

Translations of  
**MATHEMATICAL  
MONOGRAPHS**

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Volume 14

Special Chapters in the Theory of  
**Analytic Functions  
of Several Complex  
Variables**

B. A. Fuks



American Mathematical Society

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СПЕЦИАЛЬНЫЕ ГЛАВЫ  
ТЕОРИИ  
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МНОГИХ КОМПЛЕКСНЫХ  
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## PREFACE

The present volume is closely related in its contents to the author's book *Theory of analytic functions of several complex variables*, published in English translation<sup>1)</sup> by the American Mathematical Society in 1963. These two volumes together constitute the second edition, considerably revised and enlarged, of the monograph *Theory of analytic functions of several complex variables* published in 1948.

In the second edition, as well as in the first, the author does not aim to cover, even to any extent, all the material which has accumulated in the theory of analytic functions of several complex variables.

The present volume is divided into five chapters: approximation of functions and domains, coherent analytic sheaves and the solution of fundamental problems, domains analytically convex in the sense of Hartogs, holomorphic extension of domains, biholomorphic mappings. In order to understand these chapters, the reader should be familiar with the concepts contained in the first two chapters of *Theory of analytic functions of several complex variables*.

In addition, for §2 of Chapter I the reader must be acquainted with Weil's integral representations (§22, Chapter IV, (I)) and Cousin's first theorem (§25, Chapter V, (I)), for §§8–12 of Chapter II with properties of holomorphically complete complex manifolds (§14.1 and §18.3–4, Chapter III, (I)), for §14 of Chapter III with Weil's integral representations and the methods of solving Cousin's first problem (throughout §25, Chapter V, (I) and §7, Chapter II) and with properties of sequences of domains (§6, Chapter I), for §§17 and 18 of Chapter IV with the theory of plurisubharmonic functions (§13, Chapter III) and for Chapter V with properties of the kernel function in a domain (§§4 and 5, Chapter I). The remain-

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<sup>1)</sup>As Volume Eight of the *Translations of Mathematical Monographs*. In the present text a reference to this book will be indicated by the letter (I) following the number of chapter, section, formula, theorem or cited literature.

ing parts of the present volume, except for certain cross-references, are independent of one another and of Chapters III, IV and V of the first part of the book.

To shorten the volume and simplify the text the proofs of several propositions are not developed in the most general form. For example, Theorems (A) and (B) of H. Cartan are proved for complex manifolds, but not for spaces; the theorem of K. Oka on the domains convex in the sense of Hartogs is proved for the case of spaces of only two complex variables. The "edge of the wedge" theorem of N. N. Bogoljubov is also proved in a simplified form by introducing some hypotheses.

The actual text of the book itself is preceded by an introductory essay giving the most frequently used information from closely related mathematical disciplines. It is recommended that the reader refer to this essay whenever he finds it necessary.

At the request of the author, the first draft of the text of §§8–12, dealing with the theory of coherent analytic sheaves and its application to the solution of the fundamental problem, was written by D. B. Fuks, §19, dealing with the "edge of the wedge" theorem, by V. S. Vladimirov, and §24, dealing with homogeneous bounded domains, by S. G. Gindikin. The latter two sections contain new results which are due to the above-mentioned persons and are introduced here, as a rule, without reference to the original articles.

The author is indebted for advice and a number of valuable remarks to L. A. Aizenberg, who looked over the entire text while it was being prepared for the press, and to V. S. Vladimirov, who looked over the text of Chapters III and V.

To all the above persons I wish to express my profound gratitude. I also wish to take this opportunity of thanking other mathematicians who looked over various parts of the book and sent me their suggestions.

Results belonging to many mathematicians are presented in this book. It should be noted, however, that the greatest influence on its contents is due to works of S. Bergman, concerning the kernel function in a domain and its applications, of G. Bremermann, concerning domains convex in the sense of Hartogs and the holomorphic extension of domains, of H. Cartan who established, in collaboration with J.-P. Serre and others, the theory of coherent analytic sheaves and its applications to many important problems of the theory of functions, and of K. Oka, concerning approximation of functions, Cousin's problems and the solution of the *inverse problem of Hartogs*.

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1) The list given here is supplementary to the list of bibliography contained in the first part of this book (as is noted in the preface). Throughout the present volume a reference to the first list is indicated by adding (I); for instance, we write: Bremermann [1], (I), and so forth.

The present list, like the first one, makes no claim to completeness in the literature on the theory of analytic functions of several complex variables.



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