

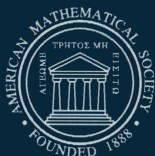
Translations of  
**MATHEMATICAL  
MONOGRAPHS**

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Volume 3

**Semigroups**

E. S. Ljapin



American Mathematical Society



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E. S. Ljapin



**American Mathematical Society**  
Providence, Rhode Island

# ПОЛУГРУППЫ

Е. С. ЛЯПИН

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## PREFACE TO THE FIRST EDITION

The present book has the goal of setting forth the basic aspects of the algebraic theory of semigroups. A semigroup (or in other words an associative system) is a set considered with respect to a binary associative operation defined in it. The concept of a semigroup is so simple and natural that it is hard to say when it first appeared. As Klein<sup>1</sup> points out, there were doubts, even in the period when the theory of groups was formulated as a separate mathematical discipline, as to whether that which we call a semigroup should be taken as the fundamental concept. However, the problems facing mathematics at that stage of its development made it necessary to choose a more restrictive concept, that of a group.

One of the most important reasons that the theory of groups played so large a role in the ensuing development of mathematics was that the algebraic theory of groups by its very nature was the abstract study of invertible transformations. The need for considering such mappings occurs in such diverse fields of mathematics, that one encounters the concept and properties of groups in very many mathematical (and not only mathematical) disciplines.

In spite of the importance and fruitfulness of the general concept of an invertible transformation, the necessity of studying the general concept of a transformation became clear with the further development of mathematics. To create a corresponding algebraic theory it was necessary to form a corresponding concept, namely the concept of a semigroup. The basic role of the algebraic theory of semigroups in mathematics evidently rests on the fact that this theory is the abstract study of general transformations.

In order to establish a basic mathematical discipline, naturalness and importance of the original concept are not sufficient. The experience of the theory of groups showed that the success of its development, granted the importance of the original ideas, is explained by the fact that the axioms permitted the construction of an extremely deep and complex theory. In this connection, the poorer axioms of the theory of semigroups raised a natural doubt at the first stages of its development: can it serve as the foundation for a sufficiently wide construction of an independent theory? At the present time, these doubts have already passed into oblivion.

After the first, frequently fragmentary, studies of semigroups, carried out in the 1920's and 1930's, the last 15 years have seen the appearance of hundreds

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<sup>1</sup> F. Klein, *Lectures on the development of mathematics in the 19th century*, Part I, Ch. VIII

of different works on the theory of semigroups, which have completely proved the possibility of a sufficiently profuse and deep independent theory.

The exposition of the principal results of these works in the form of a connected systematic theory is the aim of the present book. Of course there is no need (or possibility, for that matter) of setting forth all of the results obtained up to the present time. But the basic aspects of the contemporary abstract algebraic theory of semigroups are set forth here with sufficient completeness. Concrete semigroups, which are met in the most diverse fields of mathematics (in particular, wherever one must study one or another transformation), are completely impossible to survey completely. The material is too profuse. Nevertheless, a mathematician versed in the general theory of semigroups could apply this theory in a given concrete problem.

The present book is designed to offer this possibility to mathematicians working in different fields of pure and applied mathematics. To this end the material is set forth fairly completely and in considerable detail.

The theory as given here depends only on well-known branches of mathematics (the few places where this is not the case may be omitted). Only the simplest, universally familiar concepts and properties from the theory of sets will be used extensively.

However, we should mention here that we denote the elements of a set by including them in curly brackets, in contrast to the common practice in the theory of groups of using curly brackets in another special way. We make no distinction between a single element  $A$  of a certain set and the subset of this set consisting of the single element  $\{A\}$ .

We call the union of disjoint sets a union without intersection, or a non-intersecting union. The empty set is denoted by the symbol  $\emptyset$ . At times we shall use the concept of the empty symbol. A formula containing empty symbols should be read as if they were not present.

Throughout we shall constantly meet pairs of statements that are symmetric with respect to "left" and "right" (cancellation on the left, cancellation on the right, left ideal, right ideal, and so on). In such cases we shall usually give only one of the two assertions, counting on the reader to supply the other assertion and to see that it is correct.

The text of the book is divided into separate sections, numbered on the decimal principle. For example, a reference to II, 4.11 refers the reader to the second chapter, fourth section, 11th subsection. References within a given chapter omit the number of the chapter.

The literature on the abstract theory of semigroups is listed (with reasonable completeness) at the end of the book. Of course it does not contain works devoted completely or partially to the study of one or another concrete semigroup. As already remarked, there are simply too many such works. The bibliography also contains certain works on topological semigroups, although the theory of topological semigroups, which has already consolidated itself

into a special branch of topological algebra, is not touched on in the book itself. References to various other works not part of the general bibliography of the theory of semigroups are given in footnotes.

I remark with gratitude that in the final redaction of the manuscript I frequently made use of the valuable advice and observations of L. M. Gluškin.

Leningrad, 1958.

E. Ljapin



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## PREFACE TO THE SECOND EDITION

During the time since this book was written the theory of semigroups has continued to develop rapidly. Several hundred articles have been published on the subject and a second book has appeared, specifically devoted to semigroups: A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*. Vol. I, Math. Surveys, no. 7, Amer. Math. Soc., Providence, R. I., 1961, (Vol. II to be published in 1967). But the foundations, the general outlines and the principal directions of research have not essentially changed. Consequently, in this second edition, it has been possible to preserve the text of the first edition with only small changes, and to add a supplement containing a relatively small number of new results that are characteristic and important for an understanding of the present state of the theory.

It should be especially noted that in recent years a sharply increasing number of articles has been published on semigroups of transformations, partial transformations, and binary relations.

Also remarkable is the increase of interest in semigroups provided with an additional structure (e.g., ordered semigroups). It appears that such investigations constitute a special direction of research in algebra. This remark, which was made some time ago for topological semigroups, has been fully justified by the further development of that subject, although the theory of topological semigroups naturally remains closely connected with the general theory of semigroups.

For a clear understanding of the place of the theory of semigroups in contemporary mathematics and of its probable development in the future one must take into account the development of the following important theories, all of them so closely related in their essence to the theory of semigroups that they could from a certain point of view be included in that theory; namely, the theory of categories and the abstract theory of automata. It is also important to keep in mind the development of the general theory of binary relations. In this connection we wish to mention the extensive work of V. V. Vagner on the systematization of this theory: V. V. Vagner. *Theory of relations and algebra of partial transformations*, Theory of Semigroups and Its Applications, no. 1, Saratov Univ. Press, 1965, pp. 3-178. (Russian)

The bibliography on semigroups, which is reasonably complete for articles appearing up to 1958, has been retained for the present edition. A short supplementary bibliography, consisting of later works cited in the new material, has been added; it does not pretend to be complete.

Leningrad, 1966.

E. S. Ljapin

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## PREFACE TO THE THIRD EDITION

During the past five years, the rate of development of the theory of semigroups has continued to increase. The theory has enjoyed a substantial enrichment through the large number of new results obtained in its various subdivisions. Nevertheless, just as remarked in the preface to the preceding edition concerning the period from 1958 to 1966, it can be said again that the general aspect and character of the theory is basically unaltered. As a consequence, the earlier presentation can be retained.

However, this proposition does require in part some amendment, in the following respects.

The theory of transformations (including partial and multivalued transformations) plays a critical role in the theory of semigroups. It has importance for the connections of the theory of semigroups with other domains of mathematics, and the progress achieved in this direction would appear to merit a more extensive treatment. But one gains the impression that there is in process of formation a separate theory, though quite closely tied in with the general theory of semigroups.

New problems have appeared as a result of the permeation into the theory of semigroups of the ideas and methods of mathematical logic. There has been an expansion in the study of algorithmic questions. But this is again without question a separate line of investigation.

Within the confines of the theory of semigroups proper, most significant has been the development of, and the position taken by, the line of investigation dealing with the study of identities. Since in the preceding edition this was left almost entirely untouched, it has now appeared necessary to enlarge the presentation. A new chapter has therefore been added to deal with semigroup identities.

Also added is one more section to Chapter XI, dealing with results relating to approximation properties.

These additions have in turn made necessary additions to the Supplementary Bibliography.

Leningrad, 1972

*E. S. Ljapin*

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