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A Concise Introduction to Algebraic Varieties

Brian Osserman

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To Fu, Lingkai, and Taokai

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Preface

This book is an introduction to the theory of abstract algebraic varieties over algebraically closed fields, intended for a one-semester or one-quarter course. It provides a foundation in the basic properties of varieties, preparing the reader for either learning a more specialized topic such as toric varieties or moduli of curves, or moving on to the theory of schemes. Fundamental objects such as affine and projective varieties, regular functions and morphisms, and differential forms are introduced and interspersed with a mixture of local and global properties including nonsingularity, normality, and completeness. These ideas are combined to treat the theory of abstract nonsingular curves and linear series on them, concluding with an invitation to further directions in the form of a treatment of consequences of the Riemann-Roch theorem (a proof is not provided) and a brief discussion of Brill-Noether theory and moduli spaces of curves. The central object of the book is the abstract algebraic variety, which we define using atlases, as is commonly done for differentiable manifolds. Although this is, as far as I am aware, ahistorical,[1] it seems to strike a good balance between keeping the material down to earth while providing sufficient generality to serve as the foundation for learning about subjects such as toric varieties.

Our point of view is that abstract varieties are not motivated primarily by wanting to construct varieties that cannot be embedded in projective space, but rather because certain important constructions, such as normalization and toric varieties, are expressed most naturally in the context of abstract rather than projective varieties. Indeed, this philosophy goes back to the origins of the concept: Weil introduced abstract varieties in order to

[1] As I understand it, abstract varieties were first introduced by Weil over arbitrary fields in [**Wei46**], requiring much more technical definitions.

construct Jacobian varieties of curves over arbitrary fields [**Wei46**], [**Wei48**], which he used to prove his conjectures on zeta functions in the case of curves. Only subsequently was it proved by Chow that Weil's Jacobian varieties were in fact projective [**Cho54**]. Of course, from a practical present-day point of view it is also important that abstract varieties provide a far stronger foundation for learning about schemes.

A guiding principle of the book is the demonstration of varied techniques and approaches to proving results. In terms of approaches, we prove a number of results via reduction to commutative algebra (and include an appendix on algebra definitions and results), but we also prove other results via more geometric approaches, as for instance the proof in Chapter 7 that morphisms from projective varieties have closed images, and that regular functions on projective varieties are constant: this proof emerges out of the theory of complete varieties and extensions of morphisms from punctured nonsingular curves. We have also included illustrations of various commonly used techniques in algebraic geometry. For instance, in §3.4 we argue via factorizations of morphisms and exploit the geometry of integral ring extensions. In §7.6 we use fibrations to induct on dimension, reduce to a universal case, and make a nonemptiness argument using extension of the base field. Finally, in studying the secant variety in §8.5, we demonstrate the utility when studying parameter spaces of introducing auxiliary spaces for which conditions on the existence of certain objects are replaced with choices of those objects.[2]

Later chapters emphasize the case of algebraic curves, and while this aspect may be viewed as providing elements of a course on curves, it also serves to introduce general concepts such as divisors in a simpler setting. Perhaps more importantly, we also use results involving curves to prove statements on general varieties, including the above-described proof that regular functions on projective varieties are constant, and the proof that complex varieties are connected in the analytic topology. From this point of view, our approach can be viewed as an invitation to the circle of techniques involving studying higher-dimensional varieties through the curves on them, as is described for instance in Kollár's survey of Mori's work on the structure of threefolds [**Kol87**], and his book on rational curves on varieties [**Kol96**].

Following Hartshorne [**Har77**], we have taken a regular-function-centered approach to defining morphisms of varieties, which thus leads naturally into scheme theory. More broadly, this book is heavily influenced by—and intended to work seamlessly as a substitute for—Chapter I of [**Har77**]. Many other aspects are adapted from Shafarevich [**Sha94a**], [**Sha94b**], with some influence also from Mumford [**Mum99**] and Harris [**Har92**].

[2] In the case of the secant variety of a variety X, the condition that there exist a pair of points of X collinear with a given point is replaced by actually parametrizing such pairs of points.

Prerequisites. In any algebraic geometry course, students will benefit from having mastered as much background material as possible. In an ideal world, they would arrive having already taken an undergraduate course in algebraic geometry, a graduate course in algebra that includes a strong dose of commutative algebra, and maybe even a course in differential topology or geometry. However, it is my experience that this is rarely feasible, and this book is written around the reality that many students will arrive having learned far less. Even in a solid year-long graduate algebra course, which particular topics in commutative algebra are covered will vary drastically from course to course, so Appendix B on background in algebra is included to allow students to fill in any gaps in their own particular backgrounds as quickly and painlessly as possible. Ultimately, the only true prerequisites for this book are a strong undergraduate algebra course and a working familiarity with the basics of point-set topology, although graduate algebra is certainly strongly recommended.

Organization. Chapter 1 is an introduction to many of the ideas of algebraic geometry in the elementary context of plane curves and elliptic curves.

Chapters 2–4 constitute the local building blocks of the subject, with material on affine varieties, morphisms and rational maps, and singularities. Chapter 2 treats individual affine varieties, introducing the Zariski topology, the ideal-variety correspondence, and fundamental facts of dimension theory. Chapter 3 introduces regular functions, morphisms, and rational maps in the context of quasiaffine varieties, and proves the fundamental results describing regular functions and morphisms of affine varieties in terms of polynomial rings. A proof of Chevalley's theorem on the image of morphisms is also provided. Chapter 4 then develops the concept of singularities of affine varieties, contrasting the more geometric notion of tangent space with the more algebraic notion of Zariski cotangent space, proving the Jacobian criterion, and ultimately surveying the notions of complete local rings and of normality.

Chapters 5–7 form an introduction to the global theory, starting with the construction of abstract varieties via atlases, and continuing with projective varieties and blowups, and the basics of nonsingular curves and complete varieties. Chapter 5 lays out the foundations of the atlas approach to prevarieties and their morphisms, and develops the analogue of the Hausdorff condition which is used to define varieties. It concludes with a treatment of normalization for prevarieties. Chapter 6 introduces projective space and projective varieties as examples of varieties, makes a preliminary study of their regular functions and morphisms, and develops the theory of blowups. Chapter 7 studies extensions of morphisms from punctured nonsingular curves to projective varieties, proving quasiprojectivity of nonsingular curves and the existence of nonsingular projective compactifications of nonsingular curves. It then introduces the concept of complete varieties and proves the geometric analogue of the valuative criteria, stated in terms of extensions of morphisms from punctured nonsingular curves. Finally, these ideas are applied to the study of irreducibility of polynomials in families, leading to a proof of the fact that any two points in a variety can be connected by a chain of curves.

The main material concludes with Chapters 8–10, which develop the theory of nonsingular curves in more detail, as well as differentials on nonsingular varieties. Chapter 8 studies the role of divisors on curves in understanding morphisms to projective space, and also includes material on morphisms between nonsingular projective curves, and on testing for closed embeddings through injectivity on points and on tangent spaces. Chapter 9 develops the general theory of differential forms on nonsingular varieties, and then specializes to curves to study divisors associated to differential forms as well as the relationship between differential forms and ramification. Finally, Chapter 10 is an invitation to further topics; it initially takes the Riemann-Roch theorem as a black box and develops a variety of consequences, including the Riemann-Hurwitz theorem. It concludes with a brief sketch of Brill-Noether theory and moduli spaces of curves, although overviews of two proofs of the Riemann-Roch theorem are provided in an appendix.

Appendix A treats the analytic topology for complex varieties, relating the algebraic properties of being a variety, completeness, connectedness, and nonsingularity to the topological properties of Hausdorffness, compactness, connectedness, and being a (complex) manifold. The connectedness theorem is by far the deepest, pulling together ideas from several different parts of the book.

Finally, Appendix B is a guide to the algebra background used elsewhere in the book, collecting the definitions and statements, as well as selected proofs.

Given the goal of serving as a one-term textbook, many topics have been omitted. Obviously, sheaves, schemes, and cohomology have been left for a follow-up course. Other notable subjects which have been mostly or entirely omitted include aspects of projective geometry such as Hilbert functions and intersection theory, Bertini theorems, the theory of surfaces, Grassmannians, algebraic groups, vector bundles, and properties of morphisms. In addition, the material is focused entirely on theoretical algebraic geometry. Computational algebraic geometry has developed rapidly, and many explicit calculations are now possible and even routine. Even if one is ultimately interested in theoretical questions, a solid familiarity for what is computable can be valuable in working with examples. The development of computational techniques would take us too far afield, but the interested reader can find approachable accounts in the books of Cox-Little-O'Shea [**CLO07**] or Hassett [**Has07**].

How to use this book. This book is intended to provide roughly the right amount of material for a one-quarter or one-semester course. My own experience is that the ten chapters making up the main body of the book constitute slightly too much material to fit into a ten-week quarter, and I describe which sections are good candidates for omission below. Exercises are interspersed throughout and are a mixture of working with specific examples, developing supplemental theoretical statements, and working out (generally comparatively routine) theoretical statements which are used in proofs. In some cases (such as hyperelliptic curves, a determinantal variety, and a nonnormal surface), examples recur across multiple exercises.

Many of the choices of which particular results to include were made with subsequent applications in mind, so there is a high degree of interconnectedness, and the book is intended to be taught for the most part systematically and in the order presented. However, there is a certain amount of flexibility in how to approach the material.

First, the contents of Chapter 1 are intended to serve as the basis for a single introductory lecture giving a broad overview of many ideas in algebraic geometry in an approachable context. However, the main content of the book starts in Chapter 2 and is written independently of Chapter 1, so the latter material may freely be skipped or replaced with an alternative introduction to the subject.

There are also a few topics which are not used elsewhere or only used very narrowly, and could easily be omitted entirely if desired. These include the following, the first three of which are not used anywhere else in the book:

- the discussion of categories and geometry in §3.A;
- the treatment of completion and analytic singularity type in §4.4;
- the treatment of blowups in §6.3;
- the treatment in §4.A of the ideal-theoretic formulation of local complete intersections is used only once, in the treatment of blowups;
- both the material in §7.6 on irreducibility of polynomials in families and path-connectedness of varieties and the material in §8.5 on secant varieties and curves in projective space are used only in Appendix A;
- the material on Frobenius morphisms in §9.A is used only in Corollary 10.2.9 to remove the separability hypothesis from the statement that a curve cannot cover a curve of higher genus.

Of course, if one wishes to place a greater emphasis on higher-dimensional varieties, another natural option is to teach most of the above-mentioned material, and simply truncate the curve-specific material at the end. In this context, it is worth remarking that the general material on differential forms in §9.1 can be covered at any point after the end of Chapter 5. A similar option is to replace the invitation to further topics offered in Chapter 10 with the material on the analytic topology from Appendix A, in which case the connectedness theorem can serve as a capstone result (note however that this does require the Riemann-Roch inequality proved in Proposition 10.A.3).

While the organization largely follows a natural order for teaching, there are a few sections which are natural candidates for delayed treatment. Specifically, Chevalley's theorem is proved in §3.4, but is used only at the end of Chapter 7 and in Appendix A. Similarly, normality and normalization are treated in the affine case in §4.5 and in general in §5.4, but are also not used until Chapter 7. Both of these topics require a certain degree of sophistication and could reasonably be taught immediately prior to using them.

Acknowledgments. The mathematical perspective expressed in this book was deeply influenced by the courses I took from Ravi Vakil, Johan de Jong, and Joe Harris. This book was greatly improved in turn as a result of the many questions and comments I received from students in my courses on algebraic geometry, among whom I would especially highlight the contributions of Federico Castillo and John Challenor. I would also like to thank the reviewers for their careful readings and detailed comments, David Eisenbud for his feedback on the commutative algebra appendix, and Eriko Hironaka for her feedback as the book's editor.

Conventions. All rings (including algebras over rings) are assumed to be commutative, with multiplicative identity. In particular, an R-algebra is equivalent to a ring S, together with a homomorphism $R \to S$. We work throughout over an algebraically closed base field k except where we explicitly state otherwise.

Compact topological spaces are not assumed Hausdorff.

Newly defined terms are in bold. Words and phrases appear in quotes to indicate that they are being used in a less formal manner, to paint an intuitive picture of a situation, or to informally introduce terminology which either will be defined formally later in the text, or is only being mentioned in passing.

Brian Osserman
Davis, California
March 2021

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