

Georgii Fedoseevich VORONOÏ

Georgii Fedoseevich Voronoï was born in 1868 on his father's property Zhuravka in Poltava province. His father held the position of professor at the Nezhinsk *lycée*, and was later director of the gymnasia in Kishinev, Berdyansk, and Priluki. It was in Priluki that Voronoï completed his schooling, preserving thenceforth the very best of memories of his mathematics teacher Bogoslovskii. As a gymnasium pupil he had already demonstrated his remarkable mathematical ability. He especially liked algebra. At that time Professor Vasilii Petrovich Ermakov of Kiev University proposed in his *Journal of Elementary Mathematics* the topic of factorization of a polynomial using a property of the roots of a quadratic equation. Though still a gymnasist, Voronoï set to work on this topic, as a result of which there appeared in Yermakov's journal an article of moderate length furnished with a significant number of examples. After this Voronoï tested his prowess against a more difficult problem: to find the positive integer solutions x, y, z of the Diophantine equation

$$x^2 + y^2 + z^2 = 2mxyz,$$

for a given positive integer m . Here he did not succeed.

Voronoï continued to ponder this problem, even after entering Petersburg University in 1885. Concerning it he wrote in his diary: "I myself have lost hope of ever solving this problem."

During his first two years at the university Voronoï attended lectures faithfully, and studied Serret's algebra text, Faa de Bruno's book on binary forms, and Chebyshev's articles. The means provided by his father were insufficient, especially after the latter's retirement, and Voronoï had to resort to giving lessons, sometimes for very little payment. He found these lessons exhausting. In his diary he constantly complains about life in the student residence, which was not in the least conducive to devotion to the study of mathematics. Disturbances were constant.

At that time he wrote in his diary: "The pure mathematics lectures captivate me more and more. I prefer Professor Sokhotskii's lectures in the special course on higher algebra to all the others. . . . The main thing that concerns me is whether I have enough talent." To test himself he solved many quite difficult problems involving calculations with complicated symmetric functions, hard definite integrals, and integration of differential equations.

During his senior years Voronoï wrote his "candidate's" essay on the Bernoulli numbers, in which in particular he proved the following result, generalizing Adams' theorem:

If the index m of the m th Bernoulli number is divisible by $k = p_1^\alpha p_2^\beta \cdots p_l^\lambda$, where the p_i are primes not dividing the denominator of that Bernoulli number, then its numerator is divisible by k .

Markov expressed his liking for this work, which was published in the *Communications of the Khar'kov Mathematical Society* for 1900.

In 1889 Voronoĭ was kept at the university in order to prepare for a professorial vocation. He soon passed the exams for the Master's degree, and began to work on his dissertation "On integral algebraic numbers depending on a root of an irreducible equation of the third degree", which he defended in 1894. He was then appointed professor at Warsaw University in the department of pure mathematics.

In Warsaw Voronoĭ worked at finding a new algorithm that would allow him to solve for cubic fields the same problems that the continued fraction algorithm is used to solve for quadratic fields. In May of 1897 his doctoral dissertation, entitled "On a generalization of the continued fraction algorithm", was brilliantly defended at Petersburg University. The Academy of Sciences awarded both of his dissertations the Bunyakovskii prize.

Voronoĭ's creative life now becomes more intensive. However, he does not rush to publish his work; he waits for it to mature, revising repeatedly in an effort to find the simplest and most elegant methods through which to express his deep ideas.

One after another his marvellous articles appear: "Sur un problème du calcul des fonctions asymptotiques" (*Crelle*, Bd. 126); "Sur une fonction transcendente et ses applications à la sommation de quelques séries" (*Ann. de l'Éc. Norm. Sup.*, t. 20, 1904); "Sur quelques propriétés des formes quadratiques positives parfaites" (*Crelle*, Bd. 133, 1907); and "Recherches sur les paralléloèdres primitifs" (*Crelle*, Bde. 134, 136, 1908, 1909).

The last two of these articles, the fruit of many years of sustained thought, gave Voronoĭ much pleasure and pride.

In 1904 Voronoĭ attended the International Congress of Mathematicians in Heidelberg, where he met Minkowski, who was of course very close to him in mathematical interests, and who showed him great interest and respect.

His research on positive quadratic forms completed, Voronoĭ turned his attention to indefinite quadratic forms. However, by this time his illness (gallstones) was doing its work. In his diary Voronoĭ, then working in Novocherkassk, writes: "I am making great progress with the question under study; however, at the same time my health is becoming worse and worse. Yesterday I had for the first time a clear idea of the algorithm in the theory of forms I am investigating, but also suffered a strong attack of bilious colic, which prevented me from working in the evening and from sleeping the whole night. I am so afraid that the results of my enduring efforts, obtained with such difficulty, will perish along with me."

Unfortunately his "Note on indefinite quadratic forms", occupying 28 large pages, which Voronoĭ's friends remembered seeing, and which contained an outline of the results of this ambitious work, has hitherto not been found. Results of B. A. Venkov contained in two memoirs in *Izvestiya Akademii Nauk* to some extent fill the gap.

Voronoĭ died in 1908.

Voronoĭ worked exclusively in number theory. In his brief life he published only a few articles: the six large memoirs mentioned above, and six shorter notes. However, the depth and importance of these spacious works is such that they have had a profound influence on modern number theory. Voronoĭ was in fact the co-founder, along with Minkowski, of the geometry of numbers.

Voronoi's work of 1903 on the number of points under a hyperbola represents one of the landmarks from which the modern analytic theory of numbers took its bearings. In particular the research of Vinogradov in analytic number theory began with an elaboration of the ideas of this work. In Voronoi's work on an algorithm for computing cubic units he proposes a series of problems on the distribution of relative minima, one of which—a very difficult one—he solved himself, but the majority of which remain unsolved to this day (1947). Voronoi's work on quadratic forms and the partition of space far from exhaust the important questions raised there, and in connection with which he produced such fundamental results.

Of subsequent investigations, those most closely related to Voronoi's are the earliest of Vinogradov's articles, and certain work of Zhitomirskiĭ, Venkov, and the present author. Outside Russia almost no one has continued his line of research, so that his work remains essentially in the same state as that of Markov in number theory 30 years ago.¹²⁴

¹²⁴That is, around 1917. *Trans.*