59–85. MR2058681 (2005b:57028)

118. Peter S. Ozsváth and Zoltán Szabó, *Knot Floer homology and rational surgeries*, Algebr. Geom. Topol. **11** (2011), no. 1, 1–68. MR2764036 (2012h:57056)

119. C. D. Papakyriakopoulos, *On Dehn's lemma and the asphericity of knots*, Ann. of Math. **66** (1957), no. II, 1–26.

120. R. C. Penner and J. L. Harer, *Combinatorics of train tracks*, Annals of Mathematics Studies, vol. 125, Princeton University Press, Princeton, NJ, 1992. MR1144770 (94b:57018)

121. G. Perelman, *The entropy formula for the Ricci flow and its geometric applications*, http://arxiv.org/abs/math/0211159.

122. ———, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, http://arxiv.org/abs/math/0307245.

123. ———, *Ricci flow with surgery on three-manifolds*, http://arxiv.org/abs/math/0303109.

124. John G. Radcliffe, *Foundations of hyperbolic manifolds*, second ed., Graduate Texts in Mathematics, vol. 149, Springer Verlag, New York, 2006.

125. Yo'av Rieck and Eric Sedgwick, *Thin position for a connected sum of small knots*, Algebr. Geom. Topol. **2** (2002), 297–309 (electronic).

126. Igor Rivin, *Euclidean structures on simplicial surfaces and hyperbolic volume*, Ann. of Math. (2) **139** (1994), no. 3, 553–580. MR1283870 (96h:57010)

127. Dale Rolfsen, *Knots and links*, Mathematics Lecture Series, vol. 7, Publish or Perish Inc., Houston, TX, 1990, corrected reprint of the 1976 original.

128. C. Rourke and B. Sanderson, *Introduction to piecewise-linear topology*, Springer Study Edition, Springer-Verlag, Berlin-New York, 1982, reprint of the 1972 original.

129. Hyam Rubinstein and Martin Scharlemann, *Comparing Heegaard splittings of non-Haken 3-manifolds*, Topology **35** (1996), no. 4, 1005–1026. MR1404921 (97j:57021)

130. ———, *Transverse Heegaard splittings*, Michigan Math. J. **44** (1997), no. 1, 69–83. MR1439669 (98c:57017)

131. ———, *Comparing Heegaard splittings—the bounded case*, Trans. Amer. Math. Soc. **350** (1998), no. 2, 689–715. MR1401528 (98d:57033)

132. Martin Scharlemann, *Constructing strange manifolds with the dodecahedral space*, Duke Math. J. **43** (1976), no. 1, 33–40. MR0402760 (53 #6574)

133. ———, *Heegaard splittings of compact 3-manifolds*, Handbook of geometric topology, North-Holland, Amsterdam, 2002, pp. 921–953. MR1886684 (2002m:57027)

134. Martin Scharlemann and Jennifer Schultens, *The tunnel number of the sum of n knots is at least n*, Topology **38** (1999), no. 2, 265–270. MR1660345 (2000b:57013)

135. ———, *Annuli in generalized Heegaard splittings and degeneration of tunnel number*, Math. Ann. **317** (2000), no. 4, 783–820.

136. Martin Scharlemann and Abigail Thompson, *Thin position and Heegaard splittings of the 3-sphere*, J. Differential Geom. **39** (1994), no. 2, 343–357.

137. ———, *Thin position for 3-manifolds*, Geometric topology (Haifa, 1992), Contemp. Math., vol. 164, Amer. Math. Soc., Providence, RI, 1994, pp. 231–238.

138. Martin G. Scharlemann, *Unknotting number one knots are prime*, Invent. Math. **82** (1985), no. 1, 37–55. MR808108 (86m:57010)

139. Saul Schleimer, *Waldhausen's theorem*, Workshop on Heegaard Splittings, Geom. Topol. Monogr., vol. 12, Geom. Topol. Publ., Coventry, 2007, pp. 299–317. MR2408252

140. ______, *The end of the curve complex*, Groups Geom. Dyn. **5** (2011), no. 1, 169–176. MR2763783

141. Horst Schubert, *Knoten und Vollringe*, Acta Math. **90** (1953), 131–286.

142. ______, *Über eine numerische Knoteninvariante*, Math. Z. **61** (1954), 245–288.

143. Jennifer Schultens, *The classification of Heegaard splittings for (compact orientable surface)* $\times S^1$, Proc. London Math. Soc. (3) **67** (1993), no. 2, 425–448.

144. Jennifer Schultens and Richard Weidman, *On the geometric and the algebraic rank of graph manifolds*, Pacific J. Math. **231** (2007), no. 2, 481–510. MR2346507 (2009a:57030)

145. Peter Scott, *The geometries of 3-manifolds*, Bull. London Math. Soc. **15** (1983), no. 5, 401–487. MR705527 (84m:57009)

146. Herbert Seifert and William Threlfall, *Seifert and Threlfall: A textbook of topology*, Pure and Applied Mathematics, vol. 89, Academic Press Inc. [Harcourt Brace Jovanovich Publishers], New York, 1980, translated from the German edition of 1934 by Michael A. Goldman, with a preface by Joan S. Birman, with "Topology of 3-dimensional fibered spaces" by Seifert, translated from the German by Wolfgang Heil. MR575168 (82b:55001)

147. Zlil Sela, *The isomorphism problem for hyperbolic groups I*, Ann. Math. **141** (1995), 217–283.

148. I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*, second ed., Undergraduate Texts in Mathematics, Springer-Verlag, New York-Heidelberg-Tokyo, 1976.

149. John Stallings, *On the loop theorem*, Ann. of Math. (2) **72** (1960), 12–19. MR0121796 (22 #12526)

150. Norman Steenrod, *The topology of fibre bundles*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, 1999, reprint of the 1957 edition, Princeton Paperbacks. MR1688579 (2000a:55001)

151. Abigail Thompson, *Thin position and bridge number for knots in the 3-sphere*, Topology **36** (1997), no. 2, 505–507.

152. ______, *Algorithmic recognition of 3-manifolds*, Bull. Amer. Math. Soc. (N.S.) **35** (1998), no. 1, 57–66.

153. William P. Thurston, *The geometry and topology of three-manifolds*, http://www.msri.org/publications/books/gt3m/.

154. ______, *Three-dimensional geometry and topology*, vol. 1, Princeton Mathematical Series, vol. 35, Princeton University Press, Princeton, NJ, 1997.

155. F. Waldhausen, *Heegaard-Zerlegungen der 3-Sphäre*, Topology **7** (1968), no. 2, 195–203.

156. Friedhelm Waldhausen, *On irreducible 3-manifolds which are sufficiently large*, Ann. of Math. (2) **87** (1968), 56–88. MR0224099 (36 #7146)

Index

This book grew out of a graduate course on 3-manifolds and is intended for a mathematically experienced audience that is new to low-dimensional topology.

The exposition begins with the definition of a manifold, explores possible additional structures on manifolds, discusses the classification of surfaces, introduces key foundational results for 3-manifolds, and provides an overview of knot theory. It then continues with more specialized topics by briefly considering triangulations of 3-manifolds, normal surface theory, and Heegaard splittings. The book finishes with a discussion of topics relevant to viewing 3-manifolds via the curve complex.

With about 250 figures and more than 200 exercises, this book can serve as an excellent overview and starting point for the study of 3-manifolds.

ISBN 978-1-4704-1020-9

9 781470 410209

GSM/151

For additional information and updates on this book, visit

www.ams.org/bookpages/gsm-151