

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 8

Introduction to the Theory
of Analytic Functions
of Several Complex Variables

B. A. Fuks



American Mathematical Society

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Providence, Rhode Island

ВВЕДЕНИЕ В ТЕОРИЮ
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PREFACE

The present volume has five chapters. The first chapter deals with the fundamental properties of analytic functions in the space of several complex variables, and the second chapter with the properties of analytic functions in covering regions over a suitable space. These two chapters may be considered as a textbook for readers who are looking for basic information, in as elementary a form as possible, about the theory of functions of several complex variables.

The next three chapters deal respectively with complex spaces, integral representations of functions of several complex variables, and functions meromorphic in the whole space. They are independent¹⁾ of one another in content but each of them makes a great deal of use of the material of the first two chapters. In contradistinction to the first two chapters, the last three are to a great extent in the nature of a survey. These chapters may serve as an introduction to the current technical literature on the various branches of the theory of functions.

The actual exposition itself is preceded by an introductory essay giving the most frequently used information from closely related mathematical disciplines. It is recommended that the reader refer to this essay whenever he finds it necessary.

The present book constitutes the first part of a second edition, considerably revised and enlarged, of the author's book *Theory of analytic functions of several complex variables* published in 1948. The second part, which is to appear soon after the first, will discuss a number of special chapters in the theory of functions.

1) One exception is § 20 of Chapter IV, in which essential use is made of subsection 1, § 14 of Chapter III.

At the request of the author the first draft of the text of subsections 1–3, §23, dealing with integral representations in n -circular regions, was written by L. A. Aĭzenberg, subsections 4–6, §23, dealing with integral representations in tubular regions, by S. G. Gindikin, and section 26, dealing with methods of characterizing the growth of entire functions, by L. I. Ronkin. These sections contain a number of new results, which are due to the above mentioned persons and are introduced here, as a rule, without reference to the original articles.

An exposition of a number of original results referring to integral representations was kindly placed at my disposal by A. A. Temljakov.

I am also indebted to L. A. Aĭzenberg and D. B. Fuks, who looked over the entire text while it was being prepared for the press and gave me valuable advice.

To all the above persons I wish to express my profound gratitude.

Many sections of this book were first presented to the seminar on the theory of analytic functions at the University of Moscow. I wish to take this opportunity of thanking the members of the seminar, and several other mathematicians, who looked over various parts of the book and sent me their suggestions.

B. Fuks

March 1961

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