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CHAPTER 0

Chapter Zero

In this chapter we have gathered 25 simple problems. To solve them you do not need anything but common sense and the simplest calculational skills. These problems can be used at sessions of a mathematical circle to probe the logical and mathematical abilities of students, or as recreational questions.

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Problem 1. A number of bacteria are placed in a glass. One second later each bacterium divides in two, the next second each of the resulting bacteria divides in two again, et cetera. After one minute the glass is full. When was the glass half-full?

Problem 2. Ann, John, and Alex took a bus tour of Disneyland. Each of them must pay 5 plastic chips for the ride, but they have only plastic coins of values 10, 15, and 20 chips (each has an unlimited number of each type of coin). How can they pay for the ride?

Problem 3. Jack tore out several successive pages from a book. The number of the first page he tore out was 183, and it is known that the number of the last page is written with the same digits in some order. How many pages did Jack tear out of the book?

Problem 4. There are 24 pounds of nails in a sack. Can you measure out 9 pounds of nails using only a balance with two pans? (See Figure 1.)

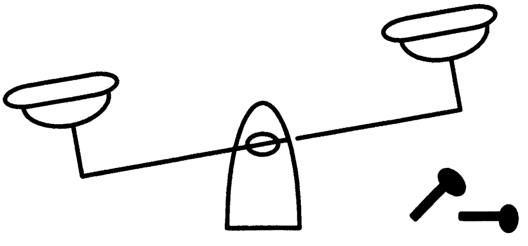

FIGURE 1

Problem 5. A caterpillar crawls up a pole 75 inches high, starting from the ground. Each day it crawls up 5 inches, and each night it slides down 4 inches. When will it first reach the top of the pole?

Problem 6. In a certain year there were exactly four Fridays and exactly four Mondays in January. On what day of the week did the 20th of January fall that year?

Problem 7. How many boxes are crossed by a diagonal in a rectangular table formed by 199×991 small squares?

Problem 8. Cross out 10 digits from the number 1234512345123451234512345 so that the remaining number is as large as possible.

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Problem 9. Peter said: "The day before yesterday I was 10, but I will turn 13 in the next year." Is this possible?

Problem 10. Pete's cat always sneezes before it rains. She sneezed today. "This means it will be raining," Pete thinks. Is he right?

Problem 11. A teacher drew several circles on a sheet of paper. Then he asked a student "How many circles are there?" "Seven," was the answer. "Correct! So, how many circles are there?" the teacher asked another student. "Five," answered the student. "Absolutely right!" replied the teacher. How many circles were really drawn on the sheet?

Problem 12. The son of a professor's father is talking to the father of the professor's son, and the professor does not take part in the conversation. Is this possible?

Problem 13. Three turtles are crawling along a straight road heading in the same direction. "Two other turtles are behind me," says the first turtle. "One turtle is behind me and one other is ahead," says the second. "Two turtles are ahead of me and one other is behind," says the third turtle. How can this be possible?

Problem 14. Three scholars are riding in a railway car. The train passes through a tunnel for several minutes, and they are plunged into darkness. When they emerge, each of them sees that the faces of his colleagues are black with the soot that flew in through the open window. They start laughing at each other, but, all of a sudden, the smartest of them realizes that his face must be soiled too. How does he arrive at this conclusion?

Problem 15. Three tablespoons of milk from a glass of milk are poured into a glass of tea, and the liquid is thoroughly mixed. Then three tablespoons of this mixture are poured back into the glass of milk. Which is greater now: the percentage of milk in the tea or the percentage of tea in the milk?

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Problem 16. Form a magic square with the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9; that is, place them in the boxes of a 3×3 table so that all the sums of the numbers along the rows, columns, and two diagonals are equal.

Problem 17. In an arithmetic addition problem the digits were replaced with letters (equal digits by same letters, and different digits by different letters). The result is: LOVES + LIVE = THERE. How many "loves" are "there"? The answer is the maximum possible value of the word THERE.

Problem 18. The secret service of The Federation intercepted a coded message from The Dominion which read: BLASE + LBSA = BASES. It is known that equal digits are coded with equal letters, and different digits with different letters. Two giant computers came up with two different answers to the riddle. Is this possible or does one of them need repair?

Problem 19. Distribute 127 one dollar bills among 7 wallets so that any integer sum from 1 through 127 dollars can be paid without opening the wallets.

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Problem 20. Cut the figure shown in Figure 2 into four figures, each similar to the original with dimensions twice as small.

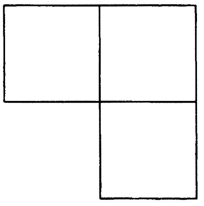

FIGURE 2

Problem 21. Matches are arranged to form the figure shown in Figure 3. Move two matches to change this figure into four squares with sides equal in length to one match.

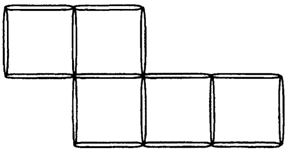

FIGURE 3

Problem 22. A river 4 meters wide makes a 90° turn (see Figure 4). Is it possible to cross the river by bridging it with only two planks, each 3.9 meters long?

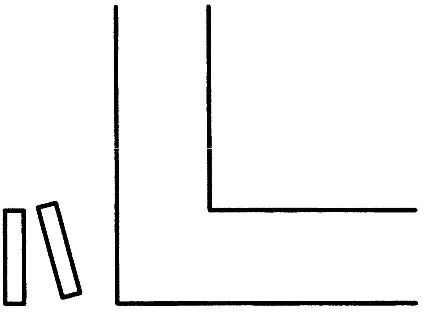

FIGURE 4

Problem 23. Is it possible to arrange six long round pencils so that each of them touches all the others?

Problem 24. Using scissors, cut a hole in a sheet of ordinary paper (say, the size of this page) through which an elephant can pass.

Problem 25. Ten coins are arranged as shown in Figure 5. What is the minimum number of coins we must remove so that no three of the remaining coins lie on the vertices of an equilateral triangle?

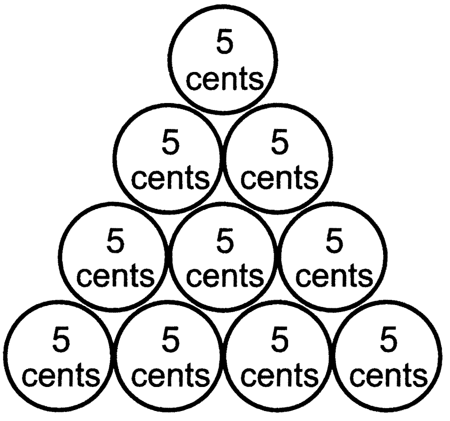

FIGURE 5