Mathematical World • Volume 8

Mathematics 1

Japanese Grade 10

Kunihiko Kodaira, Editor

American Mathematical Society
The University of Chicago School Mathematics Project

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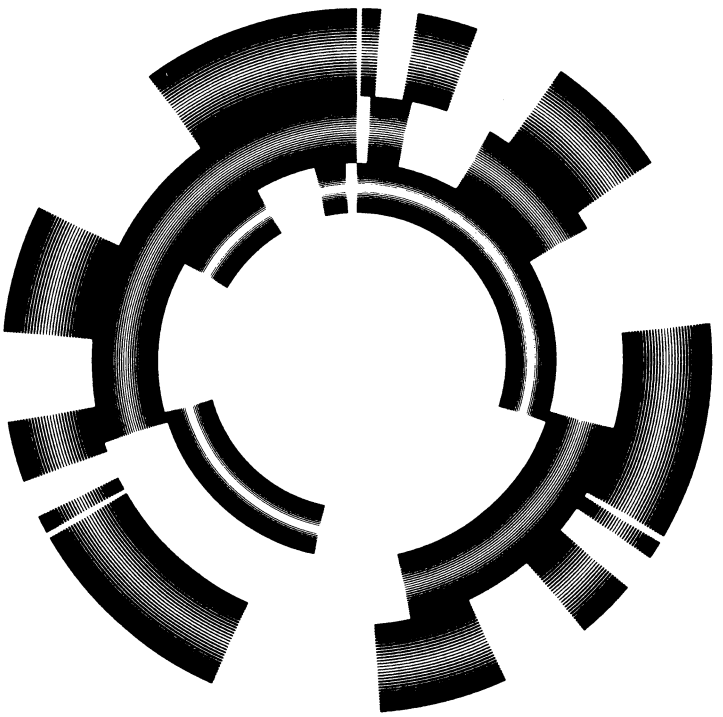

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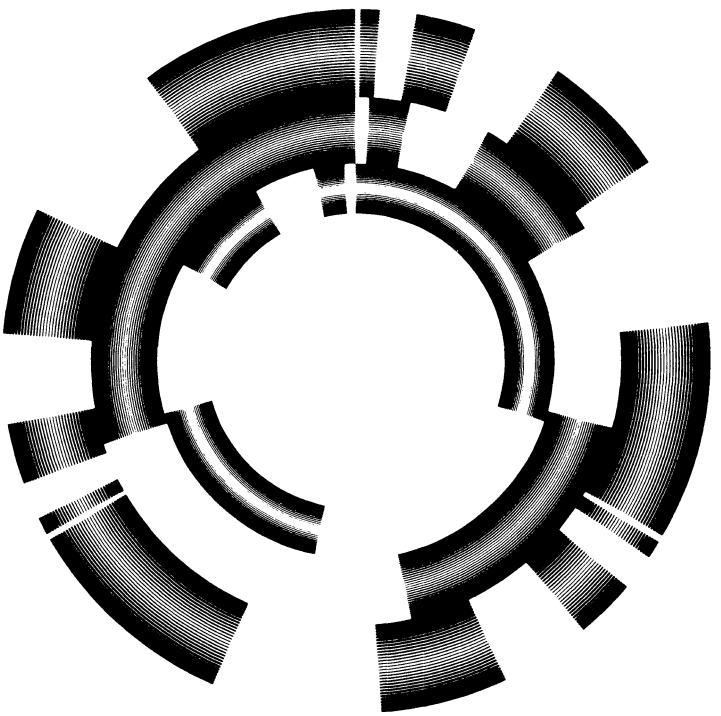

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Textbook Series Preface

The University of Chicago School Mathematics Project

This textbook is part of a series of foreign mathematics texts that have been translated by the Resource Development Component of the University of Chicago School Mathematics Project (UCSMP). Established in 1983 with major funding from the Amoco Foundation, UCSMP has been since that time the nation's largest curriculum development and implementation project in school mathematics. The international focus of its resource component, together with the project's publication experience, makes UCSMP well suited to disseminate these remarkable materials.

The textbooks were originally translated to give U.S. educators and researchers a first-hand look at the content of mathematics instruction in educationally advanced countries. More specifically, they provided input for UCSMP as it developed new instructional strategies, textbooks, and materials of its own; the resource component's translations of over 40 outstanding foreign school mathematics publications, including texts, workbooks, and teacher aids, have been used in UCSMP–related research and experimentation and in the creation of innovative textbooks.

The resource component's translations include the entire mathematics curriculum (grades 1–10) used in the former Soviet Union, standard Japanese texts for grades 7–11, and innovative elementary textbooks from Hungary and Bulgaria.

The content of Japan's compulsory national curriculum for grades 7–11 is made available for the first time in English, thanks in part to the generosity of the Japanese publisher, Tokyo Shoseki Company, Ltd., which provided the copyright permissions.

Japanese Secondary School Mathematics Textbooks

The achievement of Japanese elementary and secondary students gained world prominence largely as a result of their superb performance in the International Mathematics Studies conducted by the International Association for the Evaluation of Educational Achievement. The Second International Mathematics Study surveyed mathematics achievement in 24 countries in 1981–82 and released its findings in 1984. The results are recapitulated in a 1987 national report entitled "The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective" (A National Report on the Second International Mathematics Study, 1987).

Let us take a brief look at the schooling behind much of Japan's economic success. The Japanese school system consists of a six-year primary school, a three-year lower secondary school, and a three-year upper secondary school. The first nine grades are

compulsory, and enrollment is now 99.99%. According to 1990 statistics, 95.1% of age-group children are enrolled in upper secondary school, and the dropout rate is 2.2%. In terms of achievement, a typical Japanese student graduates from secondary school with roughly four more years of education than an average American high school graduate. The level of mathematics training achieved by Japanese students can be inferred from the following data:

Japanese Grade 7 Mathematics (New Mathematics 1) explores integers, positive and negative numbers, letters and expressions, equations, functions and proportions, plane figures, and figures in space. Chapter headings in *Japanese Grade 8 Mathematics* include calculating expressions, inequalities, systems of equations, linear functions, parallel lines and congruent figures, parallelograms, similar figures, and organizing data. *Japanese Grade 9 Mathematics* covers square roots, polynomials, quadratic equations, functions, circles, figures and measurement, and probability and statistics. The material in these three grades (lower secondary school) is compulsory for all students.

The textbook *Japanese Grade 10 Mathematics 1* covers material that is compulsory. This course, which is completed by over 97% of all Japanese students, is taught four hours per week and comprises algebra (including quadratic functions, equations, and inequalities), trigonometric functions, and coordinate geometry.

Japanese Grade 11 General Mathematics 2 is intended for the easier of the electives offered in that grade and is taken by about 40% of the students. It covers probability and statistics; vectors; exponential, logarithmic, and trigonometric functions; and differentiation and integration in an informal presentation.

The other 60% of students in grade 11 concurrently take two more extensive courses using the texts *Japanese Grade 11 Algebra and Geometry* and *Japanese Grade 11 Basic Analysis*. The first consists of fuller treatments of plane and solid coordinate geometry, vectors, and matrices. The second includes a more thorough treatment of trigonometry and an informal but quite extensive introduction to differential and integral calculus.

Some 25% of Japanese students continue with mathematics in grade 12. These students take an advanced course using the text *Probability and Statistics* and a second rigorous course with the text *Differential and Integral Calculus*.

One of the authors of these textbooks is Professor Hiroshi Fujita, who spoke at UCSMP's International Conferences on Mathematics Education in 1985, 1988, and 1991. Professor Fujita's paper on Japanese mathematics education appeared in *Developments in School Mathematics Education Around the World*, volume 1 (NCTM, 1987). The current school mathematics reform in Japan is described in the article "The Reform of Mathematics Education at the Upper Secondary School (USS) Level in Japan" by Professors Fujita, Tatsuro Miwa, and Jerry Becker in the proceedings of the Second International Conference, volume 2 of *Developments*.

Acknowledgments

It goes without saying that a publication project of this scope requires the commitment and cooperation of a broad network of institutions and individuals. In acknowledging their contributions, we would like first of all to express our deep appreciation to the Amoco Foundation. Without the Amoco Foundation's generous long-term support of the University of Chicago School Mathematics Project these books might never have been translated for use by the mathematics education community.

We are grateful to UCSMP Director Zalman Usiskin for his help and advice in making these valuable resources available to a wide audience at low cost. Robert Streit, Manager of the Resource Development Component, did an outstanding job coordinating the translation work and collaborating on the editing of most of the manuscripts. George Fowler, Steven R. Young, and Carolyn J. Ayers made a meticulous review of the translations, while Susan Chang and her technical staff at UCSMP handled the text entry and layout with great care and skill. We gratefully acknowledge the dedicated efforts of the translators and editors whose names appear on the title pages of these textbooks.

Izaak Wirszup, Director
UCSMP Resource Development Component

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FOREWORD TO THE JAPANESE EDITION

The history of mathematics goes back thousands of years, and the discipline has undergone continuous advances. In the last few decades, particularly, mathematics has made startling progress.

Geometry is said to have started in ancient Egypt and Babylonia with astronomical observations, measurement of land, and architecture. Thus, the advancement of mathematics was originally linked with science and technology; however, it gradually became independent of science and technology, and present-day mathematicians think freely about virtually everything possible. Therefore, mathematics is said to be a free creation of the human spirit.

This mathematics which is created freely by the human spirit can nonetheless be very useful when it is applied to various areas of study. For example, complex numbers were introduced as a way to imagine non-existing entities; this fact is reflected in the term "imaginary number." But present-day physics would be unthinkable without complex numbers. The realm of complex numbers is the real, existing world, and the realm of real numbers is now thought to be only a small part of that world. In this respect, even if mathematics can be said to be a free creation of the human spirit, it is not created freely by human beings. In other words, there exists a world of mathematical phenomena, just as physics is the study of natural phenomena, and mathematics is the science that studies those mathematical phenomena. Moreover, since the world of mathematical phenomena underlies the natural world, we must realize that mathematics is useful for the natural sciences. Thus, mathematics is a critical discipline; it is basic for the study of various other sciences.

This book is a textbook for students who have graduated from junior high school and entered high school to study Mathematics I. In Chapter 1 you will continue to learn basic concepts involving numbers, expressions, and fractional expressions, etc. which you also studied in junior high school. In Chapter 2 you will learn about equations and inequalities, especially a method of solving quadratic equations, and about complex numbers in connection with this method. You have already studied quadratic equations in junior high school, but in this chapter you will study quadratic equations from a broader standpoint and you will reach complex numbers. Figures are objects with completely different properties from numbers and polynomials; however, there is a close relationship between figures and polynomials. In Chapter 3 you will learn about the relation between figures and polynomials. Chapter 4 presents functions and their graphs. The fifth and last chapter deals with trigonometry and the relation between the properties of triangles and trigonometry.

To master mathematics, it is not enough to read and memorize books. It is important to think through the material, do calculations, and solve problems by yourselves. In this book we try to explain the material in a simple way by using many examples and sample problems.

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To the Student

Example	This marker designates a concrete example to help you understand the text.
Demonstration	This heading precedes a standard problem for better understanding of the material. Boxes labeled **[Solution]** and **[Proof]** give model answers.
Note:	This marker indicates an explanation to help you understand a particular point.
Problem 1	Problems for rapid mastery of current material and for introducing new material appear in the text with this label.
Exercises	At the end of each section problems are provided for practice with the material in that section.
Chapter Exercises	At the end of each chapter problems are provided for review of the entire chapter and practical application of the material. $\mathbb{A}$ problems involve primarily basic material, while $\exists$ problems are a bit more advanced.
Reference	Refers to topics from other elective courses.

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