GRADUATE STUDIES IN MATHEMATICS 204

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American Mathematical Society

10.1090/gsm/204

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2010 *Mathematics Subject Classification.* Primary 16E40, 16E05, 16E30.

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www.ams.org/bookpages/gsm-204

Library of Congress Cataloging-in-Publication Data

Cataloging-in-Publication Data has been applied for by the AMS.
See http://www.loc.gov/publish/cip/.

10 9 8 7 6 5 4 3 2 1 24 23 22 21 20 19

Contents

Introduction

Homological techniques first arose in topology, in work of Poincaré [**174**], at the end of the 19th century. They appeared in algebra several decades later in the 1940s, when Eilenberg and Mac Lane [**59–61**] introduced homology and cohomology of groups and Hochschild [**114**] introduced homology and cohomology of algebras. Since that time, both Hochschild cohomology and group cohomology, as they came to be called, have become indispensable in algebra, algebraic topology, representation theory, and other fields. They remain active areas of research, with frequent discoveries of new applications. There are excellent books on group cohomology such as [**2**, **21**, **22**, **35**, **47**, **76**]. These are good references for those working in the field and are also important resources for those learning group cohomology in order to begin using it in their research. There are fewer such resources for Hochschild cohomology, notwithstanding some informative chapters in the books [**146**, **223**]. This book aims to begin filling the gap by providing an introduction to the basic theory of Hochschild cohomology for algebras and some of its current uses in algebra and representation theory.

Hochschild cohomology records meaningful information about rings and algebras. It is used to understand their structure and deformations, and to identify essential information about their representations. This book takes a concrete approach with many early examples that reappear later in various settings.

We begin in Chapter 1 with Hochschild's own definitions from [**114**], only slightly rephrased in modern terminology and notation, and then connect to definitions based on arbitrary resolutions under suitable conditions. We present some of the important contributions of Gerstenhaber [**82**] beginning in the 1960s that lead us now to think of a Hochschild cohomology ring as a Gerstenhaber algebra, that is, it has both an associative product and a nonassociative Lie bracket. Many properties of Hochschild cohomology rings that are essential in today's applications can be seen in these classical definitions of Hochschild and Gerstenhaber. In Chapter 2 we give detailed descriptions of many equivalent definitions of the associative product (cup product) on Hochschild cohomology. In Chapter 3 we examine several different types of examples: smooth commutative algebras, Koszul algebras, algebras defined by quivers and relations, and algebras built from others such as skew group algebras and (twisted) tensor product algebras. We present the seminal Hochschild-Kostant-Rosenberg (HKR) Theorem on Hochschild homology and cohomology of smooth finitely generated commutative algebras.

Current algebraic applications and developments in the algebraic theory of Hochschild cohomology include the following, explored in detail in the rest of the book.

Some classical geometric notions such as smoothness may be viewed as essentially homological properties of commutative function algebras, allowing interpretations of them in noncommutative settings via Hochschild cohomology. We present these and related ideas in Chapter 4, including Hochschild dimension, smoothness, noncommutative differential forms, Van den Bergh duality, Calabi-Yau algebras, the Connes differential, and Batalin-Vilkovisky structures.

Understanding how some algebras may be viewed as deformations of others calls on Hochschild cohomology, as explained in Chapter 5. There we discuss formal deformations, rigidity of algebras, the Maurer-Cartan equation, Poisson brackets, and deformation quantization. We present the fundamental Poincaré-Birkhoff-Witt (PBW) Theorem as a consequence of a more general theorem on deformations of Koszul algebras. In algebraic deformation theory, the Lie structure on Hochschild cohomology arises naturally; we spend some additional time studying this important structure in detail in Chapter 6. Further probing the associative and Lie algebra structures on Hochschild cohomology and related complexes uncovers infinity algebras. There, binary operations are layered with n-ary operations which in turn have important implications for the original algebra structure. We give a brief introduction to infinity structures and their applications to Hochschild cohomology in Chapter 7.

In representation theory, support varieties may sometimes be defined in terms of Hochschild cohomology; these are geometric spaces assigned to modules that encode representation-theoretic information. Support varieties for finite-dimensional algebras are introduced and explored in Chapter 8. This theory began in the parallel setting of finite group cohomology. There are strong connections between Hochschild cohomology and group cohomology that we analyze more generally for Hopf algebras in Chapter 9. Hopf algebras are those algebras whose categories of modules are tensor categories, and include many examples of interest such as group algebras, universal enveloping algebras of Lie algebras, and quantum groups. Relationships between Hochschild cohomology and Hopf algebra cohomology lead to better understanding of both and of all their applications. Inspecting these relationships, we connect the two first appearances of homological techniques in algebra in the form of group cohomology [**59–61**] and Hochschild cohomology [**114**].

We include an appendix with needed background material from homological algebra. The appendix is largely self-contained, however, proofs are omitted, and instead the reader is referred to standard homological algebra textbooks such as [**48, 112, 151, 168, 187, 223**] for proofs and more details.

This introductory text is not intended to be a comprehensive treatment of the whole subject of Hochschild cohomology, which long ago expanded well beyond the reach of a single book. Necessarily many important topics are left out. For example, we do not treat Tate-Hochschild cohomology, relative Hochschild cohomology, Hochschild cohomology of presheaves and schemes, connections to cyclic homology and K-theory, Hochschild cohomology of abelian categories, topological Hochschild cohomology, Hochschild cohomology of differential graded and A_∞-algebras and categories, nor operads. Hochschild homology is an important subject in its own right, and we spend only a little time on it in this book. Also, here we will almost exclusively work with algebras over a field, both for simplicity of presentation in this introductory text and to take advantage of a great array of good properties and current applications for algebras over a field.

We provide a few references for the reader looking for details on some of the topics that are not in this book. This list is not meant to be complete, but rather a beginning, and further references may be found in each of these: more on Hochschild homology can be found in the standard references [**146, 223**]. Tate-Hochschild cohomology, stable Hochschild cohomology, and singular Hochschild cohomology are $\mathbb{Z}$-graded theories while Hochschild cohomology itself is $\mathbb{N}$-graded; see, for example, [**29, 74**]. Relative Hochschild cohomology and secondary Hochschild cohomology are designed for a ring and subring pair; see, for example, [**106, 115, 205**]. There is a version of Hochschild cohomology for coalgebras and bicomodules [**57**]. Hochschild cohomology is defined for presheaves of algebras and schemes, and used in algebraic geometry; see, for example, [**85, 86, 132, 213**]. Topological Hochschild homology and cohomology are related theories in algebraic topology; see, for example, [**173**]. Hochschild cohomology is used in functional analysis, with connections to properties of Banach algebras, von Neumann algebras, and locally compact groups; see, for example, [**122**, **198**]. Many important applications of the theory of Hochschild cohomology involve its connections to cyclic homology and cohomology and algebraic K-theory; see, for example, [**146**, **223**]. Hochschild homology and cohomology can be defined for some types of categories; see, for example, [**150**, **161**]. Some operads underlie much of the structure of Hochschild cohomology, a hint of which appears in the infinity structures of Chapter 7 here; see, for example, [**152**, **153**]. Formality and Deligne's Conjecture are barely touched in Chapter 7 here, and more details may be found in the references given in Section 7.6 and in [**153**]. Hochschild cohomology may be realized as the Lie algebra of the derived Picard group of an algebra; see, for example, [**128**]. Hochschild cohomology of differential graded and A_∞-algebras and categories, for example, are in [**127**, **130**].

This book is written for graduate students and working mathematicians interested in learning about Hochschild cohomology. It can serve as a reference for many facts that are currently only found in research papers, and as a bridge to some more advanced topics that are not included here. The main prerequisite for students is a graduate course in algebra. It would also be helpful to have taken further introductory courses in homological algebra or algebraic topology and in representation theory, or else to have done some reading in these subjects. However, all of the required homological algebra background is summarized in the appendix, with references, and a motivated reader might rely solely on this as homological algebra background. Beyond the first three chapters of this book, the remaining chapters are largely independent of each other, and so there are many options for basing a one-semester graduate course on this book. A one-semester course could start with a treatment of Chapter 1 and selected sections from Chapters 2 and 3, possibly including material from the appendix depending on the background of the students. Then the course could focus on a subset of the remaining chapters: a course with a focus on noncommutative geometry could continue with Chapter 4; a course with a focus on algebraic deformation theory and related structures could instead continue with Chapter 5 and the related Chapters 6 and 7, as time allowed; a course with a focus on Hopf algebras, group algebras, or support varieties could instead continue with Chapters 8 and/or 9. A full-year course might include most of the book and time for a complete introduction to or review of homological algebra based on the appendix.

This book came into being as an aftereffect of some lecture series that I gave and through interactions with many people. I first thank Universidad de Buenos Aires, and especially Andrea Solotar and her students, postdocs, and colleagues, for hosting me for several weeks in 2010. During that time I gave a short course on Hopf algebra cohomology that led to an early version of Chapter 9 on which they gave me valuable feedback. I thank the Morningside Center in Beijing and the organizers and students of a workshop there in 2011 for the opportunity to give lectures on support varieties that expanded into the current Chapter 8. I thank the Mathematisches Forschungsinstitut Oberwolfach for its hospitality during several workshops where the idea for this book began in discussions with Karin Erdmann and Henning Krause.

Most of this book was written during the academic year 2016–17 that I spent at the University of Toronto visiting Ragnar-Olaf Buchweitz and his research group. It was with deep sadness that I learned of his death the following fall. His legacy lives on and continues to grow through the mathematical writings that are still being completed by his many collaborators, as well as others he influenced. He was a great friend and mentor to so many of us.

I am grateful to the University of Toronto for hosting me in 2016–17 and to Buchweitz and his team for the many helpful conversations and stimulating seminar talks. This interaction significantly influenced some of my choices of topics for the book. I particularly thank Buchweitz and his students Benjamin Briggs and Vincent Gélinas for many pointers on smooth algebras and infinity algebras that helped me prepare Chapters 4 and 7. I also had fruitful discussions with Cris Negron and Yury Volkov in relation to the material in Chapter 6. Special thanks go to my long-time collaborator Anne Shepler for many conversations and joint projects over the years that led to my current point of view on Chapter 5.

I thank Chelsea Walton and her Spring 2017 homological algebra class at Temple University for trying out beta versions of several chapters of this book, and for their very valuable feedback. I thank my home institution Texas A&M University and especially the mathematics department for an immensely supportive work environment. I thank my husband Frank Sottile and children Maria and Samuel for their ongoing patience and support. I thank the National Science Foundation for its support through grants DMS-1401016 and DMS-1665286.

Bibliography

[1] H. Abbaspour, *On algebraic structures of the Hochschild complex*, Free loop spaces in geometry and topology, IRMA Lect. Math. Theor. Phys., vol. 24, Eur. Math. Soc., Zürich, 2015, pp. 165–222. MR3445567

[2] A. Adem and R. J. Milgram, *Cohomology of finite groups*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 309, Springer-Verlag, Berlin, 1994. MR1317096

[3] C. Aholt, *Equivalence of the Ext-algebra structures of an R-module*, Bachelor Thesis, 2008.

[4] J. Alev, M. A. Farinati, T. Lambre, and A. L. Solotar, *Homologie des invariants d'une algèbre de Weyl sous l'action d'un groupe fini* (French, with English and French summaries), J. Algebra **232** (2000), no. 2, 564–577, DOI 10.1006/jabr.2000.8406. MR1792746

[5] M. P. Allocca, *L_∞-algebra representation theory*, Ph.D. Thesis, 2010.

[6] J. L. Alperin, *Local representation theory: Modular representations as an introduction to the local representation theory of finite groups*, Cambridge Studies in Advanced Mathematics, vol. 11, Cambridge University Press, Cambridge, 1986. MR860771

[7] C. Amiot, *Preprojective algebras and Calabi-Yau duality*, Mathematisches Forschungsinstitut Oberwolfach, 2014. Oberwolfach report on joint work with O. Iyama, S. Oppermann, and I. Reiten.

[8] D. J. Anick, *On the homology of associative algebras*, Trans. Amer. Math. Soc. **296** (1986), no. 2, 641–659, DOI 10.2307/2000383. MR846601

[9] M. A. Armenta and B. Keller, *Derived invariance of the cap product in Hochschild theory* (English, with English and French summaries), C. R. Math. Acad. Sci. Paris **355** (2017), no. 12, 1205–1207, DOI 10.1016/j.crma.2017.11.005. MR3730498

[10] I. Assem, D. Simson, and A. Skowroński, *Elements of the representation theory of associative algebras 1: Techniques of representation theory*, London Mathematical Society Student Texts, vol. 65, Cambridge University Press, 2006.

[11] M. Auslander, I. Reiten, and S. O. Smalø, *Representation theory of Artin algebras*, Cambridge Studies in Advanced Mathematics, vol. 36, Cambridge University Press, Cambridge, 1995. MR1314422

[12] L. L. Avramov and S. Iyengar, *Finite generation of Hochschild homology algebras*, Invent. Math. **140** (2000), no. 1, 143–170, DOI 10.1007/s002220000051. MR1779800

[13] L. L. Avramov and S. Iyengar, *Gaps in Hochschild cohomology imply smoothness for commutative algebras*, Math. Res. Lett. **12** (2005), no. 5-6, 789–804, DOI 10.4310/MRL.2005.v12.n6.a1. MR2189239

[14] L. L. Avramov and M. Vigué-Poirrier, *Hochschild homology criteria for smoothness*, Internat. Math. Res. Notices **1** (1992), 17–25, DOI 10.1155/S1073792892000035. MR1149001

[15] BACH, *A Hochschild homology criterium for the smoothness of an algebra*, Comment. Math. Helv. **69** (1994), no. 2, 163–168. MR1282365

[16] B. Bakalov and A. Kirillov Jr., *Lectures on tensor categories and modular functors*, University Lecture Series, vol. 21, American Mathematical Society, Providence, RI, 2001. MR1797619

[17] M. J. Bardzell, A. C. Locateli, and E. N. Marcos, *On the Hochschild cohomology of truncated cycle algebras*, Comm. Algebra **28** (2000), no. 3, 1615–1639, DOI 10.1080/00927870008826917. MR1742678

[18] M. J. Bardzell, *The alternating syzygy behavior of monomial algebras*, J. Algebra **188** (1997), no. 1, 69–89, DOI 10.1006/jabr.1996.6813. MR1432347

[19] M. Barr, *Harrison homology, Hochschild homology and triples*, J. Algebra **8** (1968), 314–323, DOI 10.1016/0021-8693(68)90062-8. MR220799

[20] C. P. Bendel, D. K. Nakano, B. J. Parshall, and C. Pillen, *Cohomology for quantum groups via the geometry of the nullcone*, Mem. Amer. Math. Soc. **229** (2014), no. 1077, x+93. MR3204911

[21] D. J. Benson, *Representations and cohomology. I*: *Basic representation theory of finite groups and associative algebras*, Cambridge Studies in Advanced Mathematics, vol. 30, Cambridge University Press, Cambridge, 1991. MR1110581

[22] D. J. Benson, *Representations and cohomology. II*: *Cohomology of groups and modules*, Cambridge Studies in Advanced Mathematics, vol. 31, Cambridge University Press, Cambridge, 1991. MR1156302

[23] D. J. Benson, J. F. Carlson, and J. Rickard, *Thick subcategories of the stable module category*, Fund. Math. **153** (1997), no. 1, 59–80. MR1450996

[24] D. J. Benson and K. Erdmann, *Hochschild cohomology of Hecke algebras*, J. Algebra **336** (2011), 391–394, DOI 10.1016/j.jalgebra.2011.03.022. MR2802551

[25] D. Benson and S. Witherspoon, *Examples of support varieties for Hopf algebras with noncommutative tensor products*, Arch. Math. (Basel) **102** (2014), no. 6, 513–520, DOI 10.1007/s00013-014-0659-8. MR3227473

[26] R. Berger, *Dimension de Hochschild des algèbres graduées* (French, with English and French summaries), C. R. Math. Acad. Sci. Paris **341** (2005), no. 10, 597–600, DOI 10.1016/j.crma.2005.09.039. MR2179797

[27] F. Bergeron and N. Bergeron, *Orthogonal idempotents in the descent algebra of B_n and applications*, J. Pure Appl. Algebra **79** (1992), no. 2, 109–129, DOI 10.1016/0022-4049(92)90153-7. MR1163285

[28] P. A. Bergh and K. Erdmann, *Homology and cohomology of quantum complete intersections*, Algebra Number Theory **2** (2008), no. 5, 501–522, DOI 10.2140/ant.2008.2.501. MR2429451

[29] P. A. Bergh and D. A. Jorgensen, *Tate-Hochschild homology and cohomology of Frobenius algebras*, J. Noncommut. Geom. **7** (2013), no. 4, 907–937, DOI 10.4171/JNCG/139. MR3148613

[30] P. A. Bergh and S. Oppermann, *Cohomology of twisted tensor products*, J. Algebra **320** (2008), no. 8, 3327–3338, DOI 10.1016/j.jalgebra.2008.08.005. MR2450729

[31] P. A. Bergh and Ø. Solberg, *Relative support varieties*, Q. J. Math. **61** (2010), no. 2, 171–182, DOI 10.1093/qmath/han032. MR2646083

[32] M. Bordemann and A. Makhlouf, *Formality and deformations of universal enveloping algebras*, Internat. J. Theoret. Phys. **47** (2008), no. 2, 311–332, DOI 10.1007/s10773-007-9435-x. MR2396767

[33] A. Braverman and D. Gaitsgory, *Poincaré-Birkhoff-Witt theorem for quadratic algebras of Koszul type*, J. Algebra **181** (1996), no. 2, 315–328, DOI 10.1006/jabr.1996.0122. MR1383469

[34] B. Briggs and V. Gélinas, *The A_∞-centre of the Yoneda algebra and the characteristic action of Hochschild cohomology on the derived category*, 2017. arXiv:1702.00721.

[35] K. S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, vol. 87, Springer-Verlag, New York-Berlin, 1982. MR672956

[36] R.-O. Buchweitz, *Finite representation type and periodic Hochschild (co-)homology*, Trends in the representation theory of finite-dimensional algebras (Seattle, WA, 1997), Contemp. Math., vol. 229, Amer. Math. Soc., Providence, RI, 1998, pp. 81–109, DOI 10.1090/conm/229/03311. MR1676212

[37] R.-O. Buchweitz, *Morita contexts, idempotents, and Hochschild cohomology—with applications to invariant rings*, Commutative algebra (Grenoble/Lyon, 2001), Contemp. Math., vol. 331, Amer. Math. Soc., Providence, RI, 2003, pp. 25–53, DOI 10.1090/conm/331/05901. MR2011764

[38] R.-O. Buchweitz, E. L. Green, D. Madsen, and Ø. Solberg, *Finite Hochschild cohomology without finite global dimension*, Math. Res. Lett. **12** (2005), no. 5-6, 805–816, DOI 10.4310/MRL.2005.v12.n6.a2. MR2189240

[39] R.-O. Buchweitz, E. L. Green, N. Snashall, and Ø. Solberg, *Multiplicative structures for Koszul algebras*, Q. J. Math. **59** (2008), no. 4, 441–454, DOI 10.1093/qmath/ham056. MR2461267

[40] R.-O. Buchweitz and S. Liu, *Hochschild cohomology and representation-finite algebras*, Proc. London Math. Soc. (3) **88** (2004), no. 2, 355–380, DOI 10.1112/S0024611503014394. MR2032511

[41] D. Burghelea, *The cyclic homology of the group rings*, Comment. Math. Helv. **60** (1985), no. 3, 354–365, DOI 10.1007/BF02567420. MR814144

[42] M. C. R. Butler and A. D. King, *Minimal resolutions of algebras*, J. Algebra **212** (1999), no. 1, 323–362, DOI 10.1006/jabr.1998.7599. MR1670674

[43] A. Čap, H. Schichl, and J. Vanžura, *On twisted tensor products of algebras*, Comm. Algebra **23** (1995), no. 12, 4701–4735, DOI 10.1080/00927879508825496. MR1352565

[44] J. F. Carlson, *Periodic modules over modular group algebras*, J. London Math. Soc. (2) **15** (1977), no. 3, 431–436, DOI 10.1112/jlms/s2-15.3.431. MR0472985

[45] J. F. Carlson, *The varieties and the cohomology ring of a module*, J. Algebra **85** (1983), no. 1, 104–143, DOI 10.1016/0021-8693(83)90121-7. MR723070

[46] J. F. Carlson, *The variety of an indecomposable module is connected*, Invent. Math. **77** (1984), no. 2, 291–299, DOI 10.1007/BF01388448. MR752822

[47] J. F. Carlson, L. Townsley, L. Valeri-Elizondo, and M. Zhang, *Cohomology rings of finite groups*, Algebra and Applications, vol. 3, Kluwer Academic Publishers, Dordrecht, 2003. With an appendix: Calculations of cohomology rings of groups of order dividing 64 by Carlson, Valeri-Elizondo and Zhang. MR2028960

[48] H. Cartan and S. Eilenberg, *Homological algebra*, Princeton University Press, Princeton, N. J., 1956. MR0077480

[49] S. Chouhy and A. Solotar, *Projective resolutions of associative algebras and ambiguities*, J. Algebra **432** (2015), 22–61, DOI 10.1016/j.jalgebra.2015.02.019. MR3334140

[50] C. Cibils, *On the Hochschild cohomology of finite-dimensional algebras*, Comm. Algebra **16** (1988), no. 3, 645–649, DOI 10.1080/00927878808823591. MR917337

[51] C. Cibils, *Hochschild cohomology algebra of radical square zero algebras*, Algebras and modules, II (Geiranger, 1996), CMS Conf. Proc., vol. 24, Amer. Math. Soc., Providence, RI, 1998, pp. 93–101. MR1648617

[52] C. Cibils, E. Marcos, M. J. Redondo, and A. Solotar, *Cohomology of split algebras and of trivial extensions*, Glasg. Math. J. **45** (2003), no. 1, 21–40, DOI 10.1017/S0017089502008959. MR1972690

[53] C. Cibils and A. Solotar, *Hochschild cohomology algebra of abelian groups*, Arch. Math. (Basel) **68** (1997), no. 1, 17–21, DOI 10.1007/PL00000389. MR1421841

[54] A. B. Conner, *A(infinity)-structures, generalized Koszul properties, and combinatorial topology*, ProQuest LLC, Ann Arbor, MI, 2011. Thesis (Ph.D.)–University of Oregon. MR2942011

[55] J. Cuntz and D. Quillen, *Algebra extensions and nonsingularity*, J. Amer. Math. Soc. **8** (1995), no. 2, 251–289, DOI 10.2307/2152819. MR1303029

[56] R. K. Dennis and K. Igusa, *Hochschild homology and the second obstruction for pseudoisotopy*, Algebraic K-theory, Part I (Oberwolfach, 1980), Lecture Notes in Math., vol. 966, Springer, Berlin-New York, 1982, pp. 7–58. MR689365

[57] Y. Doi, *Homological coalgebra*, J. Math. Soc. Japan **33** (1981), no. 1, 31–50.

[58] V. Dolgushev, D. Tamarkin, and B. Tsygan, *The homotopy Gerstenhaber algebra of Hochschild cochains of a regular algebra is formal*, J. Noncommut. Geom. **1** (2007), no. 1, 1–25, DOI 10.4171/JNCG/1. MR2294189

[59] S. Eilenberg and S. MacLane, *Group extensions and homology*, Ann. of Math. (2) **43** (1942), 757–831, DOI 10.2307/1968966. MR0007108

[60] S. Eilenberg and S. Mac Lane, *Cohomology theory in groups*, Bull. Amer. Math. Soc. **50** (1944), no. 1, 53.

[61] S. Eilenberg and S. MacLane, *Cohomology theory in abstract groups. I*, Ann. of Math. (2) **48** (1947), 51–78, DOI 10.2307/1969215. MR0019092

[62] D. Eisenbud, *Homological algebra on a complete intersection, with an application to group representations*, Trans. Amer. Math. Soc. **260** (1980), no. 1, 35–64, DOI 10.2307/1999875. MR570778

[63] K. Erdmann, *Nilpotent elements in Hochschild cohomology*, Geometric and topological aspects of the representation theory of finite groups, Springer Proc. Math. Stat., vol. 242, Springer, Cham, 2018, pp. 51–66. MR3901156

[64] K. Erdmann and M. Hellstrøm-Finnsen, *Hochschild cohomology of some quantum complete intersections*, J. Algebra Appl. **17** (2018), no. 11, 1850215, 22, DOI 10.1142/S0219498818502158. MR3879091

[65] K. Erdmann, M. Holloway, R. Taillefer, N. Snashall, and Ø. Solberg, *Support varieties for selfinjective algebras*, K-Theory **33** (2004), no. 1, 67–87, DOI 10.1007/s10977-004-0838-7. MR2199789

[66] K. Erdmann and T. Holm, *Twisted bimodules and Hochschild cohomology for self-injective algebras of class A_n*, Forum Math. **11** (1999), no. 2, 177–201, DOI 10.1515/form.1999.002. MR1680594

[67] K. Erdmann, T. Holm, and N. Snashall, *Twisted bimodules and Hochschild cohomology for self-injective algebras of class A_n. II*, Algebr. Represent. Theory **5** (2002), no. 5, 457–482, DOI 10.1023/A:1020551906728. MR1935856

[68] K. Erdmann and N. Snashall, *On Hochschild cohomology of preprojective algebras. I, II*, J. Algebra **205** (1998), no. 2, 391–412, 413–434, DOI 10.1006/jabr.1998.7547. MR1632808

[69] K. Erdmann and N. Snashall, *Preprojective algebras of Dynkin type, periodicity and the second Hochschild cohomology*, Algebras and modules, II (Geiranger, 1996), CMS Conf. Proc., vol. 24, Amer. Math. Soc., Providence, RI, 1998, pp. 183–193. MR1648626

[70] K. Erdmann and Ø. Solberg, *Radical cube zero selfinjective algebras of finite complexity*, J. Pure Appl. Algebra **215** (2011), no. 7, 1747–1768, DOI 10.1016/j.jpaa.2010.10.010. MR2771643

[71] P. Etingof, S. Gelaki, D. Nikshych, and V. Ostrik, *Tensor categories*, Mathematical Surveys and Monographs, vol. 205, American Mathematical Society, Providence, RI, 2015. MR3242743

[72] P. Etingof and D. Kazhdan, *Quantization of Lie bialgebras. I*, Selecta Math. (N.S.) **2** (1996), no. 1, 1–41, DOI 10.1007/BF01587938. MR1403351

[73] P. Etingof and V. Ostrik, *Finite tensor categories*, Mosc. Math. J. **4** (2004), no. 3, 627–654, 782–783.

[74] C.-H. Eu and T. Schedler, *Calabi-Yau Frobenius algebras*, J. Algebra **321** (2009), no. 3, 774–815, DOI 10.1016/j.jalgebra.2008.11.003. MR2488552

[75] L. Evens, *The cohomology ring of a finite group*, Trans. Amer. Math. Soc. **101** (1961), 224–239, DOI 10.2307/1993372. MR137742

[76] L. Evens, *The cohomology of groups*, Oxford Mathematical Monographs, The Clarendon Press, Oxford University Press, New York, 1991. Oxford Science Publications. MR1144017

[77] M. Farinati, *Hochschild duality, localization, and smash products*, J. Algebra **284** (2005), no. 1, 415–434, DOI 10.1016/j.jalgebra.2004.09.009. MR2115022

[78] J. Feldvoss and S. Witherspoon, *Support varieties and representation type of self-injective algebras*, Homology Homotopy Appl. **13** (2011), no. 2, 197–215, DOI 10.4310/HHA.2011.v13.n2.a13. MR2854335

[79] E. M. Friedlander and A. Suslin, *Cohomology of finite group schemes over a field*, Invent. Math. **127** (1997), no. 2, 209–270, DOI 10.1007/s002220050119. MR1427618

[80] A. M. Garsia, *Combinatorics of the free Lie algebra and the symmetric group*, Analysis, et cetera, Academic Press, Boston, MA, 1990, pp. 309–382. MR1039352

[81] E. Gawell and Q. R. Xantcha, *Centers of partly (anti-)commutative quiver algebras and finite generation of the Hochschild cohomology ring*, Manuscripta Math. **150** (2016), no. 3-4, 383–406, DOI 10.1007/s00229-015-0816-9. MR3514736

[82] M. Gerstenhaber, *The cohomology structure of an associative ring*, Ann. of Math. (2) **78** (1963), 267–288, DOI 10.2307/1970343. MR0161898

[83] M. Gerstenhaber and S. D. Schack, *Relative Hochschild cohomology, rigid algebras, and the Bockstein*, J. Pure Appl. Algebra **43** (1986), no. 1, 53–74, DOI 10.1016/0022-4049(86)90004-6. MR862872

[84] M. Gerstenhaber and S. D. Schack, *A Hodge-type decomposition for commutative algebra cohomology*, J. Pure Appl. Algebra **48** (1987), no. 3, 229–247, DOI 10.1016/0022-4049(87)90112-5. MR917209

[85] M. Gerstenhaber and S. D. Schack, *Algebraic cohomology and deformation theory*, Deformation theory of algebras and structures and applications (Il Ciocco, 1986), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., vol. 247, Kluwer Acad. Publ., Dordrecht, 1988, pp. 11–264, DOI 10.1007/978-94-009-3057-5_2. MR981619

[86] M. Gerstenhaber and S. D. Schack, *The cohomology of presheaves of algebras. I. Presheaves over a partially ordered set*, Trans. Amer. Math. Soc. **310** (1988), no. 1, 135–165, DOI 10.2307/2001114. MR965749

[87] A. Giaquinto, *Topics in algebraic deformation theory*, Higher structures in geometry and physics, Progr. Math., vol. 287, Birkhäuser/Springer, New York, 2011, pp. 1–24, DOI 10.1007/978-0-8176-4735-3_1. MR2762537

[88] V. Ginzburg, *Lectures on noncommutative geometry*, 2005. arXiv:0506603.

[89] V. Ginzburg, *Calabi-Yau algebras*, 2006. arXiv:0612139.

[90] V. Ginzburg and D. Kaledin, *Poisson deformations of symplectic quotient singularities*, Adv. Math. **186** (2004), no. 1, 1–57, DOI 10.1016/j.aim.2003.07.006. MR2065506

[91] V. Ginzburg and S. Kumar, *Cohomology of quantum groups at roots of unity*, Duke Math. J. **69** (1993), no. 1, 179–198, DOI 10.1215/S0012-7094-93-06909-8. MR1201697

[92] E. Golod, *The cohomology ring of a finite p-group* (Russian), Dokl. Akad. Nauk SSSR **125** (1959), 703–706. MR0104720

[93] K. R. Goodearl and R. B. Warfield Jr., *An introduction to noncommutative Noetherian rings*, London Mathematical Society Student Texts, vol. 16, Cambridge University Press, Cambridge, 1989. MR1020298

[94] J. Goodman and U. Krähmer, *Untwisting a twisted Calabi-Yau algebra*, J. Algebra **406** (2014), 272–289, DOI 10.1016/j.jalgebra.2014.02.018. MR3188338

[95] N. S. Gopalakrishnan and R. Sridharan, *Homological dimension of Ore-extensions*, Pacific J. Math. **19** (1966), 67–75. MR200324

[96] I. G. Gordon, *Cohomology of quantized function algebras at roots of unity*, Proc. London Math. Soc. (3) **80** (2000), no. 2, 337–359, DOI 10.1112/S002461150001217X. MR1734320

[97] E. L. Green and N. Snashall, *The Hochschild cohomology ring modulo nilpotence of a stacked monomial algebra*, Colloq. Math. **105** (2006), no. 2, 233–258, DOI 10.4064/cm105-2-6. MR2237910

[98] E. L. Green, N. Snashall, and Ø. Solberg, *The Hochschild cohomology ring of a selfinjective algebra of finite representation type*, Proc. Amer. Math. Soc. **131** (2003), no. 11, 3387–3393, DOI 10.1090/S0002-9939-03-06912-0. MR1990627

[99] E. L. Green, N. Snashall, and Ø. Solberg, *The Hochschild cohomology ring modulo nilpotence of a monomial algebra*, J. Algebra Appl. **5** (2006), no. 2, 153–192, DOI 10.1142/S0219498806001648. MR2223465

[100] E. L. Green and Ø. Solberg, *An algorithmic approach to resolutions*, J. Symbolic Comput. **42** (2007), no. 11-12, 1012–1033, DOI 10.1016/j.jsc.2007.05.002. MR2368070

[101] L. Grimley, *Brackets on Hochschild cohomology of noncommutative algebras*, Ph.D. Thesis, 2016.

[102] L. Grimley, V. C. Nguyen, and S. Witherspoon, *Gerstenhaber brackets on Hochschild cohomology of twisted tensor products*, J. Noncommut. Geom. **11** (2017), no. 4, 1351–1379, DOI 10.4171/JNCG/11-4-4. MR3743225

[103] J. A. Guccione and J. J. Guccione, *Hochschild homology of twisted tensor products*, *K*-Theory **18** (1999), no. 4, 363–400, DOI 10.1023/A:1007890230081. MR1738899

[104] G. Halbout, *Globalization of Tamarkin's formality theorem*, Lett. Math. Phys. **71** (2005), no. 1, 39–48, DOI 10.1007/s11005-004-5926-3. MR2136736

[105] D. Happel, *Hochschild cohomology of finite-dimensional algebras*, Séminaire d'Algèbre Paul Dubreil et Marie-Paul Malliavin, 39ème Année (Paris, 1987/1988), Lecture Notes in Math., vol. 1404, Springer, Berlin, 1989, pp. 108–126, DOI 10.1007/BFb0084073. MR1035222

[106] T. J. Harding and F. J. Bloore, *Isomorphism of de Rham cohomology and relative Hochschild cohomology of differential operators*, Ann. Inst. H. Poincaré Phys. Théor. **58** (1993), no. 4, 433–452. MR1241705

[107] D. K. Harrison, *Commutative algebras and cohomology*, Trans. Amer. Math. Soc. **104** (1962), 191–204, DOI 10.2307/1993575. MR142607

[108] R. Hermann, *Exact sequences, Hochschild cohomology, and the Lie module structure over the M-relative center*, J. Algebra **454** (2016), 29–69, DOI 10.1016/j.jalgebra.2016.01.021. MR3473419

[109] R. Hermann, *Homological epimorphisms, recollements and Hochschild cohomology—with a conjecture by Snashall-Solberg in view*, Adv. Math. **299** (2016), 687–759, DOI 10.1016/j.aim.2016.05.022. MR3519480

[110] R. Hermann, *Monoidal categories and the Gerstenhaber bracket in Hochschild cohomology*, Mem. Amer. Math. Soc. **243** (2016), no. 1151, v+146, DOI 10.1090/memo/1151. MR3518219

[111] E. Herscovich, *Using torsion theory to compute the algebraic structure of Hochschild (co)homology*, Homology Homotopy Appl. **20** (2018), no. 1, 117–139, DOI 10.4310/HHA.2018.v20.n1.a8. MR3775352

[112] P. J. Hilton and U. Stammbach, *A course in homological algebra*, Graduate Texts in Mathematics, vol. 4, Springer-Verlag, New York-Berlin, 1971. MR0346025

[113] V. Hinich, *Tamarkin's proof of Kontsevich formality theorem*, Forum Math. **15** (2003), no. 4, 591–614, DOI 10.1515/form.2003.032. MR1978336

[114] G. Hochschild, *On the cohomology groups of an associative algebra*, Ann. of Math. (2) **46** (1945), 58–67, DOI 10.2307/1969145. MR0011076

[115] G. Hochschild, *Relative homological algebra*, Trans. Amer. Math. Soc. **82** (1956), 246–269, DOI 10.2307/1992988. MR80654

[116] G. Hochschild, B. Kostant, and A. Rosenberg, *Differential forms on regular affine algebras*, Trans. Amer. Math. Soc. **102** (1962), 383–408, DOI 10.2307/1993614. MR142598

[117] T. Holm, *The Hochschild cohomology ring of a modular group algebra: the commutative case*, Comm. Algebra **24** (1996), no. 6, 1957–1969, DOI 10.1080/00927879608825682. MR1386022

[118] T. Holm, *Hochschild cohomology rings of algebras* $k[X]/(f)$, Beiträge Algebra Geom. **41** (2000), no. 1, 291–301. MR1745598

[119] J. Huebschmann, *On the construction of* A_∞*-structures*, Georgian Math. J. **17** (2010), no. 1, 161–202.

[120] J. Huebschmann and J. Stasheff, *Formal solution of the master equation via HPT and deformation theory*, Forum Math. **14** (2002), no. 6, 847–868, DOI 10.1515/form.2002.037. MR1932522

[121] T. W. Hungerford, *Algebra*, Springer-Verlag, 1974.

[122] B. E. Johnson, *Cohomology in Banach algebras*, American Mathematical Society, Providence, R.I., 1972. Memoirs of the American Mathematical Society, No. 127. MR0374934

[123] T. V. Kadeishvili, *The algebraic structure in the homology of an* $A(\infty)$*-algebra* (Russian, with English and Georgian summaries), Soobshch. Akad. Nauk Gruzin. SSR **108** (1982), no. 2, 249–252 (1983). MR720689

[124] H. Kajiura and J. Stasheff, *Homotopy algebras inspired by classical open-closed string field theory*, Comm. Math. Phys. **263** (2006), no. 3, 553–581, DOI 10.1007/s00220-006-1539-2. MR2211816

[125] C. Kassel, *Quantum groups*, Graduate Texts in Mathematics, vol. 155, Springer-Verlag, New York, 1995. MR1321145

[126] B. Keller, *A-infinity algebras in representation theory*, Representations of algebra. Vol. I, II, Beijing Norm. Univ. Press, Beijing, 2002, pp. 74–86. MR2067371

[127] B. Keller, *Derived invariance of higher structures on the Hochschild complex*, 2003. http://www.math.jussieu.fr/~keller/publ/dih.pdf.

[128] B. Keller, *Hochschild cohomology and derived Picard groups*, J. Pure Appl. Algebra **190** (2004), no. 1-3, 177–196, DOI 10.1016/j.jpaa.2003.10.030. MR2043327

[129] B. Keller, *Deformation quantization after Kontsevich and Tamarkin*, Déformation, quantification, théorie de Lie, Panor. Synthèses, vol. 20, Soc. Math. France, Paris, 2005, pp. 19–62. MR2274224

[130] B. Keller, *On differential graded categories*, International Congress of Mathematicians. Vol. II, Eur. Math. Soc., Zürich, 2006, pp. 151–190. MR2275593

[131] Y. Kobayashi, *Gröbner bases on path algebras and the Hochschild cohomology algebras*, Sci. Math. Jpn. **64** (2006), no. 1, 103–129. MR2242347

[132] M. Kontsevich, *Homological algebra of mirror symmetry*, Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994), Birkhäuser, Basel, 1995, pp. 120–139. MR1403918

[133] M. Kontsevich, *Deformation quantization of Poisson manifolds*, Lett. Math. Phys. **66** (2003), no. 3, 157–216, DOI 10.1023/B:MATH.0000027508.00421.bf. MR2062626

[134] M. Kontsevich and Y. Soibelman, *Deformations of algebras over operads and the Deligne conjecture*, Conférence Moshé Flato 1999, Vol. I (Dijon), Math. Phys. Stud., vol. 21, Kluwer Acad. Publ., Dordrecht, 2000, pp. 255–307. MR1805894

[135] N. Kowalzig and U. Krähmer, *Batalin-Vilkovisky structures on* Ext *and* Tor, J. Reine Angew. Math. **697** (2014), 159–219.

[136] U. Krähmer, *Poincaré duality in Hochschild (co)homology*, New techniques in Hopf algebras and graded ring theory, Contactforum, 2007, pp. 117–125.

[137] U. Krähmer, *Notes on Koszul algebras*, 2011. Semantic Scholar pdf, semanticscholar.org.

[138] T. Lada and M. Markl, *Strongly homotopy Lie algebras*, Comm. Algebra **23** (1995), no. 6, 2147–2161, DOI 10.1080/00927879508825335. MR1327129

[139] T. Lambre, G. Zhou, and A. Zimmermann, *The Hochschild cohomology ring of a Frobenius algebra with semisimple Nakayama automorphism is a Batalin-Vilkovisky algebra*, J. Algebra **446** (2016), 103–131, DOI 10.1016/j.jalgebra.2015.09.018. MR3421088

[140] J. Le and G. Zhou, *On the Hochschild cohomology ring of tensor products of algebras*, J. Pure Appl. Algebra **218** (2014), no. 8, 1463–1477, DOI 10.1016/j.jpaa.2013.11.029. MR3175033

[141] M. Linckelmann, *On the Hochschild cohomology of commutative Hopf algebras*, Arch. Math. (Basel) **75** (2000), no. 6, 410–412, DOI 10.1007/s000130050523. MR1799425

[142] M. Linckelmann, *Finite generation of Hochschild cohomology of Hecke algebras of finite classical type in characteristic zero*, Bull. Lond. Math. Soc. **43** (2011), no. 5, 871–885, DOI 10.1112/blms/bdr024. MR2854558

[143] Y. Liu and G. Zhou, *The Batalin-Vilkovisky structure over the Hochschild cohomology ring of a group algebra*, J. Noncommut. Geom. **10** (2016), no. 3, 811–858, DOI 10.4171/JNCG/249. MR3554837

[144] A. C. Locateli, *Hochschild cohomology of truncated quiver algebras*, Comm. Algebra **27** (1999), no. 2, 645–664, DOI 10.1080/00927879908826454. MR1671958

[145] J.-L. Loday, *Opérations sur l'homologie cyclique des algèbres commutatives* (French), Invent. Math. **96** (1989), no. 1, 205–230, DOI 10.1007/BF01393976. MR981743

[146] J.-L. Loday, *Cyclic homology*, 2nd ed., Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 301, Springer-Verlag, Berlin, 1998. Appendix E by María O. Ronco; Chapter 13 by the author in collaboration with Teimuraz Pirashvili. MR1600246

[147] S. A. Lopes and A. Solotar, *Lie structure on the Hochschild cohomology of a family of subalgebras of the Weyl algebra*, 2019. arXiv:1903.01226.

[148] J. López-Peña, *Factorization structures. A Cartesian product for noncommutative geometry*, Ph.D. Thesis, 2007.

[149] M. Lorenz, *On the homology of graded algebras*, Comm. Algebra **20** (1992), no. 2, 489–507, DOI 10.1080/00927879208824353. MR1146311

[150] W. Lowen and M. Van den Bergh, *Hochschild cohomology of abelian categories and ringed spaces*, Adv. Math. **198** (2005), no. 1, 172–221, DOI 10.1016/j.aim.2004.11.010. MR2183254

[151] S. MacLane, *Homology*, 1st ed., Die Grundlehren der mathematischen Wissenschaften, vol. 114, Springer-Verlag, Berlin-New York, 1967. MR0349792

[152] M. Markl, *Deformation theory of algebras and their diagrams*, CBMS Regional Conference Series in Mathematics, vol. 116, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 2012. MR2931635

[153] M. Markl, S. Shnider, and J. Stasheff, *Operads in algebra, topology and physics*, Mathematical Surveys and Monographs, vol. 96, American Mathematical Society, Providence, RI, 2002. MR1898414

[154] R. Martínez-Villa, *Introduction to Koszul algebras*, Rev. Un. Mat. Argentina **48** (2007), no. 2, 67–95 (2008). MR2388890

[155] M. Mastnak, J. Pevtsova, P. Schauenburg, and S. Witherspoon, *Cohomology of finite-dimensional pointed Hopf algebras*, Proc. Lond. Math. Soc. (3) **100** (2010), no. 2, 377–404, DOI 10.1112/plms/pdp030. MR2595743

[156] H. Matsumura, *Commutative ring theory*, Cambridge Studies in Advanced Mathematics, vol. 8, Cambridge University Press, Cambridge, 1986. Translated from the Japanese by M. Reid. MR879273

[157] J. E. McClure and J. H. Smith, *A solution of Deligne's Hochschild cohomology conjecture*, Recent progress in homotopy theory (Baltimore, MD, 2000), Contemp. Math., vol. 293, Amer. Math. Soc., Providence, RI, 2002, pp. 153–193, DOI 10.1090/conm/293/04948. MR1890736

[158] J. C. McConnell and J. C. Robson, *Noncommutative Noetherian rings*, Pure and Applied Mathematics (New York), John Wiley & Sons, Ltd., Chichester, 1987. With the cooperation of L. W. Small; A Wiley-Interscience Publication. MR934572

[159] J. Meinel, V. C. Nguyen, B. Pauwels, M. J. Redondo, and A. Solotar, *The Gerstenhaber structure on the Hochschild cohomology of a class of special biserial algebras*, 2018. arXiv:1803.10310.

[160] L. Menichi, *Connes-Moscovici characteristic map is a Lie algebra morphism*, J. Algebra **331** (2011), 311–337, DOI 10.1016/j.jalgebra.2010.12.025. MR2774661

[161] B. Mitchell, *Rings with several objects*, Advances in Math. **8** (1972), 1–161, DOI 10.1016/0001-8708(72)90002-3. MR0294454

[162] S. Montgomery, *Hopf algebras and their actions on rings*, CBMS Regional Conference Series in Mathematics, vol. 82, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1993. MR1243637

[163] I. Mori and S. P. Smith, *The classification of 3-Calabi-Yau algebras with 3 generators and 3 quadratic relations*, Math. Z. **287** (2017), no. 1–2, 215–241.

[164] C. Negron, *Spectral sequences for the cohomology rings of a smash product*, J. Algebra **433** (2015), 73–106, DOI 10.1016/j.jalgebra.2015.02.021. MR3341818

[165] C. Negron and S. Witherspoon, *An alternate approach to the Lie bracket on Hochschild cohomology*, Homology Homotopy Appl. **18** (2016), no. 1, 265–285, DOI 10.4310/HHA.2016.v18.n1.a14. MR3498646

[166] C. Negron and S. Witherspoon, *The Gerstenhaber bracket as a Schouten bracket for polynomial rings extended by finite groups*, Proc. Lond. Math. Soc. (3) **115** (2017), no. 6, 1149–1169, DOI 10.1112/plms.12057. MR3741848

[167] S. Oppermann, *Hochschild cohomology and homology of quantum complete intersections*, Algebra Number Theory **4** (2010), no. 7, 821–838, DOI 10.2140/ant.2010.4.821. MR2776874

[168] M. S. Osborne, *Basic homological algebra*, Graduate Texts in Mathematics, vol. 196, Springer-Verlag, New York, 2000. MR1757274

[169] S. Pan and G. Zhou, *Stable equivalences of Morita type and stable Hochschild cohomology rings*, Arch. Math. (Basel) **94** (2010), no. 6, 511–518, DOI 10.1007/s00013-010-0137-x. MR2653667

[170] A. Parker and N. Snashall, *A family of Koszul self-injective algebras with finite Hochschild cohomology*, J. Pure Appl. Algebra **216** (2012), no. 5, 1245–1252, DOI 10.1016/j.jpaa.2011.12.011. MR2875343

[171] M. Penkava and A. Schwarz, *A_∞ algebras and the cohomology of moduli spaces*, Lie groups and Lie algebras: E. B. Dynkin's Seminar, Amer. Math. Soc. Transl. Ser. 2, vol. 169, Amer. Math. Soc., Providence, RI, 1995, pp. 91–107, DOI 10.1090/trans2/169/07. MR1364455

[172] J. Pevtsova and S. Witherspoon, *Varieties for modules of quantum elementary abelian groups*, Algebr. Represent. Theory **12** (2009), no. 6, 567–595, DOI 10.1007/s10468-008-9100-y. MR2563183

[173] T. Pirashvili and F. Waldhausen, *Mac Lane homology and topological Hochschild homology*, J. Pure Appl. Algebra **82** (1992), no. 1, 81–98, DOI 10.1016/0022-4049(92)90012-5. MR1181095

[174] H. Poincaré, *Analysis situs*, J. de l'École Polytechnique **1** (1895), no. 2, 1–123.

[175] A. Polishchuk and L. Positselski, *Quadratic algebras*, University Lecture Series, vol. 37, American Mathematical Society, Providence, RI, 2005. MR2177131

[176] L. E. Positselski, *Nonhomogeneous quadratic duality and curvature* (Russian, with Russian summary), Funktsional. Anal. i Prilozhen. **27** (1993), no. 3, 57–66, 96, DOI 10.1007/BF01087537; English transl., Funct. Anal. Appl. **27** (1993), no. 3, 197–204. MR1250981

[177] S. B. Priddy, *Koszul resolutions*, Trans. Amer. Math. Soc. **152** (1970), 39–60, DOI 10.2307/1995637. MR265437

[178] C. Psaroudakis, Ø. Skartsæterhagen, and Ø. Solberg, *Gorenstein categories, singular equivalences and finite generation of cohomology rings in recollements*, Trans. Amer. Math. Soc. Ser. B **1** (2014), 45–95, DOI 10.1090/S2330-0000-2014-00004-6. MR3274657

[179] D. Quillen, *On the (co-) homology of commutative rings*, Applications of Categorical Algebra (Proc. Sympos. Pure Math., Vol. XVII, New York, 1968), Amer. Math. Soc., Providence, R.I., 1970, pp. 65–87. MR0257068

[180] D. Quillen, *Algebra cochains and cyclic cohomology*, Inst. Hautes Études Sci. Publ. Math. **68** (1988), 139–174 (1989). MR1001452

[181] D. Quillen, *Cyclic cohomology and algebra extensions*, K-Theory **3** (1989), no. 3, 205–246, DOI 10.1007/BF00533370. MR1040400

[182] D. E. Radford, *Hopf algebras*, Series on Knots and Everything, vol. 49, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2012. MR2894855

[183] M. J. Redondo and L. Román, *Comparison morphisms between two projective resolutions of monomial algebras*, Rev. Un. Mat. Argentina **59** (2018), no. 1, 1–31. MR3825761

[184] V. S. Retakh, *Homotopy properties of categories of extensions* (Russian), Uspekhi Mat. Nauk **41** (1986), no. 6(252), 179–180. MR890505

[185] J. Rickard, *Derived equivalences as derived functors*, J. London Math. Soc. (2) **43** (1991), no. 1, 37–48, DOI 10.1112/jlms/s2-43.1.37. MR1099084

[186] M. Ronco, *On the Hochschild homology decompositions*, Comm. Algebra **21** (1993), no. 12, 4699–4712, DOI 10.1080/00927879308824824. MR1242856

[187] J. J. Rotman, *An introduction to homological algebra*, 2nd ed., Universitext, Springer, New York, 2009. MR2455920

[188] K. Sanada, *On the Hochschild cohomology of crossed products*, Comm. Algebra **21** (1993), no. 8, 2727–2748, DOI 10.1080/00927879308824703. MR1222741

[189] T. Schedler, *Deformations of algebras in noncommutative geometry*, Noncommutative algebraic geometry, Math. Sci. Res. Inst. Publ., vol. 64, Cambridge Univ. Press, New York, 2016, pp. 71–165. MR3618473

[190] W. F. Schelter, *Smooth algebras*, J. Algebra **103** (1986), no. 2, 677–685, DOI 10.1016/0021-8693(86)90160-2. MR864437

[191] M. Schlessinger and J. Stasheff, *The Lie algebra structure of tangent cohomology and deformation theory*, J. Pure Appl. Algebra **38** (1985), no. 2-3, 313–322, DOI 10.1016/0022-4049(85)90019-2. MR814187

[192] H.-J. Schneider, *Lectures on Hopf algebras*, Universidad Nacional de Córdoba, 2006. http://www.famaf.unc.edu.ar/series/pdf/pdfBMat/BMat31.pdf.

[193] F. Schuhmacher, *Deformation of L_∞-algebras*, 2004. arXiv:math.QA/0405485.

[194] S. Schwede, *An exact sequence interpretation of the Lie bracket in Hochschild cohomology*, J. Reine Angew. Math. **498** (1998), 153–172, DOI 10.1515/crll.1998.048. MR1629858

[195] A. V. Shepler and S. Witherspoon, *Poincaré-Birkhoff-Witt theorems*, Commutative algebra and noncommutative algebraic geometry. Vol. I, Math. Sci. Res. Inst. Publ., vol. 67, Cambridge Univ. Press, New York, 2015, pp. 259–290. MR3525474

[196] A. Shepler and S. Witherspoon, *Resolutions for twisted tensor products*, Pacific J. Math. **298** (2019), no. 2, 445–469, DOI 10.2140/pjm.2019.298.445. MR3936025

[197] S. F. Siegel and S. J. Witherspoon, *The Hochschild cohomology ring of a group algebra*, Proc. London Math. Soc. (3) **79** (1999), no. 1, 131–157, DOI 10.1112/S0024611599011958. MR1687539

[198] A. M. Sinclair and R. R. Smith, *Hochschild cohomology of von Neumann algebras*, London Mathematical Society Lecture Note Series, vol. 203, Cambridge University Press, Cambridge, 1995. MR1336825

[199] E. Sköldberg, *A contracting homotopy for Bardzell's resolution*, Math. Proc. R. Ir. Acad. **108** (2008), no. 2, 111–117, DOI 10.3318/PRIA.2008.108.2.111. MR2475805

[200] N. Snashall, *Support varieties and the Hochschild cohomology ring modulo nilpotence*, Proceedings of the 41st Symposium on Ring Theory and Representation Theory, Symp. Ring Theory Represent. Theory Organ. Comm., Tsukuba, 2009, pp. 68–82. MR2512413

[201] N. Snashall and Ø. Solberg, *Support varieties and Hochschild cohomology rings*, Proc. London Math. Soc. (3) **88** (2004), no. 3, 705–732, DOI 10.1112/S002461150301459X. MR2044054

[202] N. Snashall and R. Taillefer, *The Hochschild cohomology ring of a class of special biserial algebras*, J. Algebra Appl. **9** (2010), no. 1, 73–122, DOI 10.1142/S0219498810003781. MR2642814

[203] Ø. Solberg, *Support varieties for modules and complexes*, Trends in representation theory of algebras and related topics, Contemp. Math., vol. 406, Amer. Math. Soc., Providence, RI, 2006, pp. 239–270, DOI 10.1090/conm/406/07659. MR2258047

[204] R. Sridharan, *Filtered algebras and representations of Lie algebras*, Trans. Amer. Math. Soc. **100** (1961), 530–550, DOI 10.2307/1993527. MR130900

[205] M. D. Staic, *Secondary Hochschild cohomology*, Algebr. Represent. Theory **19** (2016), no. 1, 47–56, DOI 10.1007/s10468-015-9561-8. MR3465889

[206] J. D. Stasheff, *Homotopy associativity of H-spaces. I, II*, Trans. Amer. Math. Soc. 108 (1963), 275-292; ibid. **108** (1963), 293–312, DOI 10.1090/s0002-9947-1963-0158400-5. MR0158400

[207] J. Stasheff, *The intrinsic bracket on the deformation complex of an associative algebra*, J. Pure Appl. Algebra **89** (1993), no. 1-2, 231–235, DOI 10.1016/0022-4049(93)90096-C. MR1239562

[208] D. Ştefan, *Hochschild cohomology on Hopf Galois extensions*, J. Pure Appl. Algebra **103** (1995), no. 2, 221–233, DOI 10.1016/0022-4049(95)00101-2. MR1358765

[209] D. Ştefan and C. Vay, *The cohomology ring of the 12-dimensional Fomin-Kirillov algebra*, Adv. Math. **291** (2016), 584–620, DOI 10.1016/j.aim.2016.01.001. MR3459024

[210] M. Suárez-Álvarez, *Algebra structure on the Hochschild cohomology of the ring of invariants of a Weyl algebra under a finite group*, J. Algebra **248** (2002), no. 1, 291–306, DOI 10.1006/jabr.2001.9013. MR1879019

[211] M. Suárez-Álvarez, *The Hilton-Heckmann argument for the anti-commutativity of cup products*, Proc. Amer. Math. Soc. **132** (2004), no. 8, 2241–2246, DOI 10.1090/S0002-9939-04-07409-X. MR2052399

[212] M. Suárez-Álvarez, *A little bit of extra functoriality for Ext and the computation of the Gerstenhaber bracket*, J. Pure Appl. Algebra **221** (2017), no. 8, 1981–1998, DOI 10.1016/j.jpaa.2016.10.015. MR3623179

[213] R. G. Swan, *Hochschild cohomology of quasiprojective schemes*, J. Pure Appl. Algebra **110** (1996), no. 1, 57–80, DOI 10.1016/0022-4049(95)00091-7. MR1390671

[214] D. Tamarkin, *Another proof of M. Kontsevich formality theorem*, 1998. arXiv:math.QA/9803025.

[215] T. Tradler, *The Batalin-Vilkovisky algebra on Hochschild cohomology induced by infinity inner products* (English, with English and French summaries), Ann. Inst. Fourier (Grenoble) **58** (2008), no. 7, 2351–2379. MR2498354

[216] M. Van den Bergh, *A relation between Hochschild homology and cohomology for Gorenstein rings*, Proc. Amer. Math. Soc. **126** (1998), no. 5, 1345–1348, DOI 10.1090/S0002-9939-98-04210-5. Erratum, Proc. Amer. Math. Soc. 130 (2002), no. 9, 2809–2810. MR1443171; MR1900889

[217] B. B. Venkov, *Cohomology algebras for some classifying spaces* (Russian), Dokl. Akad. Nauk SSSR **127** (1959), 943–944. MR0108788

[218] M. Vigué-Poirrier, *Décompositions de l'homologie cyclique des algèbres différentielles graduées commutatives* (French, with English summary), K-Theory **4** (1991), no. 5, 399–410, DOI 10.1007/BF00533212. MR1116926

[219] Y. V. Volkov, *BV differential on Hochschild cohomology of Frobenius algebras*, J. Pure Appl. Algebra **220** (2016), no. 10, 3384–3402, DOI 10.1016/j.jpaa.2016.04.005. MR3497967

[220] Y. Volkov, *Gerstenhaber bracket on the Hochschild cohomology via an arbitrary resolution*, Proc. Edinb. Math. Soc. (2) **62** (2019), no. 3, 817–836, DOI 10.1017/s0013091518000901. MR3974969

[221] A. A. Voronov, *Homotopy Gerstenhaber algebras*, Conférence Moshé Flato 1999, Vol. II (Dijon), Math. Phys. Stud., vol. 22, Kluwer Acad. Publ., Dordrecht, 2000, pp. 307–331. MR1805923

[222] A. A. Voronov and M. Gerstenkhaber, *Higher-order operations on the Hochschild complex* (Russian, with Russian summary), Funktsional. Anal. i Prilozhen. **29** (1995), no. 1, 1–6, 96, DOI 10.1007/BF01077036; English transl., Funct. Anal. Appl. **29** (1995), no. 1, 1–5. MR1328534

[223] C. A. Weibel, *An introduction to homological algebra*, Cambridge Studies in Advanced Mathematics, vol. 38, Cambridge University Press, Cambridge, 1994. MR1269324

[224] S. Witherspoon, *Varieties for modules of finite dimensional Hopf algebras*, Geometric and topological aspects of the representation theory of finite groups, Springer Proc. Math. Stat., vol. 242, Springer, Cham, 2018, pp. 481–495. MR3901173

[225] F. Xu, *Hochschild and ordinary cohomology rings of small categories*, Adv. Math. **219** (2008), no. 6, 1872–1893, DOI 10.1016/j.aim.2008.07.014. MR2455628

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