Five-Minute Mathematics

Ehrhard Behrends
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Ehrhard Behrends

Translated by David Kramer
Contents

Foreword xi
Translator’s Note xv
Preface to the English Edition xix
Preface to the German Edition xxii
Acknowledgments xxiii
Introduction xxv

Chapter 1. You Can’t Beat the Odds 1
Chapter 2. Magical Mathematics: The Integers 5
Chapter 3. How Old Is the Captain? 8
Chapter 4. Vertiginously Large Prime Numbers 11
Chapter 5. Loss Plus Loss Equals Win 14
Chapter 6. When It Comes to Large Numbers, Intuition Fails 17
Chapter 7. The Key for Encryption Is in the Telephone Book 22
Chapter 8. The Village Barber Who Shaves Himself  
Chapter 9. Quit While You’re Ahead?  
Chapter 10. Can a Monkey Create Great Literature?  
Chapter 11. The Birthday Paradox  
Chapter 12. Horror Vacui  
Chapter 13. Sufficient Difficulties with the Logic of Mathematics Are in Fact a Necessity  
Chapter 14. To Change or Not to Change? The Monty Hall Problem  
Chapter 15. In Hilbert’s Hotel There Is Always a Vacancy  
Chapter 16. That Fascinating Number Pi  
Chapter 17. How Random Events Become Calculable Quantities  
Chapter 18. A One-Million-Dollar Prize: How Are the Prime Numbers Distributed?  
Chapter 19. The Five-Dimensional Cake  
Chapter 20. One Night Stand  
Chapter 21. Fly Me to the Moon  
Chapter 22. Using Residues  
Chapter 23. Top Secret!  
Chapter 24. Magical Mathematics: Order amidst Chaos  
Chapter 25. How Does One Approach Genius?  
Chapter 26. On Semitones and Twelfth Roots  
Chapter 27. Why Am I Always Standing in the Wrong Line?
## Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>Zero: An Undeservedly Underrated Number</td>
<td>109</td>
</tr>
<tr>
<td>29</td>
<td>I Love to Count!</td>
<td>112</td>
</tr>
<tr>
<td>30</td>
<td>Genius Autodidact: The Indian Mathematician Ramanujan</td>
<td>118</td>
</tr>
<tr>
<td>31</td>
<td>I Hate Mathematics Because. . .</td>
<td>121</td>
</tr>
<tr>
<td>32</td>
<td>The Traveling Salesman: A Modern Odyssey</td>
<td>124</td>
</tr>
<tr>
<td>33</td>
<td>Squaring the Circle</td>
<td>127</td>
</tr>
<tr>
<td>34</td>
<td>A Step into the Infinite</td>
<td>134</td>
</tr>
<tr>
<td>35</td>
<td>Mathematics in Your CD Player</td>
<td>139</td>
</tr>
<tr>
<td>36</td>
<td>The Logarithm: A Dying Breed</td>
<td>142</td>
</tr>
<tr>
<td>37</td>
<td>Prizeworthy Mathematics</td>
<td>145</td>
</tr>
<tr>
<td>38</td>
<td>Why Axioms of All Things?</td>
<td>148</td>
</tr>
<tr>
<td>39</td>
<td>Proof by Computer?</td>
<td>151</td>
</tr>
<tr>
<td>40</td>
<td>The Lottery: The Small Prizes</td>
<td>155</td>
</tr>
<tr>
<td>41</td>
<td>Formulas = Concentrated Thought</td>
<td>158</td>
</tr>
<tr>
<td>42</td>
<td>Endless Growth</td>
<td>161</td>
</tr>
<tr>
<td>43</td>
<td>How Do Quanta Compute?</td>
<td>165</td>
</tr>
<tr>
<td>44</td>
<td>Extremes!</td>
<td>169</td>
</tr>
<tr>
<td>45</td>
<td>Infinitely Small?</td>
<td>172</td>
</tr>
<tr>
<td>46</td>
<td>Mathematical Observations at the Fire Department</td>
<td>176</td>
</tr>
<tr>
<td>47</td>
<td>The First Mathematical Proof Is 2,500 Years Old</td>
<td>179</td>
</tr>
<tr>
<td>48</td>
<td>There Is Transcendence in Mathematics, Though It Has Nothing to Do with Mysticism</td>
<td>183</td>
</tr>
</tbody>
</table>
Chapter 49. Is Every Even Number the Sum of Two Primes? 188
Chapter 50. Why We Invert Conditional Probabilities Incorrectly 192
Chapter 51. Millionaire or Billionaire? 196
Chapter 52. Mathematics and Chess 199
Chapter 54. The Search for Mersenne Primes 206
Chapter 55. Berlin, Eighteenth Century: A Beautiful Formula Is Discovered 210
Chapter 56. The First Really Complicated Number 213
Chapter 57. P = NP: In Mathematics, Is Luck Sometimes Unnecessary? 216
Chapter 58. Happy 32nd Birthday! 219
Chapter 59. Buffon’s Needle 222
Chapter 60. Running Hot and Cold: Controlled Cooling Solves Optimization Problems 226
Chapter 61. Who Didn’t Pay? 230
Chapter 62. What Can Statistics Tell Us? 233
Chapter 63. Arbitrage 236
Chapter 64. Farewell to Risk: Options 239
Chapter 65. Is Mathematics a Reflection of the World? 242
Chapter 66. Mathematics That You Can Hear 246
Chapter 67. Chance as Composer 251
Contents

Chapter 68. Do Dice Have a Guilty Conscience? 255
Chapter 69. Strawberry Ice Cream Can Kill You! 258
Chapter 70. Prosperity for All 261
Chapter 71. No Risk, Thank You! 264
Chapter 72. A Nobel Prize in Mathematics? 268
Chapter 73. Chance as Reckoner: Monte Carlo Methods 272
Chapter 74. Fuzzy Logic 276
Chapter 75. Secret Messages in the Bible? 279
Chapter 76. How Knotted Can a Knot Be? 283
Chapter 77. How Much Mathematics Does a Person Need? 287
Chapter 78. Big, Bigger, Biggest 290
Chapter 79. It Is Probably Correct 293
Chapter 80. Is the World a Crooked Place? 296
Chapter 81. Is There a Mathematical Bureau of Standards? 299
Chapter 82. The Butterfly That Fluttered By 303
Chapter 83. Guaranteed to Make You Rich 307
Chapter 84. Don’t Trust Anyone over Thirty 310
Chapter 85. Equality in Mathematics 312
Chapter 86. Magical Invariants 314
Chapter 87. Mathematics Goes to the Movies 318
Chapter 88. The Lazy Eight: Infinity 320
Chapter 89. Books Need Bigger Margins! 323
Chapter 90. Visualizing Internal Organs with Mathematics 327
Chapter 91. A Brain in the Computer 330
Chapter 92. Cogito, Ergo Sum 335
Chapter 93. Does the World Have a Hole? 339
Chapter 94. Complex Numbers Are Not So Complex as Their Name Suggests 342
Chapter 95. M. C. Escher and Infinity 347
Chapter 96. A One at the Beginning Is Much More Likely Than a Two 351
Chapter 97. The Leipzig Town Hall and the Sunflower 354
Chapter 98. Information Optimally Packaged 360
Chapter 99. Four Colors Suffice! 364
Chapter 100. Mathematics Makes Billionaires 369
Further Reading 373
Index 377
Foreword: “Five-Minute Mathematics” in *Die Welt*

by Dr. Norbert Lossau, editor-in-chief for science at *Die Welt* and author of the column “Five-Minute Physics”

Most people do not harbor any particular affection for mathematics. They perceive numbers and formulas as difficult, confusing, abstract, irrelevant. And perhaps it is true that one needs a certain predisposition, analogous to what we call musicality, to develop a passionate interest in mathematics.

Nevertheless, I am convinced that many mathematical skeptics would readily take an interest in the queen of the sciences if only someone would build them a bridge into the fascinating realm of mathematics. Teachers could build such bridges by packaging mathematical lessons in suspense-filled stories from daily life. What would be the response, for example, if the discussion of abstract curves were motivated by a search for the optimal terms of a stock option? Or if one used geometry to calculate the amount of living space in a geometrically complicated dwelling and the number of rolls of wallpaper needed for redecorating? And when it comes to prime numbers, a
Foreword: “Five-Minute Mathematics” in *Die Welt*

story about cryptography and the challenges of breaking secret codes is certain to get the attention of many pupils.

The science of mathematics is central to our lives. It is to be found everywhere one looks: from the scanner at the checkout counter, the calculation of mortgage interest, and the PIN code on your debit card to computer tomography in medicine and the design of automobiles and airplanes. Mathematics makes possible space probes to distant planets and brings robots to life. It is the driving force behind technological advances and—if one allows oneself to be drawn into the subject—simply unbelievably fascinating.

Even if no bridge was laid down in far-off schooldays, there are still opportunities for grownups to approach the subject. For one thing, the coverage of science and technology in the media has improved enormously in recent years, although alas, such cannot be said for coverage of mathematics. Only a few newspapers and television outlets report regularly, or even sporadically, on topics related to mathematics even though there is much that is worth reporting. It would appear that many editors consider the subject of mathematics taboo.

*Die Welt* does not suffer from such fears and is not afraid, for example, of devoting a double-page spread to the number \(\pi\) (25 February 2006).

With the weekly column “Five-Minute Mathematics” from the pen of Professor Ehrhard Behrends, the newspaper has provided a stable editorial forum for the publication of one hundred columns on mathematical topics. From the large number of readers’ responses, we know that the column has generated considerable interest. Mathematics has been communicated—packaged in motivational stories—concisely and succinctly, comprehensibly and competently. And wonder of wonders, the unpalatable subject of mathematics has suddenly developed a pleasing taste.

“Five-Minute Mathematics” deserves to reach readers beyond the subscribers to the newspaper *Die Welt*, and we are pleased that with this book, the publishing house Vieweg Verlag is making this series of one hundred columns available to a wider public.
Professor Behrends is a bridge-builder, offering a way across the moat haunted by the dragons of math anxiety into fortress mathematics. He has the ability to package mathematical content so skillfully that there are few traces remaining of arid abstraction. If the status of mathematics is finally gradually to rise in the public’s perception, we need more writers like him, and of course more publications that will offer forums to these authors.
Translator’s Note

Translating Ehrhard Behrends’s hundred mathematical vignettes has been a great pleasure. It was also occasionally a challenge. The phrase “lost in translation” exists for a reason, since no translation can reproduce every nuance of the original. Fortunately, mathematics is a universal language, and I was not confronted with the types of issues faced by the translator of poetry or fiction. I was able to carry over most of the content intact. Some choices had to be made, however. Should I, for example, change references to the currency of the European Union, the euro, to dollars? I decided that the English-speaking audience could deal with a foreign currency, and so euros were retained. As of the moment of writing, one euro can be purchased for about US$1.42. On Monday, 12 May 2003, when the first of these articles appeared, the euro was trading at US$1.15.

I also retained geographical references to Germany and references to the German national lottery. On the other hand, for the English reader’s benefit, kilograms and meters have been converted to pounds and feet, and degrees Celsius to degrees Fahrenheit. References to odds of obtaining particular hands in the card game skat have been reinterpreted in terms of poker.

That was the easy stuff. How was your translator to deal with the chapter title “Die Mädchenhandelsschule,” in which the way that
items in a mathematical expression are grouped is explained by analogy to natural language? A Handelsschule is a business school, and so a Mädchenhandelsschule is a business school for girls. Unless, that is, one misreads it as (Mädchenhandels)schule, in which case one has something along the lines of a school for trading in girls. The German language is not associative! Neither is the English language, and I sought to come up with equivalent expressions. Lest the reader entertain any doubt, the discussion of the 1958 hit song “Flying Purple People Eater” in Chapter 20 of this volume does not appear in the German original.¹

No man is an island, and my work benefited from the collaboration of a number of people, whom I would like to thank here. Everyone at the American Mathematical Society with whom I worked was unstintingly helpful and friendly. I would like to thank Ed Dunne, who invited me to undertake this project, and Cristin Zanella, who kept me in the loop and answered all my queries. Thanks also to Barbara Beeton for her friendly and intelligent TeXnical support and to Arlene O’Sean for her careful copyediting.

I owe special thanks to two individuals who read the translation as it was being produced. One of them, Professor Ehrhard Behrends, is of course the author of these articles, and his careful reading made it possible for me to correct a number of typographical errors and clarify some fuzzy points.

My second reader was Christina Kramer, one of my several sisters, who as a professor not of mathematics but of Slavic linguistics brought to the book the analytical skills of the linguist and the intelligence of the “educated reader” along with the innocent eye of the mathematical neophyte. She regrets that she is perhaps one of those for whom Chapter 31, “I Hate Mathematics Because...,” was written. Christina called my attention to a number of linguistic anomalies and corrected quite a few typographical errors. She also pointed out several places where an additional phrase or sentence could rescue the reader with a background similar to hers from total befuddlement.

¹Is it a (purple people) eater or a purple (people eater)?
Finally, I would like to thank my dog, Orpheus, without whom this translation would have gotten done much sooner, but at the price of my not having taken nearly as many salubrious walks.
Preface to the English Edition

A few weeks after my book *Fünf Minuten Mathematik* went to press, I received a proposal from the American Mathematical Society to publish an English-language edition. I must say that I was pleased at the prospect, although it would mean quite a bit of work for me, since I would surely have to revise some of the chapters extensively, and one or two would have to be completely rewritten. After all, would English-language readers have any interest in the German lottery? Or be able to make heads or tails out of it even if they were? And were there enough math-haters out there to justify reprinting the chapter “I Hate Mathematics”? And how could anyone possibly translate the examples using colloquial German that I chose as illustrations of mathematical laws?

Having read the English translation, I see that I had no need for worry. The reason: the translator, David Kramer, to whom I would like to offer here my most heartfelt thanks. Of course he translated my German sentences into perfect English, which is no more than one would expect from a pro. But he has achieved much more, for through an intensive email correspondence between the two of us over several weeks, I have seen how the book has benefited as well from
a number of amplifying remarks (from which future German editions will also profit).

David has also added a number of observations of particular interest to the English-speaking world, and if that weren’t enough, he found a number of typographical errors in the German edition that had somehow eluded everyone else.

In my opinion, his masterpiece is the translation of Chapter 20. In second place are some of the chapter and section titles, such as “The Butterfly That Fluttered By” and “Both a Borrower and a Lender Be.” You can’t do much better than that.

I wish you, dear reader, much pleasure in perusing these “Five-Minute” morsels of mathematics, and I hope that those among you who may be skeptical about anything at all to do with mathematics will be disabused of the beliefs that everything interesting has already been discovered and that mathematics is nothing more than a bone-dry collection of facts and techniques.
During the years 2003 and 2004 there appeared the first and, so far, only regular column on mathematics in a newspaper read throughout Germany. “Fünf Minuten Mathematik” appeared every Monday in Die Welt, and the Berliner Morgenpost reprinted the column several weeks later.

By the end of two years, one hundred columns had been published, covering a wide variety of topics. Regular readers of the column obtained an overview of cryptography and coding theory, as well as insights into the fascination of prime numbers and the infinite, mathematics in the CD player and CAT scan, the notorious Monty Hall problem and other mysteries of probability theory, to name but a few of the subjects covered.

This book contains all one hundred articles. They have been carefully revised and expanded with explanatory texts, tables, and figures that have more than doubled the original length.

Everyone with an interest in learning more about aspects of contemporary mathematics that can be explained without assuming any specialized knowledge will find something of interest in these pages.
The author especially hopes to convince readers who were traumatized by school mathematics that the subject is not the boring, dry-as-dust subject that they remember, but a wellspring of fascination and excitement.
Acknowledgments

The American Mathematical Society gratefully acknowledges the kindness of the following institutions and individuals in granting the following permissions:

Elke Behrends

Graphics of the rice mountain in Chapter 6, the monkey in Chapter 10, and Hilbert’s Hotel in Chapter 15.

Bertrand Russell Archives, McMaster University Library

Photograph of Bertrand Russell at the blackboard in Chapter 8.

Vagn Lundsgaard Hansen

Photograph of the bridge in Chapter 53.

Robin Wilson

Photograph of Andrew Wiles in Chapter 89 taken and owned by Robin Wilson.

The author wishes to express his gratitude to the American Mathematical Society’s editorial and production departments for the careful preparation of this book.
Introduction

The story of this book begins on 25 January 2002, when the German Mathematical Society decided to hold a dinner to bring together the society’s officers and a group of journalists. The agenda was a conversation about the image of mathematics in the world at large. One of the participants was Dr. Norbert Lossau, science editor for the newspaper *Die Welt*, with whom I met again several months later. Out of these conversations arose the idea of a regular column on mathematics.

I put together an extensive proposal, in which about 150 possible topics were sketched. My suggestion of “Five-Minute Mathematics” as the title of the column was accepted, the graphic designers came up with a logo, and the first column appeared in the Monday, 12 May 2003, edition of *Die Welt*. And so it went week after week, with the rhythm being broken only when Monday was a holiday and the newspaper did not appear. After two years and one hundred columns, “Five-Minute Mathematics” yielded to another column.

In my selection of topics I have attempted to think particularly of readers who left school long ago and perhaps have no concrete memory traces of the subject yet would like to learn something about mathematics. Should the quadratic formula and curve sketching be
the limit of what is worth learning about mathematics? Where is mathematics to be found in the “real world”?

In two years I was able to cover a wide spectrum of topics, as can be seen from a perusal of the table of contents. There is the contemporary and there is the classical; there are hors d’oeuvres and main courses. And in many places the reader will learn how mathematics penetrates our lives, whether in the lottery, cryptography, computer-aided tomography (CAT), and the evaluation of securities options.

Even before the final column appeared, I received a proposal from the publisher Vieweg to collect the columns in a book. There were many good reasons to begin at once. First, many regular readers of the column had inquired about such a book. Second, a newspaper column is confined to a fixed size, so that every column had to have the same length, regardless of the topic.¹ For some of the topics, the space limitation meant that important information had to be omitted, leaving the author with a guilty conscience. Therefore I am pleased that the book format allows such limitations to be overcome. And finally, the luxury of space in a book means that the word can be supplemented by the image: photographs, drawings, graphs, tables.

In writing the column there were three aspects that I considered important:

Mathematics is useful: It should be made clear why our technologically based world could not function without mathematics. A label reading “mathematics inside” could be placed on many a product.

Mathematics is fascinating: Aside from its utility, mathematics offers a very special intellectual appeal. The irrepresible compulsion to see the solution of a problem through to the end can release enormous amounts of energy.

Without mathematics one cannot understand the world: According to Galileo, “The book of nature is written in the language of mathematics.” At his time, that was no more than a vision. Today it is known that mathematics is the bridge that leads us across the

¹At least that is what the author was told. Every now and then, the exigencies of the page layout required that the column be trimmed a bit.
unknown into realms that lie beyond the limits of human perception. Without mathematics, it would be impossible, in Goethe’s words, “to know what holds the world together from the inside.”

I would like to thank Dr. Lossau for allowing me for two years to present mathematical topics to readers of Die Welt. I retain wonderful memories of our collaboration.

I wish also to thank Elke Behrends for many photographs, particularly the photomontages appearing in Chapters 6, 10, and 15. I am also grateful to colleagues Vagn Hansen (Copenhagen) and Robin Wilson (Oxford) for the images that they provided (Chapters 53 and 89). Finally, I would like to thank Tina Scherer and Albrecht Weis for their extirpation of so many typographical errors during proofreading so that you, dear reader, will not have to be annoyed by them.
Further Reading

In recent years, a number of popular books on mathematics have appeared. Some of these have been mentioned in this book. Here is a small selection of additional books that may be of interest.

M. Aigner and G. Ziegler.  
This book collects some of the most beautiful proofs in mathematics. Recommended to all who have some background in mathematics.

A. Beutelspacher.  
*Cryptology*. Mathematical Association of America, 1996.  
For all who wish to learn more about cryptography.

J. Bewersdorff.  
Several chapters of *Five-Minute Mathematics* are devoted to games of chance and gambling. This book examines the mathematics of games from chess to Monopoly.

A. Doxiadis.  
This wonderfully written novel is about a man obsessed with the solution to Goldbach’s conjecture. There is no better description of how someone can be changed by succumbing to the fascination of a mathematical problem.
U. Dudley. 
A must for those interested in number mysticism.

R. Kanigel. 
An extensive biography of the mathematical genius Ramanujan.

R. Kaplan. 
Anyone wishing to know more about zero should read this book.

P. Ribenboim. 
*Five-Minute Mathematics* has much to say about large prime numbers. In this book the subject is systematically developed.

M. du Sautoy. 
Yet more about primes!

S. Singh. 
This book is one of the best popular books on mathematics. The reader learns much about mathematics in general and about mathematicians in a presentation of the Fermat problem, which was finally solved by Andrew Wiles.

S. Singh. 
Just as full of information and detailed as his book on Fermat’s last theorem, in this book Singh discusses the history of cryptography from ancient times to the present day. Needless to say, the RSA method is described in full. However, one learns as well how the Enigma code operated and how its secrets were cracked by the British.

R. Taschner. 
Many of the topics presented in *Five-Minute Mathematics* can also be found in Taschner’s book. The author has translated his extensive knowledge
into a book that is exciting to read. The major themes are number and symbol, number and music, number and time, number and space, number and politics, number and matter, number and spirit.
Index

Abel Prize, 146, 269, 311
Abel, Niels Henrik, 268, 269, 271, 310
algebraic number, 186
American option, 241
Apollonius, 81, 82
arbitrage, 236–238
associative law, 76–78, 111
axiom, 8, 175, 180, 184, 199, 296

Bach, J. S., 252, 253
Banach–Tarski paradox, 244
barber who shaves himself, 27
Bayesian formula, 192–194
Benford’s law, 351
Bible, 279–282
billionaire, 196, 197, 369
binomial coefficient, 116
birthday paradox, 37, 38, 40, 41
Black–Scholes formula, 237
Bolyai, János, 297
Buffon, Georges-Louis Leclerc, Comte de, 222
butterfly, 303
call option, 241
Cantor, Georg, 42, 59, 230
Cardano’s formula, 270
Cartesian coordinate system, 333, 336
casino, 33, 272
CD player, 139, 140
chain letter, 18, 261, 262
chaos theory, 15
chess, 199
Clay Mathematics Institute, 71, 146, 218, 339
C line, 348
coding theory, 360, 361
combinatorics, 112, 227
commutative law, 78, 79, 85
complex number, 344
complex number, 343–346
complex plane, 344
composition, 251–254
computer proof, 152, 365
computer-aided tomography, 327–329
conditional probability, 51, 192
conic section, 81
construction with straightedge and compass, 98, 99, 127, 128, 131–133, 182, 184, 296, 335
continued fraction, 356, 357
Coxeter, Harold Scott MacDonald, 348
cryptography, 84, 86, 87
cubing the sphere, 133
Dalí, Salvador, 75
Descartes, René, 25, 335, 336, 338
diagonal method, 292
dimension, 72, 73
distributive law, 79
e, 187, 211
Einstein, Albert, 297, 341
Elements (Euclid), 127, 148, 151, 180, 281
empty set, 42–44
epsilon, 173, 174
equal temperament, 102, 103
equality in mathematics, 312
error, types 1 and 2, 178
error-correcting code, 361, 363
Escher, M. C., 347–349
Euclid, 11–13, 97, 127, 148, 149, 281
Euler’s formula, 211, 212
Euler, Leonhard, 210
European option, 241
existence proof, 230, 231
exponential function, 211
exponential growth, 210
extreme value problem, 170
Fermat conjecture, 311, 323–326
Fermat prime, 98, 99
Fermat, Pierre de, 323, 324
Ferrari, Ludovico, 271
Fibonacci sequence, 355, 357, 359
Fields Medal, 145, 146, 268, 311
fifth-degree equation, 269, 271
four color problem, 151, 364–366
four-dimensional
cube, 75
spacetime, 243
Fourier analysis, 246
Fourier, Joseph, 246
fuzzy logic, 276
Galileo, 200, 202, 203
Gauss, Carl Friedrich, 96–99, 137
Gilbreath magic trick, 94
Gödel, Kurt, 150
Goldbach conjecture, 120, 188–191, 200
Goldbach, Christian, 188
golden ratio, 354–359
Google, 287, 352, 353, 369–372
great circle, 298
Hadamard, Jacques, 69
Hamming code, 363
Hamming, Richard Wesley, 362
Hardy, Godfrey Harold, 119
hedge, 264, 265, 267
hedgehog, 265
hexadecimal system, 220, 221
hierarchy of infinities, 291
hierarchy of numbers, 61, 183, 184
Hilbert’s Hotel, 58
Hilbert, David, 58, 149
horror vacui, 42
hypercube, 75
i, 211, 344
induction, 134, 135, 137, 152, 154, 184
induction hypothesis, 135, 137, 138
induction step, 135
infinite descent, 325, 326
infinitesimal quantities, 172–174
integers, 185
invariant, 94, 314–317
inverse problem, 328
irrational number, 200, 213, 214, 281, 325
KAM theory, 357
Kant, Immanuel, 97, 180
Kepler, Johannes, 82, 281
knot, 283–286
knot invariant, 285
law of small numbers, 282
Leibniz, Gottfried, 170, 172, 173, 175
linear problem, 306
Lobachevsky, Nikolai Ivanovich, 297
logarithm, 142–144, 187, 204
lottery, 112, 113, 115
Lucas–Lehmer test, 209
Index

Mersenne, Marin, 207
millionaire, 196, 198
modulo, 83
monkey at the typewriter, 34, 251
Monte Carlo method, 272–274, 293
Monty Hall problem, 48, 50, 54, 56, 106, 193
Mozart, Wolfgang Amadeus, 251–253
music, 247, 251–253
Musil, Robert, 343
mysticism, 279–281, 318
Nash, John Forbes, 319
natural number, 173, 184–186, 291, 301
neural network, 330, 331, 333
neusis construction, 131
Nevanlinna Prize, 166
Newton, Isaac, 170, 172, 180
nonconstructive existence proof, 230, 231
non-Euclidean geometry, 97, 297, 348
NP problem, 126, 218
null hypothesis, 234
option, 264–267
American, 241
European, 241
put, 240
P problem, 217, 218
Pacioli icosahedron, 358
paradox
Banach–Tarski, 244
birthday, 37, 38, 40, 41
Parrondo, 14
permutation, 16
Russell’s, 28, 29
Parrondo paradox, 14
perceptron, 331–333
Perelman, Grigoriı̆, 341
permutation paradox, 16
pigeonhole principle, 114, 231, 232, 282
Poincaré, Jules Henri, 340, 341
Poincaré problem, 146, 339–341
poker, 3, 115, 116, 176
machine, 12, 13
record, 11, 207
test, 208
theorem, 68–70
conditional, 51, 192
public key cryptography, 23, 24, 87, 166
purple people eater, 77
put option, 240
Pythagoras, 101, 279, 281, 282
Pythagorean theorem, 179, 337
quadratic formula, 270, 354
quantum computer, 165, 295
quantum computer, 23, 165, 166, 168, 294, 295
qubit, 166–168
queuing theory, 106
Radon, Johann, 328
Ramanujan, Srinivasa, 118–120
rational number, 185, 186, 213, 291
Reidemeister, Kurt, 285
Riemann hypothesis, 71
Riemann, Bernhard, 82, 297
Ries, Adam, 158
RSA algorithm, 87, 166
Russell’s paradox, 28, 29
Russell, Bertrand, 28, 29
sampling theorem, 140, 141
Serre, Jean-Pierre, 146, 269
set theory, 42
seventeen-gon, 98
Shor’s algorithm, 294
Shor, Peter, 166, 294
simply connected, 340
simulated annealing, 126, 226–228
slide rule, 143
spherical trigonometry, 204, 298
squaring the circle, 61, 127, 128,
132, 133, 184, 335
statistics, 177, 233, 258–260, 287
straightedge and compass
construction, 98, 99, 127, 128,
131–133, 182, 184, 296, 335
Taylor, Richard, 341
Thales’ theorem, 153, 179–182
tiling, 347, 348
transcendental number, 61, 128,
183, 184, 186, 187, 230, 335,
365
trapezoid, 45, 46, 296, 300, 301, 347
traveling salesman problem, 217,
218, 227
Vallée Poussin, Charles Jean de la,
69
Weierstrass, Karl, 173
Wiener, Norbert, 277
Wiles, Andrew, 146, 311, 324–326,
341
wind chill, 9
Xenakis, Iannis, 102, 252
Zadeh, Lotfi Asker, 276
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