

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 18

Introduction to the
Theory of Linear
Nonselfadjoint
Operators
in Hilbert Space

I. C. Gohberg
M. G. Kreĭn



American Mathematical Society

Recent Titles in This Series

- 142 N. V. Krylov, Introduction to the theory of diffusion processes, 1995
141 A. A. Davydov, Qualitative theory of control systems, 1994
140 Aizik I. Volpert, Vitaly A. Volpert, and Vladimir A. Volpert, Traveling wave solutions of parabolic systems, 1994
139 I. V. Skrypnik, Methods for analysis of nonlinear elliptic boundary value problems, 1994
138 Yu. P. Razmyslov, Identities of algebras and their representations, 1994
137 F. I. Karpelevich and A. Ya. Kreinin, Heavy traffic limits for multiphase queues, 1994
136 Masayoshi Miyanishi, Algebraic geometry, 1994
135 Masaru Takeuchi, Modern spherical functions, 1994
134 V. V. Prasolov, Problems and theorems in linear algebra, 1994
133 P. I. Naumkin and I. A. Shishmarev, Nonlinear nonlocal equations in the theory of waves, 1994
132 Hajime Urakawa, Calculus of variations and harmonic maps, 1993
131 V. V. Sharko, Functions on manifolds: Algebraic and topological aspects, 1993
130 V. V. Vershinin, Cobordisms and spectral sequences, 1993
129 Mitsuo Morimoto, An introduction to Sato's hyperfunctions, 1993
128 V. P. Orevkov, Complexity of proofs and their transformations in axiomatic theories, 1993
127 F. L. Zak, Tangents and secants of algebraic varieties, 1993
126 M. L. Agranovskii, Invariant function spaces on homogeneous manifolds of Lie groups and applications, 1993
125 Masayoshi Nagata, Theory of commutative fields, 1993
124 Masahisa Adachi, Embeddings and immersions, 1993
123 M. A. Akivis and B. A. Rosenfeld, Élie Cartan (1869–1951), 1993
122 Zhang Guan-Hou, Theory of entire and meromorphic functions: Deficient and asymptotic values and singular directions, 1993
121 I. B. Fesenko and S. V. Vostokov, Local fields and their extensions: A constructive approach, 1993
120 Takeyuki Hida and Masuyuki Hitsuda, Gaussian processes, 1993
119 M. V. Karasev and V. P. Maslov, Nonlinear Poisson brackets. Geometry and quantization, 1993
118 Kenkichi Iwasawa, Algebraic functions, 1993
117 Boris Zilber, Uncountably categorical theories, 1993
116 G. M. Fel'dman, Arithmetic of probability distributions, and characterization problems on abelian groups, 1993
115 Nikolai V. Ivanov, Subgroups of Teichmüller modular groups, 1992
114 Seizô Itô, Diffusion equations, 1992
113 Michail Zhitomirskii, Typical singularities of differential 1-forms and Pfaffian equations, 1992
112 S. A. Lomov, Introduction to the general theory of singular perturbations, 1992
111 Simon Gindikin, Tube domains and the Cauchy problem, 1992
110 B. V. Shabat, Introduction to complex analysis Part II. Functions of several variables, 1992
109 Isao Miyadera, Nonlinear semigroups, 1992
108 Takeo Yokonuma, Tensor spaces and exterior algebra, 1992
107 B. M. Makarov, M. G. Goluzina, A. A. Lodkin, and A. N. Podkorytov, Selected problems in real analysis, 1992
106 G.-C. Wen, Conformal mappings and boundary value problems, 1992
105 D. R. Yafaev, Mathematical scattering theory: General theory, 1992
104 R. L. Dobrushin, R. Kotecký, and S. Shlosman, Wulff construction: A global shape from local interaction, 1992
103 A. K. Tsikh, Multidimensional residues and their applications, 1992
102 A. M. Il'in, Matching of asymptotic expansions of solutions of boundary value problems, 1992

(Continued in the back of this publication)

This page intentionally left blank

Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 18

Introduction to the
Theory of Linear
Nonselfadjoint Operators
in Hilbert Space

I. C. Gohberg
M. G. Kreĭn



American Mathematical Society
Providence, Rhode Island

И. Ц. ГОХБЕРГ
М. Г. КРЕЙН

**ВВЕДЕНИЕ В ТЕОРИЮ ЛИНЕЙНЫХ
НЕСАМОСПРЯЖЕННЫХ ОПЕРАТОРОВ
В ГИЛБЕРТОВОМ ПРОСТРАНСТВЕ**

Translated from the Russian by
A. Feinstein

Издательство «Наука»
Главная Редакция
Физико-Математической Литературы
Москва 1965

2000 *Mathematics Subject Classification*. Primary 47-02;
Secondary 47Bxx, 47D50.

Library of Congress Card Number 67-22348
Standard Book Number 0-8218-1568-7
International Standard Serial Number 0065-9282

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Assistant to the Publisher, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@ams.org.

© Copyright 1969 by the American Mathematical Society. All rights reserved.
Translation authorized by the
All-Union Agency for Author's Rights, Moscow.
Printed in the United States of America.

The American Mathematical Society retains all rights
except those granted to the United States Government.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at URL: <http://www.ams.org/>

12 11 10 9 8 7 6 04 03 02 01 00

TABLE OF CONTENTS

Introduction	xiii
Chapter I. General Theorems on Bounded Nonselfadjoint Operators . .	1
§1. Notation and some known results	1
§2. Normal points of a bounded operator	8
§3. Stability of the root multiplicities	13
§4. Some spectral properties of completely continuous operators . .	15
§5. A theorem on holomorphic operator-functions and its corollaries	19
Chapter II. s -Numbers of Completely Continuous Operators	24
§1. Minimax properties of the eigenvalues of selfadjoint com- pletely continuous operators	24
§2. s -numbers of completely continuous operators and their simplest properties.	26
§3. Inequalities relating the s -numbers, eigenvalues and diagonal elements of completely continuous operators	33
§4. Inequalities for the s -numbers of sums and products of completely continuous operators.	46
§5. Some generalizations of the preceding inequalities	51
§6. Inequalities for the eigenvalues of linear operators with completely continuous imaginary component	56
§7. s -numbers of bounded operators	59
Chapter III. Symmetrically-normed Ideals of the Ring of Bounded Linear Operators	65
§1. Two-sided ideals of the ring of bounded linear operators	66
§2. Symmetrically-normed ideals	68
§3. Symmetric norming functions.	71
§4. Symmetrically-normed ideals generated by a symmetric norming function	80
§5. A criterion for an operator to belong to an s.n. ideal \mathfrak{S}_*	83
§6. Separable s.n. ideals.	87
§7. The symmetrically-normed ideals \mathfrak{S}_p	91
§8. Nuclear operators	95
§9. Hilbert-Schmidt operators.	106

§10. Tests for the nuclearity of integral operators and formulas for calculating the trace.	112
§11. Functions adjoint to s.n. functions	125
§12. Symmetrically-normed ideals adjoint to separable symmetrically-normed ideals	128
§13. The three lines theorem for operator-functions which wander in \mathfrak{S}_p spaces	136
§14. The symmetrically-normed ideals \mathfrak{S}_{II} and $\mathfrak{S}_{II}^{(0)}$	139
§15. The symmetrically-normed ideals \mathfrak{S}_r and their connection with \mathfrak{S}_{II} and $\mathfrak{S}_{II}^{(0)}$	145
§16. Another interpolation theorem	150
§17. Conical norms in the real Banach spaces of operators \mathfrak{S}	152
Chapter IV. Infinite Determinants and Related Analytic Methods . .	156
§1. The characteristic determinant of a nuclear operator.	156
§2. Regularized characteristic determinants for the operators from \mathfrak{S}_p	166
§3. Perturbation determinants.	171
§4. A lemma on the growth of the perturbation determinant of a dissipative operator.	175
§5. A theorem on the perturbation determinant of a dissipative operator	178
§6. The determinants $D_{A^*, A}(\lambda)$ and $D_{A^{\mathcal{R}}, A}(\lambda)$ for a dissipative operator A with nuclear imaginary component	180
§7. Dissipative Volterra operators with nuclear imaginary component.	183
§8. Nondissipative operators with nuclear imaginary component .	188
§9. An asymptotic property of the spectrum of an operator with nuclear imaginary component.	198
§10. A theorem on Volterra operators with a finite-dimensional imaginary component	203
§11. Further theorems on relations between the Hermitian components of Volterra operators.	209
Chapter V. Theorems on the Completeness of the System of Root Vectors	222
§1. Lemmas on dissipative operators	222
§2. Tests for the completeness of the system of root vectors for dissipative operators with nuclear imaginary component	226
§3. Tests for the completeness of the system of root vectors of a contraction operator.	231

§4. Theorems on tests for the completeness of the systems of root vectors of dissipative operators with nuclear imaginary component.	237
§5. Estimation of the growth of the resolvents of operators of various classes.	242
§6. Theorems on the completeness of the systems of root vectors of higher classes of operators.	245
§7. Two lemmas on the resolvents of normal operators.	252
§8. Theorems on the completeness of the system of root vectors of a weakly perturbed selfadjoint operator	256
§9. Theorems on the multiple completeness of the system of eigenvectors and associated vectors of an operator bundle . . .	265
§10. Tests for the completeness of the systems of root vectors of unbounded operators.	275
§11. Asymptotic properties of the spectrum of a weakly perturbed positive operator	278
§12. Selfadjoint quadratic bundles	291
Chapter VI. Bases. Tests for the Existence of Bases, Consisting of Root Vectors of a Dissipative Operator	306
§1. Bases of a Hilbert space.	306
§2. Bases equivalent to orthonormal bases (Riesz bases).	309
§3. Bases quadratically close to orthonormal bases (Bari bases) . .	319
§4. Tests for the existence of a basis, consisting of eigenvectors of a dissipative operator.	328
§5. Basis of subspaces	332
§6. Tests for the existence of a basis consisting of the root subspaces of a dissipative operator.	345
Bibliography	352
Subject Index.	363

This page intentionally left blank

AUTHORS' PREFACE TO THE ENGLISH EDITION

We were happy to learn that immediately after the publication of our book, the American Mathematical Society had undertaken the publication of an English edition.

We wish to thank the translation editor, Dr. S. H. Gould, for the energy and initiative which he has displayed.

We wish to express our gratitude to Dr. A. Feinstein, who took upon himself the task of translating this book and who has fulfilled it with considerable responsibility in the shortest time. We owe to him, as well as to Professor T. Ando, the correction of a number of inaccuracies, a list of which would, to our surprise, prove to be rather long.

February 23, 1967.

I.C. Gohberg, M.G. Krein

This page intentionally left blank

PREFACE

The history of the writing of this book is not altogether usual. However, if all authors shared with their readers the history of the origins of their books, then in all probability we would have to deny this statement.

In the spring of 1959 we decided to revise our paper [1] on the theory of defect numbers, root numbers and indices of linear operators, attempting mainly to include in it various results from the theory of perturbations of selfadjoint operators. In particular, we had decided to complete it with the theory of perturbation determinants.

At this point it was discovered that the method of perturbation determinants also made it possible to obtain new results for nonself-adjoint operators. We decided to discuss these results, in connection with which we began to think about what to do in this area, which was new to us. Thus step by step we changed our original plan. . . . In roughly a year we sent to the editors of the journal "Uspehi Matematičeskikh Nauk" a review paper (in it, however, no space was found for perturbation determinants), in which we attempted to give something of a "bird's eye view" of the current state of investigation in the theory of nonselfadjoint operators. The editors of the journal "voted down" the paper, for which we wish to express our deep and sincere gratitude.

The fact is that the size of the original paper (twelve printer's sheets) exceeded by three times the norm established for review papers, and the editors thought it advisable to recommend the publication of the paper as a separate monograph in the series "Modern problems of mathematics".

Naturally, this suggestion induced us to extend the paper somewhat and, in particular, to restore to its rights the theory of perturbation determinants. We thought that all of this would require not more than two or three months. In actuality the work dragged out for three years. It should be said that even while we were working on the paper we had already established close scientific contact with Soviet mathematicians active in this field. Our colleagues gave us help and information concerning their results (before publication), and thus guarded us against the illusion of completeness. If to this one adds our desire to reflect in this book the investigations into which we ourselves were drawn by this work, one can understand our naïve delusion with respect to dates.

We hope that we are properly understood: we bear no grudge towards those who have given us copies of their papers before publication, who have communicated new, improved versions of proofs and have furnished us with all sorts of oral and written information. On the contrary, we are deeply grateful to all those with whose help we have been able, we hope, to achieve a certain degree of harmony in discussing the wide range of investigations which have come before our eyes, concerned with a large new domain of functional analysis—the theory of linear nonselfadjoint operators. Among all our colleagues who have helped us we wish particularly to mention M. S. Brodskii, S. G. Kreĭn, B. Ja. Levin, V. B. Lidskii, Ju. I. Ljubič, A. S. Markus, V. I. Macaev, and L. A. Sahnovič.

However, we have still not completed our account of how this book came to be written. We hope to continue this account in our monograph on the theory of abstract triangular representations of linear operators and its applications to the theory of canonical differential equations.

The editor of this book, F. V. Širokov, time and again “intruded” with his criticism in depth of the presentation of one topic or another. It often turned out that following such an intrusion we began to better understand even our own researches, and as a result the presentation of them gained in simplicity and clarity.

We wish to express our deep gratitude to F. V. Širokov for his considerable efforts and bold “excesses” of editorial authority.

Odessa, Arcadia
September 1, 1964

I. C. GOHBERG
M. G. KREĬN

INTRODUCTION

The theory of nonselfadjoint operators in Hilbert space is a young branch of functional analysis. In recent times it has attracted the ever increasing attention of mathematicians and physicists, and sometimes of engineers also. The aim of this book is to present a number of achievements in this field, most of them related to the theory of completely continuous operators.

For a long period the investigations of D. Hilbert and F. Riesz, generalizing Fredholm's theory of integral equations, remained the most essential part of this theory.

In contrast with the theory of selfadjoint operators, no spectral decomposition nor even any theorems on the completeness of the system of root vectors had been obtained until recently in the abstract theory of nonselfadjoint operators.

At the same time, important progress was made in the theory of nonselfadjoint boundary value problems for ordinary differential equations by Birkhoff (in 1908) and J. D. Tamarkin (in 1911). These authors started from the methods of Cauchy and Poincaré, based on the study of the analytic properties of the resolvent of the problem.

But as for boundary value problems for partial differential equations, in view of the difficulty in constructing the resolvent, there was for a long time no similar progress.

This situation changed only in 1951, thanks to progress in the abstract theory of nonselfadjoint operators. In this year the work of M. V. Keldyš appeared, in which he established theorems on the completeness of the eigenvectors and associated vectors and theorems concerning the asymptotic properties of the eigenvalues for a wide class of polynomial bundles of nonselfadjoint operators. These theorems made it possible to obtain important results in boundary value problems for partial differential equations; they also led to strong new results for ordinary differential equations.

At the same time, the asymptotic theorems of M. V. Keldyš for abstract operators made it possible to generalize the original investigations of T. Carleman on the asymptotic behavior of the eigenvalues for boundary value problems for second order elliptic differential operators.

In the same period (1951—1956) M. S. Livšic and a group of his co-workers began the development of a new and profound technique in the theory of nonselfadjoint operators.

The guiding light of these investigations was the idea of generalizing to abstract operators the algebraic result of I. Schur on the reduction of a matrix to triangular form by a unitary transformation. These investigations led M. S. Livšic to the creation of a theory of characteristic matrix-functions and triangular models of a nonselfadjoint operator. The realization of this idea in the abstract theory of operators in infinite-dimensional space entailed many difficulties. An essential role in overcoming these difficulties was played by the deep work of V. P. Potapov in the theory of analytic matrix-functions, which in turn was stimulated by investigations on nonselfadjoint operators.

New progress in the theory of the triangular representation of an operator has taken place in recent years (1958—1962).

Already in 1954 N. Aronszajn and K. Smith established the existence of a nontrivial invariant subspace for every completely continuous operator in a Banach space. Later, in 1958, L. A. Sahnovič first deduced from this geometric fact the existence of triangular models for a wider class of operators, and with the help of these models obtained new analytic results in the theory of nonselfadjoint operators.

At approximately the same period, there appeared in papers of M. S. Brodskii an integral giving an abstract triangular representation of a nonselfadjoint operator. The naturalness and transparency of this representation attracted to it the attention of a new group of mathematicians. In a short time the theory of the new integral and its applications was substantially developed (M. S. Brodskii, I. C. Gohberg and M. G. Krein, V. I. Macaev). This integral led in a natural way to the singling out of remarkable classes of Banach spaces, formed from completely continuous operators and representing ideals of the ring of bounded operators. Certain of these spaces were introduced earlier by J. von Neumann and R. Schatten, and many arose precisely in connection with the theory of the integral giving a triangular representation of operators.

Let us mention, finally, that the “abstract triangular integral” made it possible to solve, for a wide class of operators, the problem of their “factorization” (Gohberg and Krein).

Although geometric and algebraic ideas lie at the foundation of the theory of the abstract triangular integral, the solution of a number of fundamental questions of this theory was achieved only thanks to the

application of various (and sometimes new) tools from the theory of analytic functions.

It should be generally emphasized that the development of the theory of linear operators has drawn more and more widely on the theory of functions and, in particular, analytic functions (the theory of the growth and the distribution of the zeros and poles of entire and meromorphic functions, various generalizations and refinements of the Phragmén-Lindelöf theorem (for example the theorem of Levinson-Sjoberg), the theory of subharmonic functions, Tauberian theorems and others).

The basic channel for the use of the methods of the theory of functions is the natural and by now classical tradition of studying the spectral properties of a linear operator by studying its resolvent as an analytic operator-function. Fairly recently another channel appeared for the use of these methods; it is connected with the perturbation determinants of linear operators (M. G. Kreĭn, B. Ja. Levin, V. I. Macaev).

On the basis of the study of the resolvent, it has been possible in recent years to obtain strong tests for the existence of a "sufficiently complete" set of invariant subspaces of a (not necessarily bounded) operator (J. Wermer, F. Wolf, E. Bishop, V. I. Macaev and Ju. I. Ljubič). This, in turn, made it possible to develop the theory of abstract triangular representations, extending it to a wide class of bounded and even unbounded operators (L. A. Sahnovič, M. S. Brodskii, Gohberg and Kreĭn).

The study of the resolvent has also made it possible to establish theorems concerning the formation of bases from the root vectors of a nonselfadjoint operator and theorems on the summation of expansions in root vectors (M. A. Naimark, V. B. Lidskii, A. S. Markus). Moreover, a justification has been obtained for the Fourier method for equations of evolution with a nonselfadjoint operator of one class or another (Lidskii).

Let us mention that the technique for obtaining bounds for the resolvent of a nonselfadjoint operator, developed by T. Carleman, E. Hille and J. D. Tamarkin, M. V. Keldyš, V. B. Lidskii and others, has been substantially improved in recent papers of V. I. Macaev.

In this brief review, as in the book itself, we have not touched upon the interesting direction in the spectral theory of nonselfadjoint operators developed by N. Dunford [1] and a group of his students and followers.

The theory of the abstract triangular representation of operators and the problem of the factorization of abstract operators, and also their applications, will be discussed in another monograph by the authors.

In the present book these problems are left almost untouched, although in various places we allude to isolated facts from the theory of the triangular representation. The fact is that complete results are sometimes attained only by the interaction of three types of methods: methods based on the study of the growth of the resolvent, methods of the theory of perturbation determinants and, lastly, methods of the theory of the triangular representation of an operator.

In recent times appreciable development has taken place in the theory of special operators which become selfadjoint or unitary upon the introduction of a certain indefinite metric in the Hilbert space. The investigations in this theory deserve discussion in a separate monograph; in the present book we have not touched upon them, except for a few cases related to them (Chapter V, §12).

Going on to a brief indication of the contents of the individual chapters, we mention, first of all, that a substantial part of the material is discussed here, with adequate proofs, for the first time. The material as a whole is systematized for the first time in this monograph.

In the first chapter, well-known results of the general theory of bounded nonselfadjoint operators are recalled. As a rule, these results are not specific for Hilbert spaces; they could have been formulated for operators in a Banach space.

In the third chapter we discuss the theory of symmetrically-normed ideals of the ring of bounded operators in a Hilbert space. This chapter includes the basic content of the elegant book of R. Schatten [2], devoted to the von Neumann-Schatten theory of "norm ideals" and "cross-norms". Together with the "veteran ideals" (the ideals of the nuclear operators, Hilbert-Schmidt operators and others), we single out and study new ideals which play important roles in various questions of the theory of nonselfadjoint operators. We are able to develop a treatment of the theory of symmetrically-normed ideals, starting from higher principles, thanks to the systematic use of the theory of the s -numbers of completely continuous operators worked out by H. Weyl, Ky Fan, A. Horn and others. The theory of s -numbers forms the basic content of the second chapter. General theorems on s -numbers of linear operators are also used systematically in the following two chapters.

The fourth chapter is devoted to the theory of perturbation determinants and some of its applications. The use of the extensive apparatus of the modern theory of analytic functions distinguishes this chapter from the others. This apparatus is regarded as auxiliary, and in most cases results of the theory of functions are presented without proof.

In the fifth chapter we discuss various theorems on the completeness of the system of root (eigen- and associated) vectors of a completely continuous operator (operator bundle). In the selection of the material the authors have attempted, on the one hand, to present sufficiently original and strong results and, on the other, to present as fully as possible the various existing methods.

Here we also discuss various results on the growth of the resolvent and theorems on the asymptotic properties of the spectrum (of operators of various classes).

In this chapter, apparently, the fundamental results of M. V. Keldyš are for the first time treated in a sufficiently detailed form.

Considerable attention has also been devoted to theorems on the completeness of the system of root vectors of a dissipative operator.

The last section is devoted to the study of the spectral properties of a selfadjoint quadratic bundle. Here we use the results of almost all the other sections of this chapter. However, as recent investigations have shown (M. G. Kreĭn and H. Langer), for the construction of a complete theory of quadratic selfadjoint bundles it is natural to use various theorems of the theory of operators in spaces with an indefinite metric. In view of this, certain results are presented here without proof.

As a supplement to the theorems on completeness, the authors have thought it appropriate to discuss in Chapter VI the simplest tests for the existence of a basis (of one kind or another) made up of the root vectors of a given linear operator (results of B. R. Mukminov, I. M. Glazman and A. S. Markus). The theory of bases in a Hilbert space is not discussed in texts on functional analysis; for this reason it too is presented in this chapter.

The presentation in this book is carried out in the spirit of the abstract theory of operators: it is illustrated by various applications to the theory of integral equations.

The reader who has some experience in the theory of boundary value problems for differential equations, or an acquaintance with the theory of linear vibrating systems with a finite or infinite number of degrees of freedom, will easily discover how many of the results discussed here find immediate application in each of these fields. A clearer presentation of the possibilities and prospects existing here can be obtained from the interesting survey article by M. V. Keldyš and V. B. Lidskiĭ [1] (cf. also the survey by C. L. Dolph [1] and the report by M. G. Kreĭn and H. Langer [2]).

This page intentionally left blank

BIBLIOGRAPHY

S. AGMON

1. *On the eigenfunctions and on the eigenvalues of general elliptic boundary value problems*, Comm. Pure Appl. Math. 15 (1962), 119-147. MR 26 # 5288.

N. I. AHIEZER AND I. M. GLAZMAN

1. *Theory of linear operators in Hilbert space*, GITTL, Moscow, 1950; 2nd rev. ed., "Nauka", Moscow, 1966; English transl., Ungar, New York, 1961. MR 13, 358; MR 34 # 6527.

N. I. AHIEZER AND M. G. KREIN

1. *Some questions in the theory of moments*, Naučn. Tehn. Izdat. Ukrain., Kharkov, 1938; English transl., Transl. Math. Monographs, vol. 2, Amer. Math. Soc., Providence, R. I., 1962. MR 29 # 5073.

D. È. ALLAHVERDIEV

1. *On the completeness of the systems of eigen-elements and adjoined elements of non-self-adjoint operators close to normal ones*, Dokl. Akad. Nauk SSSR 115 (1957), 207-210. (Russian) MR 20 # 1227.
2. *On the rate of approximation of completely continuous operators by finite-dimensional operators*, Azerbaidžan. Gos. Univ. Učen. Zap. Ser. Fiz.-Mat. i Him. Nauk. 2 (1957), 27-35. (Russian)

A. R. AMIR-MOEZ

1. *Extreme properties of eigenvalues of a hermitian transformation and singular values of the sum and product of linear transformations*, Duke Math. J. 23 (1956), 463-476. MR 18, 105.

A. R. AMIR-MOEZ AND A. HORN

1. *Singular values of a matrix*, Amer. Math. Monthly 65 (1958), 742-478. MR 20 # 7037.

N. ARONSZAJN AND K. T. SMITH

1. *Invariant subspaces of completely continuous operators*, Ann. of Math. (2) 60 (1954) 345-350; Russian transl., Matematika 2 (1958), no. 1, 97-102. MR 16, 488.

N. G. ASKEROV, S. G. KREIN AND G. I. LAPTEV

1. *On a class of non-selfadjoint boundary-value problems*, Dokl. Akad. Nauk SSSR 155 (1964), 499-502 = Soviet Math. Dokl. 5 (1964), 424-427. MR 28 # 3347.

K. I. BABENKO

1. *On conjugate functions*, Dokl. Akad. Nauk SSSR 62 (1948), 157-160. (Russian) MR 10, 249.

S. BANACH

1. *Théorie des opérations linéaires*, Monogr. Mat., PWN, Warsaw, 1932; reprint, Chelsea, New York, 1955; Ukrainian transl., Kiev, 1948. MR 17, 175.

N. K. BARI

1. *Sur les bases dans l'espace de Hilbert*, Dokl. Akad. Nauk SSSR 54 (1946), 379-382. MR 8, 513.
2. *Biorthogonal systems and bases in Hilbert space*, Moskov. Gos. Univ. Učen. Zap. 148, Matematika 4 (1951), 69-107. (Russian) MR 14, 289.
3. *Trigonometric series*, Fizmatgiz, Moscow, 1961; English transl., *A treatise on trigonometric series*. Vols. I, II, Macmillan, New York, 1964. MR 23 # A3411; MR 30, 1347.

H. BATEMAN

1. *A formula for the solving function of a certain integral equation of the second kind*, Messengers of Mathematics 37 (1908), 179-187.

- R. BELLMAN
 1. *Multiplicative inequalities obtained from additive inequalities. Notes on matrix theory.* XV, Amer. Math. Monthly **65** (1958), 693-694. MR **20** # 5214.
- E. BISHOP
 1. *A duality theorem for an arbitrary operator*, Pacific J. Math. **9** (1959), 379-397. MR **22** # 8339.
- R. P. BOAS, JR.
 1. *A general moment problem*, Amer. J. Math. **63** (1941), 361-370. MR **2**, 281.
- M. S. BRODSKII
 1. *Characteristic matrix functions of linear operators*, Mat. Sb. **39** (81) (1956), 179-200. (Russian) MR **18**, 220.
 2. *On a problem of I. M. Gel'fand*, Uspehi Mat. Nauk **12** (1957), no. 2 (74), 129-132. (Russian) MR **20** # 1229.
 3. *Integral representations of bounded non-selfadjoint operators with a real spectrum*, Dokl. Akad. Nauk SSSR **126** (1959), 1166-1169. (Russian) MR **21** # 7438
 4. *Triangular representation of some operators with completely continuous imaginary part*, Dokl. Akad. Nauk SSSR **133** (1960), 1271-1274 = Soviet Math. Dokl. **1** (1960), 952-955. MR **26** # 6778.
 5. *On the triangular representation of completely continuous operators with one-point spectra*, Uspehi Mat. Nauk **16** (1961), no. 1 (97), 135-141. (Russian) MR **24** # A426.
- M. S. BRODSKII, I. C. GOHBERG, M. G. KREIN AND V. I. MACAEV
 1. *Some investigations in the theory of non-selfadjoint operators*, Proc. Fourth All-Union Math. Congress. Vol. II: Sectional Lectures, "Nauka", Leningrad, 1964, pp. 261-271; English transl., Amer. Math. Soc. Transl. (2) **65** (1967), 237-251.
- M. S. BRODSKII AND M. S. LIVŠIĆ
 1. *Spectral analysis of non-selfadjoint operators and intermediate systems*, Uspehi Mat. Nauk **13** (1958), no. 1 (79), 3-85; English transl., Amer. Math. Soc. Transl. (2) **13** (1960), 265-346. MR **20** # 7221; MR **22** # 3982.
- A. P. CALDERÓN AND A. ZYGMUND
 1. *A note on the interpolation of sublinear operations*, Amer. J. Math. **78** (1956), 282-288. MR **18**, 586.
- J. W. CALKIN
 1. *Two-sided ideals and congruences in the ring of bounded operators in Hilbert space*, Ann. of Math. (2) **42** (1941), 839-873. MR **3**, 208.
- T. CARLEMAN
 1. *Über die Fourierkoeffizienten einer stetigen Funktion*, Acta Math. **41** (1918), 377-384.
 2. *Zur Theorie der linearen Integralgleichungen*, Math. Z. **9** (1921), 196-217.
- M. CARTWRIGHT
 1. *On certain integral functions of order one*, Quart. J. Math. **7** (1936), 46-55.
- S. H. CHANG
 1. *On the distribution of the characteristic values and singular values of linear integral equations*, Trans. Amer. Math. Soc. **67** (1949), 351-367. MR **11**, 523.
- J. DIXMIER
 1. *Les algèbres d'opérateurs dans l'espace hilbertien. Algèbres de von Neumann*, Cahiers Scientifiques, Fasc. XXV, Gauthier-Villars, Paris, 1957. MR **20** # 1234.
- M. D. DOL'BERG
 1. *The development of a positive kernel in a bilinear series*, Dokl. Akad. Nauk SSSR **120** (1958), 945-948. (Russian) MR **24** # A400.
 2. *On links of greatest rigidity*, Har'kov. Gos. Univ. Učen. Zap. (4) **25** (1957), 179-190. (Russian)
- C. L. DOLPH
 1. *Recent developments in some non-self-adjoint problems of mathematical physics*, Bull. Amer. Math. Soc. **67** (1961), 1-69. MR **25** # 5612.

- R. DUFFIN
1. *A minimax theory for overdamped networks*, J. Rational Mech. Anal. 4 (1955), 221-233. MR 16, 979.
- N. DUNFORD
1. *A survey of the theory of spectral operators*, Bull. Amer. Math. Soc. 64 (1958), 217-274. MR 21 # 3616.
- N. DUNFORD AND J. T. SCHWARTZ
1. *Linear operators*. I: *General theory*, Pure and Appl. Math., vol. 7, Interscience, New York and London, 1958; Russian transl., IL, Moscow, 1962. MR 22 # 8302.
2. *Linear operators*. II: *Spectral theory. Self adjoint operators in Hilbert space*, Interscience, New York and London, 1963. MR 32 # 6181.
- M. K. FACE
1. *The rectification of bases in Hilbert space*, Dokl. Akad. Nauk SSSR 74 (1950), 1053-1056. (Russian) MR 14, 184.
- K. FAN
1. *On a theorem of Weyl concerning eigenvalues of linear transformations*. I, Proc. Nat. Acad. Sci. U.S.A. 35 (1949), 652-655. MR 11, 600.
2. *ibid.*, II, Proc. Nat. Acad. Sci. U.S.A. 36 (1950), 31-35. MR 11, 526.
3. *Maximum properties and inequalities for the eigenvalues of completely continuous operators*, Proc. Nat. Acad. Sci. U.S.A. 37 (1951), 760-766. MR 13, 661.
4. *A minimum property of the eigenvalues of a Hermitian transformation. Eigenvalues of a sum of Hermitian matrices*, Amer. Math. Monthly 60 (1953), 48-50.
- I. FREDHOLM
1. *Sur une classe d'équations fonctionnelles*, Acta Math. 27 (1903), 365-390.
- F. R. GANTMAHER
1. *Theory of matrices*, GITTL, Moscow, 1953; 2nd ed., "Nauka", Moscow, 1966; English transl., Vols. 1, 2, Chelsea, New York, 1959. MR 16, 438; MR 21 # 6372c.
- F. R. GANTMAHER AND M. G. KREIN
1. *Oscillation matrices and kernels and small oscillations of mechanical systems*, 2nd ed., GITTL, Moscow, 1950; German transl., Akademie-Verlag, Berlin, 1960. MR 14, 178; MR 22 # 5161.
- V. F. GAPOŠKIN
1. *A generalization of the theorem of M. Riesz on conjugate functions*, Mat. Sb. 46 (88) (1958), 359-372. (Russian) MR 20 # 6000.
- I. M. GEL'FAND
1. *Remark on the work of N. K. Bari "Biorthogonal systems and bases in Hilbert space"*, Moskov. Gos. Univ. Učen. Zap. 148, Matematika 4 (1951), 224-225. (Russian) MR 14, 289.
- I. M. GEL'FAND AND N. JA. VILENKIN
1. *Generalized functions*. Vol. 4: *Applications of harmonic analysis*, Fizmatgiz, Moscow, 1961; English transl., Academic Press, New York, 1964. MR 26 # 4173; MR 30 # 4152.
- A. O. GEL'FOND
1. *On the growth of the eigenvalues of homogeneous equations*; Russian suppl. to translation of *Linear integral equations* by W. V. Lovitt, McGraw-Hill, New York, 1924; reprint, Dover, New York, 1950; Russian transl., GITTL, Moscow, 1957.
2. *Sur l'ordre de $D(\lambda)$* , C. R. Acad. Sci. Paris 192 (1931), 828.
- JU. P. GINZBURG
1. *On J -contractive operator-functions*, Dokl. Akad. Nauk SSSR 117 (1957), 171-173. (Russian) MR 20 # 1203.
- I. M. GLAZMAN
1. *On expansibility in a system of eigenelements of dissipative operators*, Uspehi Mat. Nauk 13 (1958), no. 3 (81), 179-181. (Russian) MR 20 # 4193.
- I. C. GOHBERG
1. *On linear operators depending analytically upon a parameter*, Dokl. Akad. Nauk SSSR 78 (1951), 629-632. (Russian) MR 13, 46.

I. C. GOHBERG AND M. G. KREIN

1. *The basic propositions on defect numbers, root numbers and indices of linear operators*, Uspehi Mat. Nauk 12 (1957), no. 2 (74), 43-118; English transl., Amer. Math. Soc. Transl. (2) 13 (1960), 185-264. MR 20 # 3459; MR 22 # 3984.
2. *Completely continuous operators with a spectrum concentrated at zero*, Dokl. Akad. Nauk SSSR 128 (1959), 227-230. (Russian) MR 24 # A1022.
3. *On the theory of triangular representations of nonselfadjoint operators*, Dokl. Akad. Nauk SSSR 137 (1961), 1034-1038 = Soviet Math. Dokl. 2 (1961), 392-396. MR 25 # 3370.
4. *Volterra operators with imaginary component in one class or another*, Dokl. Akad. Nauk SSSR 139 (1961), 779-781 = Soviet Math. Dokl. 2 (1961), 983-985. MR 25 # 3371.
5. *On the problem of factoring operators in a Hilbert space*, Dokl. Akad. Nauk SSSR 147 (1962), 279-282 = Soviet Math. Dokl. 3 (1962), 1578-1582. MR 26 # 6777.
6. *Criteria for completeness of the system of root vectors of a contraction*, Ukrain. Mat. Z. 16 (1964), 78-82; English transl., Amer. Math. Soc. Transl., (2) 54 (1966), 119-124. MR 29 # 2651.
7. *Theory and applications of Volterra operators in Hilbert space*, "Nauka", Moscow, 1967; English transl., Transl. Math. Monographs, vol. 24, Amer. Math. Soc., Providence, R. I. (to appear).
8. *On the factorization of operators in Hilbert space*, Acta Sci. Math. Szeged 25 (1964), 90-123; English transl., Amer. Math. Soc. Transl. (2) 51 (1966), 155-188. MR 29 # 6313.

I. C. GOHBERG AND A. S. MARKUS

1. *On the stability of bases in Banach and Hilbert spaces*, Izv. Moldavsk. Fil. Akad. Nauk SSSR 1962, no. 5, 17-35. (Russian)
2. *On some inequalities between the eigenvalues and matrix elements of linear operators*, Izv. Moldavsk. Fil. Akad. Nauk SSSR 1962, no. 5, 103-108. (Russian)
3. *Some relations between eigenvalues and matrix elements of linear operators*, Mat. Sb. 64 (106) (1964), 481-496; English transl., Amer. Math. Soc. Transl. (2) 52 (1966), 201-216. MR 30 # 457.

S. H. GOULD

1. *Variational methods for eigenvalue problems*, 2nd ed., Mathematical Expositions, no. 10, Univ. of Toronto Press, Toronto and Oxford Univ. Press, London, 1966. MR 35 # 559.

H. L. HAMBURGER

1. *Über die Zerlegung des Hilbertschen Raumes durch vollstetige lineare Transformationen*, Math. Nachr. 4 (1951), 56-69. MR 12, 718.

G. H. HARDY

1. *Divergent series*, Clarendon Press, Oxford, 1949; Russian transl., IL, Moscow, 1951. MR 11, 25; MR 16, 690.

G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA

1. *Inequalities*, Cambridge Univ. Press, New York, 1934; 2nd ed., 1952; Russian transl., IL, Moscow, 1948. MR 13, 727; MR 18, 722.

W. K. HAYMAN

1. *Questions of regularity connected with the Phragmén-Lindelöf principle*, J. Math. Pures Appl. (9) 35 (1956), 115-126. MR 17, 1073.

E. HILLE AND J. D. TAMARKIN

1. *On the characteristic values of linear integral equations*, Acta Math. 57 (1931), 1-76.

A. HORN

1. *On the singular values of a product of completely continuous operators*, Proc. Nat. Acad. Sci. U.S.A. 36 (1950), 374-375. MR 13, 565.
2. *On the eigenvalues of a matrix with prescribed singular values*, Proc. Amer. Math. Soc. 5 (1954), 4-7. MR 15, 847.
3. *Doubly stochastic matrices and the diagonal of a rotation matrix*, Amer. J. Math. 76 (1954), 620-630. MR 16, 105.

- I. S. IOHVIDOV
 1. *On the spectra of Hermitian and unitary operators in a space with indefinite metric*, Dokl. Akad. Nauk SSSR 71 (1950), 225-228. (Russian) MR 12, 33.
- I. S. IOHVIDOV AND M. G. KREIN
 1. *Spectral theory of operators in spaces with an indefinite metric*. I, Trudy Moskov. Mat. Obsč. 5 (1956), 367-432; English transl., Amer. Math. Soc. Transl. (2) 13 (1960), 105-175. MR 18, 320; MR 22 # 3983.
- T. KATO
 1. *Fractional powers of dissipative operators*, J. Math. Soc. Japan 13 (1961), 246-274. MR 25 # 1453.
- M. V. KELDYŠ
 1. *On the characteristic values and characteristic functions of certain classes of non-self-adjoint equations*, Dokl. Akad. Nauk SSSR 77 (1951), 11-14. (Russian) MR 12, 835.
 2. *On a Tauberian theorem*, Trudy Mat. Inst. Steklov. 38 (1951), 77-86. (Russian) MR 13, 738.
- M. V. KELDYŠ AND V. B. LIDSKII
 1. *On the spectral theory of non-selfadjoint operators*, Proc. Fourth All-Union Math. Congress (Leningrad, 1961), Vol. I, Izdat. Akad. Nauk SSSR, Leningrad, 1963, pp. 101-120. (Russian) MR 30 # 1414.
- H. VON KOCH
 1. *Sur quelques points de la théorie des déterminants infinis*, Acta Math. 24 (1900), 89-122.
 2. *Sur la convergence des déterminants infinis*, Rend. Circ. Mat. Palermo 28 (1909), 255-266.
- A. N. KOLMOGOROV
 1. *Über die beste Annäherung von Funktionen einer gegebenen Funktionenklasse*, Ann. of Math. 37 (1936), 107-110.
- B. I. KORENBLIUM
 1. *A general Tauberian theorem for the ratio of functions*, Dokl. Akad. Nauk SSSR 88 (1953), 745-748. (Russian) MR 14, 866.
- M. G. KREIN
 1. *On the characteristic numbers of differentiable symmetric kernels*, Mat. Sb. 2 (1937), 725-732. (Russian)
 2. *On "loaded" integral equations the distribution functions of which are not monotonic*, Memorial volume to D. A. Grave, Moscow, 1940, pp. 88-103. (Russian) MR 2, 312.
 3. *A contribution to the theory of entire functions of exponential type*, Izv. Akad. Nauk SSSR Ser. Mat. 11 (1947), 309-326. (Russian) MR 9, 179.
 4. *On inverse problems for a nonhomogeneous cord*, Dokl. Akad. Nauk SSSR 82 (1952), 669-672. (Russian) MR 14, 649.
 5. *On a generalization of investigations of Stieltjes*, Dokl. Akad. Nauk SSSR 87 (1952), 881-884. (Russian) MR 14, 868.
 6. *On the trace formula in perturbation theory*, Mat. Sb. 33 (75) (1953), 597-626. (Russian) MR 15, 720.
 7. *On Bari bases of a Hilbert space*, Uspehi Mat. Nauk 12 (1957), no. 3 (75), 333-341. (Russian) MR 19, 1184.
 8. *Criteria for completeness of the system of root vectors of a dissipative operator*, Uspehi Mat. Nauk 14 (1959), no. 3 (87), 145-152; English transl., Amer. Math. Soc. Transl. (2) 26 (1963), 221-229. MR 22 # 9856; MR 27 # 1852.
 9. *A contribution to the theory of linear non-selfadjoint operators*, Dokl. Akad. Nauk SSSR 130 (1960), 254-256 = Soviet Math. Dokl. 1 (1960), 38-40. MR 24 # A1024.
 10. *On linear completely continuous operators in functional spaces with two norms*, Akad. Nauk Ukrain. RSR. Zbirnik Prac' Inst. Mat. 1947, no. 9, 104-129. (Ukrainian)
 11. *A new application of the fixed-point principle in the theory of operators in a space with indefinite metric*, Dokl. Akad. Nauk SSSR 154 (1964), 1023-1026 = Soviet Math. Dokl. 5 (1964), 224-228. MR 29 # 6314.

12. *On perturbation determinants and a trace formula for unitary and selfadjoint operators*, Dokl. Akad. Nauk SSSR 144 (1962), 268-271 = Soviet Math. Dokl. 3 (1962), 707-710. MR 25 # 2446.
 13. *On the theory of weighted integral equations*. Bul. Akad. Štiințe RSS Moldoven. 1965, no. 7, 40-46. (Russian) MR 33 # 6313.
- M. G. KREIN, M. A. KRASNOSEL'SKII AND D. P. MIL'MAN
1. *On the defect numbers of linear operators in a Banach space and on some geometric questions*, Akad. Nauk Ukrain RSR. Zbirnik Prac' Inst. Mat. 1948, no. 11, 97-112. (Ukrainian)
- M. G. KREIN AND H. K. LANGER
1. *On the theory of quadratic pencils of self-adjoint operators*, Dokl. Akad. Nauk SSSR 154 (1964). 1258-1261 = Soviet Math. Dokl. 5 (1964), 266-269. MR 29 # 6315.
 2. *On some mathematical principles of the linear theory of damped oscillations of continua*, Trudy Internat. Sympos. on Applications of the Theory of Functions of a Complex Variable in the Mechanics of Continuous Media, "Nauka", Moscow, 1965. (Russian)
- M. G. KREIN AND M. A. RUTMAN
1. *Linear operators leaving invariant a cone in a Banach space*, Uspehi Mat. Nauk 3 (1948), no. 1 (23), 3-95; English transl., Amer. Math. Soc. Transl. (1) 10 (1962), 199-325. MR 10, 256; MR 12, 341.
- S. G. KREIN
1. *Oscillations of a viscous fluid in a container*, Dokl. Akad. Nauk SSSR 159 (1964), 262-265 = Soviet Math. Dokl. 5 (1964), 1467-1471. MR 31 # 6461.
- S. T. KURODA
1. *On a theorem of Weyl-von Neumann*, Proc. Japan Acad. 34 (1958), 11-15. MR 21 # 1537.
 2. *On a generalization of the Weinstein-Aronszajn formula and the infinite determinant*, Sci. Papers Coll. Gen. Ed. Univ. Tokyo 11 (1961), 1-12. MR 25 # 1456.
- H. LANGER
1. *Ein Zerspaltungssatz für Operatoren im Hilbertraum*, Acta Math. Acad. Sci. Hungar. 12 (1961), 441-445. MR 25 # 3378.
 2. *Über die Wurzeln eines maximalen dissipativen Operators*, Acta Math. Acad. Sci. Hungar. 13 (1962), 415-424. MR 26 # 1757.
 3. *On J -Hermitian operators*, Dokl. Akad. Nauk SSSR 134 (1960), 263-266 = Soviet Math. Dokl. 1 (1960), 1052-1055. MR 25 # 1457.
- B. JA. LEVIN
1. *Distribution of zeros of entire functions*, GITTL, Moscow, 1956; English transl., Transl. Math. Monographs, vol. 5, Amer. Math. Soc., Providence, R. I., 1964. MR 19, 402; MR 28 # 217.
 2. *On completely continuous non-selfadjoint operators*, Sb. Trudy Har'kov. Inst. Inž. ž.-d. Transport. im. S. M. Kirov, no. 35 (1959), 5-23. (Russian)
- V. B. LIDSKII
1. *On the eigenvalues of sums and products of symmetric matrices*, Dokl. Akad. Nauk SSSR 75 (1950), 769-772. (Russian)
 2. *On the completeness of the system of eigenfunctions and associated functions of a non-selfadjoint differential operator*, Dokl. Akad. Nauk SSSR 110 (1956), 172-175. (Russian)
 3. *On the completeness of the system of eigenelements and associated elements of a completely continuous operator*, Dokl. Akad. Nauk SSSR 113 (1957), 234-236. (Russian)
 4. *Theorems on the completeness of the system of characteristic and adjoined elements of operators having a discrete spectrum*, Dokl. Akad. Nauk SSSR 119 (1958), 1088-1091; English transl., Amer. Math. Soc. Transl. (2) 47 (1965), 37-41. MR 20 # 5430.
 5. *Conditions for completeness of the system of root subspaces of non-selfadjoint operators with discrete spectra*, Trudy Moskov. Mat. Obšč. 8 (1959), 83-120; English transl., Amer. Math. Soc. Transl. (2) 34 (1963), 241-281. MR 21 # 6539.
 6. *Non-selfadjoint operators with a trace*, Dokl. Akad. Nauk SSSR 125 (1959), 485-587; English transl., Amer. Math. Soc. Transl. (2) 47 (1965), 43-46. MR 21 # 3769.

7. *Summation of series over principal vectors of non-selfadjoint operators*, Dokl. Akad. Nauk SSSR 132 (1960), 275-278 = Soviet Math. Dokl. 1 (1960), 540-543. MR 22 # 9862.
 8. *The Fourier series expansion in terms of the principal functions of a non-selfadjoint elliptic operator*, Mat. Sb. 57 (99) (1962), 137-150. (Russian) MR 26 # 460.
 9. *Summability of series in terms of the principal vectors of non-selfadjoint operators*, Trudy Moskov. Mat. Obsč. 11 (1962), 3-35; English transl., Amer. Math. Soc. Transl. (2) 40 (1964), 193-228. MR 26 # 1760.
- M. S. LIVŠIC
1. *On a certain class of operators in a Hilbert space*, Mat. Sb. 19 (61) (1946), 239-262; English transl., Amer. Math. Soc. Transl. (2) 13 (1960), 61-83. MR 8, 588; MR 22 # 3981a.
 2. *On spectral resolution of linear non-selfadjoint operators*, Mat. Sb. 34 (76) (1954), 145-199; English transl., Amer. Math. Soc. Transl. (2) 5 (1957), 67-114. MR 16, 48; MR 18, 748.
 3. *Theory of non-selfadjoint operators and its applications*, Trudy Third All-Union Math. Congress, vol. 3, 1956, pp. 269-276. (Russian)
- V. È. LIANCE
1. *Some properties of idempotent operators*, Teor. i Prikl. Mat. L'vov, 1 (1959), 16-22. (Russian)
- JU. I. LJUBIČ
1. *Almost periodic functions in the spectral analysis of operators*, Dokl. Akad. Nauk SSSR 132 (1960), 518-520 = Soviet Math. Dokl. 1 (1960), 593-595.
 2. *Completeness conditions for the system of eigenvectors of a correct operator*, Uspehi Mat. Nauk 18 (1963), no. 1 (109), 165-171. (Russian) MR 27 # 4081.
- JU. I. LJUBIČ AND V. I. MACAEV
1. *To the spectral theory of linear operators in a Banach space*, Dokl. Akad. Nauk SSSR 131 (1960), 21-23 = Soviet Math. Dokl. 1 (1960), 184-186.
 2. *Operators with separable spectrum*, Mat. Sb. 56 (98) (1962), 433-468; English transl., Amer. Math. Soc. Transl. (2) 47 (1965), 89-129. MR 25 # 2450.
- L. A. LJUSTERNIK AND V. I. SOBOLEV
1. *Elements of functional analysis*, GITTL, Moscow, 1951; 2nd rev. ed., "Nauka", Moscow, 1965; English transl., Hindustan, Delhi and Gordon & Breach and Ungar, New York, 1961. MR 14, 54; MR 25 # 5361; MR 25 # 5362.
- E. R. LORCH
1. *Bicontinuous linear transformations in certain vector spaces*, Bull. Amer. Math. Soc. 45 (1939), 564-569. MR 1, 58.
- V. I. MACAEV
1. *On the growth of entire functions that admit a certain estimate from below*, Dokl. Akad. Nauk SSSR 132 (1960), 283-286 = Soviet Math. Dokl. 1 (1960), 548-552. MR 23 # A328.
 2. *A class of completely continuous operators*, Dokl. Akad. Nauk SSSR 139 (1961), 548-551 = Soviet Math. Dokl. 2 (1961), 972-975. MR 24 # A1617.
 3. *Volterra operators obtained from self-adjoint operators by perturbation*, Dokl. Akad. Nauk SSSR 139 (1961), 810-813 = Soviet Math. Dokl. 2 (1961), 1013-1016. MR 25 # 457.
 4. *A method of estimation for the resolvents of non-selfadjoint operators*, Dokl. Akad. Nauk SSSR 154 (1964), 1034-1037 = Soviet Math. Dokl. 5 (1964), 236-241. MR 28 # 1495.
 5. *Several theorems on completeness of root subspaces of completely continuous operators*, Dokl. Akad. Nauk SSSR 155 (1964), 273-276 = Soviet Math. Dokl. 5 (1964), 396-399. MR 28 # 2444.
- V. I. MACAEV AND JU. A. PALANT
1. *On the powers of a bounded dissipative operator*, Ukrain. Mat. Ž. 14 (1962), 329-337. (Russian) MR 26 # 4184.

- G. W. MACKEY
 1. *Commutative Banach algebras*, Multigraphed Lecture Notes, Harvard, 1952.
- A. I. MAL'CEV
 1. *Foundations of linear algebra*, OGIZ, Moscow, 1948, 1956; English transl., Freeman, San Francisco, Calif., 1963. MR 11, 412; MR 29 # 3477.
- A. S. MARKUS
 1. *On holomorphic operator-functions*, Dokl. Akad. Nauk SSSR 119 (1958), 1099-1102. (Russian) MR 21 # 3772.
 2. *A basis of root vectors of a dissipative operator*, Dokl. Akad. Nauk SSSR 132 (1960), 524-527 = Soviet Math. Dokl. 1 (1960), 599-602. MR 22 # 9846.
 3. *The root vector expansion of a weakly perturbed self-adjoint operator*, Dokl. Akad. Nauk SSSR 142 (1962), 538-541 = Soviet Math. Dokl. 3 (1962), 104-108. MR 27 # 1837.
 4. *Characteristic numbers and singular numbers of the sum and product of linear operators*, Dokl. Akad. Nauk SSSR 146 (1962), 34-36 = Soviet Math. Dokl. 3 (1962), 1238-1240. MR 25 # 5395.
 5. *Some tests for the completeness of the system of root vectors of a linear operator and the summability of series in this system*, Dokl. Akad. Nauk SSSR 155 (1964), 753-756 = Soviet Math. Dokl. 5 (1964), 505-509. MR 28 # 5345.
 6. *Eigenvalues and singular values of the sum and product of linear operators*, Uspehi Mat. Nauk 19 (1964), no. 4 (118), 93-123 = Russian Math. Surveys 19 (1964), no. 4, 91-120. MR 29 # 6318.
- S. MAZURKIEWICZ
 1. *Sur le déterminant de Fredholm. II: Les noyaux dérivables*, C. R. Séances Soc. Sci. Varsovie 8 (1915), 805-810.
- L. MIRSKY
 1. *Matrices with prescribed characteristic roots and diagonal elements*, J. London Math. Soc. 33 (1958), 14-21. MR 19, 1034.
 2. *Remarks on an existence theorem in matrix theory due to A. Horn*, Monatsh. Math. 63 (1959), 241-243. MR 21 # 2662.
- B. S. MITJAGIN
 1. *Normed ideals of intermediate type*, Izv. Akad. Nauk SSSR Ser. Mat. 28 (1964), 819-832; English transl., Amer. Math. Soc. Transl. (2) 63 (1967), 180-194. MR 30 # 4142.
- B. S. MITJAGIN AND A. S. ŠVARC
 1. *Functors in categories of Banach spaces*, Uspehi Mat. Nauk 19 (1964), no. 2 (116), 65-130 = Russian Math. Surveys 19 (1964), no. 2, 65-127. (Russian) MR 29 # 3866.
- B. R. MUKMINOV
 1. *On expansion with respect to the eigenfunctions of dissipative kernels*, Dokl. Akad. Nauk SSSR 99 (1954), 499-502. (Russian) MR 16, 830.
- M. A. NAIMARK
 1. *On some criteria for completeness of the system of eigen and adjointed vectors of a linear operator in Hilbert space*, Dokl. Akad. Nauk SSSR 98 (1954), 727-730. (Russian) MR 16, 1032.
 2. *Linear differential operators*, GITTL, Moscow, 1954; German transl., Akademie-Verlag, Berlin, 1960; English transl., Ungar, New York, 1968. MR 16, 702.
 3. *Spectral analysis of non-self-adjoint operators*, Uspehi Mat. Nauk 11 (1956), no. 6 (72), 183-202; English transl., Amer. Math. Soc. Transl. (2) 20 (1962), 55-75. MR 21 # 3645; MR 25 # 464.
- J. VON NEUMANN
 1. *Some matrix inequalities and metrization of metric space*, Izv. Mat. Meh. Tomsk. Univ. 1 (1937), 286-300.
- W. ORLICZ
 1. *Über unbedingte Konvergenz in Funktionenräumen*, Studia Math. II 4 (1933), 41-47.
- A. OSTROWSKI
 1. *Sur quelques applications des fonctions convexes et concaves au sens de I. Schur*, J. Math. Pures Appl. 9 (31) (1952), 253-292. MR 14, 625.

- JU. A. PALANT
 1. *A test for the completeness of the system of eigenvectors and adjointed vectors of a polynomial bundle of operators*, Dokl. Akad. Nauk SSSR 141 (1961), 558-560 = Soviet Math. Dokl. 2 (1961), 1507-1509. MR 26 # 1737.
- V. I. PARASKA
 1. *On asymptotics of eigenvalues and singular numbers of linear operators which increase smoothness*, Mat. Sb. 68 (110) (1965), 623-631. (Russian) MR 33 # 7892.
- R. S. PHILLIPS
 1. *Dissipative operators and hyperbolic systems of partial differential equations*, Trans. Amer. Math. Soc. 90 (1959), 193-254. MR 21 # 3669.
- V. T. POLJACKII
 1. *On the reduction of quasi-unitary operators to a triangular form*, Dokl. Akad. Nauk SSSR 113 (1957), 756-759. (Russian) MR 19, 873.
- G. PÓLYA
 1. *Remark on Weyl's note "Inequalities between the two kinds of eigenvalues of a linear transformation"*, Proc. Nat. Acad. Sci. U.S.A. 36 (1950), 49-51. MR 11, 526.
- L. S. PONTRJAGIN
 1. *Hermitian operators in spaces with indefinite metric*, Izv. Akad. Nauk SSSR Ser. Mat. 8 (1944), 243-280. (Russian) MR 6, 273.
- V. P. POTAPOV
 1. *The multiplicative structure of J -contractive matrix functions*, Trudy Moskov. Mat. Obsč. 4 (1955), 125-236; English transl., Amer. Math. Soc. Transl. (2) 15 (1960), 131-243. MR 17, 958; MR 22 # 5733.
- I. I. PRIVALOV
 1. *Boundary properties of analytic functions*, GITTL, Moscow, 1950; German transl., Hochschulbücher für Math., Band 25, VEB Deutscher Verlag, Berlin, 1956. MR 13, 926; MR 18, 727.
- F. RIESZ AND B. SZ.-NAGY
 1. *Leçons d'analyse fonctionnelle*, Akad. Kiadó, Budapest, 1952; 2nd ed., 1953; Russian transl., IL, Moscow, 1954; English transl., *Functional analysis*, Ungar, New York, 1955. MR 14, 286; MR 15, 132; MR 16, 837; MR 17, 175.
- M. RIESZ
 1. *Sur les maxima des formes bilinéaires et sur les fonctionnelles linéaires*, Acta Math. 49 (1926), 465-497.
 2. *Sur les fonctions conjuguées*, Math. Z. 27 (1927), 218-244.
- L. A. SAHNOVIĆ
 1. *The reduction of non-selfadjoint operators to triangular form*, Izv. Vysš. Učebn. Zaved. Matematika 1959, no. 1 (8), 180-186. (Russian) MR 25 # 460.
 2. *A study of the "triangular form" of non-selfadjoint operators*, Izv. Vysš. Učebn. Zaved. Matematika 1959, no. 4 (11), 141-149; English transl., Amer. Math. Soc. Transl. (2) 54 (1966), 75-84. MR 25 # 461.
- R. SCHATTEN
 1. *A theory of cross-spaces*, Ann. of Math. Studies, no. 26, Princeton Univ. Press, Princeton, N. J., 1950. MR 12, 186.
 2. *Norm ideals of completely continuous operators*, Ergebnisse der Math. und ihrer Grenzgebiete, Heft 27, Springer-Verlag, Berlin, 1960. MR 22 # 9878.
- I. SCHUR
 1. *Über die charakteristischen Wurzeln einer linearen Substitution mit einer Anwendung auf die Theorie der Integralgleichungen*, Math. Ann. 66 (1909), 488-510.
- V. I. SMIRNOV
 1. *On the boundary values of functions which are regular outside a disc*, Ž. Leningrad. Fiz.-Mat. Obsč. 2 (1929), no. 2, 22-37. (Russian)
- W. F. STINESPRING
 1. *A sufficient condition for an integral operator to have a trace*, J. Reine Angew. Math. 200 (1958), 200-207. MR 20 # 5431.

- M. H. STONE
1. *Linear transformations in Hilbert space, and their applications to analysis*, Amer. Math. Soc. Colloq. Publ., vol. 15, Amer. Math. Soc., Providence, R. I., 1932; reprint, 1966.
- A. V. ŠTRAUS
1. *Characteristic functions of linear operators*, Izv. Akad. Nauk SSSR Ser. Mat. 24 (1960), 43-74; English transl., Amer. Math. Soc. Transl. (2) 40 (1964), 1-37. MR 25 # 4363.
- B. SZ.-NAGY
1. *Perturbations des transformations autoadjointes dans l'espace de Hilbert*, Comment. Math. Helv. 19 (1947), 347-366. MR 8, 589.
 2. *Perturbations des transformations linéaires fermées*, Acta Sci. Math. Szeged 14 (1951), 125-137. MR 13, 849.
- B. SZ.-NAGY AND C. FOIAŞ
1. *Sur les contractions de l'espace de Hilbert*. III, Acta Sci. Math. Szeged 19 (1958), 26-45. MR 21 # 2188.
 2. *Sur les contractions de l'espace de Hilbert*. IV, Acta Sci. Math. Szeged 21 (1960), 251-259. MR 23 # A3445.
 3. *Sur les contractions de l'espace de Hilbert*. V: *Translations bilatérales*, Acta Sci. Math. Szeged 23 (1962), 106-129. MR 26 # 2890.
 4. *Sur les contractions de l'espace de Hilbert*. VI: *Calcul fonctionnel*, Acta Sci. Math. Szeged 23 (1962), 130-167. MR 26 # 2891.
- V. M. TIHOMIROV
1. *Diameters of sets in functional spaces and the theory of best approximations*, Uspehi Mat. Nauk 15 (1960), no. 3 (93), 81-120 = Russian Math. Surveys 15 (1960), no. 3, 75-111. MR 22 # 8268.
- E. C. TITCHMARSH
1. *Theory of functions*, Oxford Univ. Press, Oxford, 1939; Russian transl., GITTL, Moscow, 1951.
- B. E. VEIC
1. *On some properties of unconditional convergence bases*, Uspehi Mat. Nauk 17 (1962), no. 6 (108), 135-142. (Russian) MR 26 # 4149.
- J. WERMER
1. *The existence of invariant subspaces*, Duke Math. J. 19 (1952), 615-622. MR 14, 384.
- H. WEYL
1. *Das asymptotische Verteilungsgesetz der Eigenwerte linearer partieller Differentialgleichungen*, Math. Ann. 71 (1912), 441-479.
 2. *Inequalities between the two kinds of eigenvalues of a linear transformation*, Proc. Nat. Acad. Sci. U.S.A. 35 (1949), 408-411. MR 11, 37.
- H. WIELANDT
1. *An extremum property of sums of eigenvalues*, Proc. Amer. Math. Soc. 6 (1955), 106-110. MR 16, 785.
- F. WOLF
1. *Operators in Banach space which admit a generalized spectral decomposition*, Nederl. Akad. Wetensch. Proc. Ser. A. 60 = Indag. Math. 19 (1957), 302-311. MR 19, 869.

This page intentionally left blank

SUBJECT INDEX

- A-completely continuous operator, 275
- Abel's summation method, 248
- Absolute convergence of the series of eigenvalues of the product of two Hilbert-Schmidt operators, 94
- Absolute multiplicity of a characteristic number of a bundle, 262
- Adjoint cone, 152-153
- Adjointness of each space in the triple $\mathfrak{S}_{11}^{(0)}$, \mathfrak{S}_* , \mathfrak{S}_{11} to the preceding one, 149
- Adjointness of the functions ϕ_{11} and ϕ_* , 148
- Ahiezer, N. I., 182
- Algebraic multiplicity of an eigenvalue of an operator, 5
- Allahverdiev, D. È., 28, 244, 252, 269, 275
- Almost normed sequences of vectors, 308
- Analytic description of Hilbert-Schmidt operators, 108ff
- Analytic tests for the nuclearity of integral operators, 112ff
- Angle between subspaces, minimal, 339
- Approximation property of the s -numbers of
 - a bounded operator, 61
 - a completely continuous operator, 28-29
- Aronszajn, N., viii, 197
- Askerov, H. G., 304
- Associated vector of an eigenvector of a bundle, 266
- Asymptotic behavior of
 - the difference $N(r; A_{\mathcal{Q}}) - N(r; A)$ for a completely continuous operator A with nuclear imaginary component, 200
 - the number of characteristic numbers of the real component of a Volterra operator with nuclear imaginary component (dissipative operator), 184, (arbitrary operator), 192
 - the s -numbers of a dissipative Volterra operator with nuclear imaginary component, 185
- Asymptotic estimate for the s -numbers of an operator from the ideal \mathfrak{S}_p , 95
- Asymptotic theorem of Ky Fan on s -numbers, 32

- Babenko, K. I., 319
- Banach, S., 306, 307, 333
- Bari basis, see Basis, Bari
- Bari, N. K., 303, 309-311, 317, 319
- Basis
 - Bari, 303, 309 (def.)
 - tests for, 320-324
 - equivalent to an orthonormal one, 309
 - of a Hilbert space, 306
 - permutable, 314
 - quadratically close to an orthogonal one, 319
 - Riesz, 264, 265, 302, 304, 309
 - Bari's theorem on, 310-311
 - Glazman's theorem on, 328
 - Lorch's theorem on, 315

- Schur, 16 (cf. Schur system)
of subspaces, 332
 connection with vector bases, 344, 345
 equivalent to an orthogonal one, 334
 Gel'fand's Theorem, 335
 orthogonal, 334
 rectifiable, 334
 unconditional, 314
- Bendixson's Theorem, 176
- Bernstein's Theorem, 118
- Binet-Cauchy Theorem, 33
- Binormalizing s.n. function, 88
- Binormalizing sequence, 141
 regular, 143
- Birkhoff, G. D., vii
- Bishop, E., ix
- Boas, R. P., 311
- Bound
 $\|sp A\| \leq \|A\|_1$ for a nuclear operator, 104
 $\|A^r\|_p \leq (\|A\|_p)^r$, 93
 $\sum_{j=1}^{\infty} |(A\phi_j, \phi_j)|^p \leq (\|A\|_p)^p$ for an operator A from \mathfrak{S}_p , 94
on the distance of an invertible operator A from the set of unitary operators, 323
on the growth of the Fredholm resolvent of a nuclear operator, 242-243
on the growth of the resolvent close to the set of values of the quadratic form (Af, f) , 246
on the $\|\cdot\|_1$ -norm of a diagonal-cell operator, 105-106
on the $\|\cdot\|_p$ -norm of a diagonal-cell operator, 94
on the $\|\cdot\|_q$ -norm of the weak limit of a sequence of completely continuous operators, 86
on the rate of growth of the perturbation determinant of a dissipative operator in a sectorial region about the imaginary axis, 176, 177
on the sum of the moduli of the imaginary parts of the eigenvalues of an operator with completely continuous imaginary component, 57
for the sum of the powers of the s -numbers of the powers of an operator, 50
for the sum of the p th powers of the moduli of the imaginary parts of the eigenvalues of an operator with imaginary component from \mathfrak{S}_p , 94
for the sum of the p th powers of the moduli of the eigenvalues
 in terms of the norm $\|A\|_p$, 93
 in terms of the sum of the p th powers of the s -numbers, 41
for the sum of the s -numbers of the weak limit of a sequence of completely continuous operators, 83-84
for the sum of values of a convex function at the moduli of the imaginary parts of the eigenvalues of an operator with completely continuous imaginary component, 58
on an s.n. function on the sequence of moduli of the eigenvalues of an operator A from the ideal \mathfrak{S}_ϕ , 83
on an s.n. function on the sequence of moduli of the imaginary parts of the eigenvalues of an operator A with imaginary component from the ideal \mathfrak{S}_ϕ , 83
- Brodskii, M. S., viii, ix, 149, 155, 180, 187
- Bundle,
 operator, 265
 eigenvector of, 265
 characteristic number of, 265
 quadratic, 291
 regular point of, 293-294

- spectrum of, 294
- strongly damped, 301-302
- weakly damped, 300-301

- Calkin's Theorem, 66
- Carleman, T., vii, ix, 171
- Cartwright, M., 189, 203, 211
- Cauchy, A. L., vii
- Cayley transform, 231
- Chain of orthoprojectors, 197
 - maximal, 197
 - eigen-, 197
- Characteristic determinant
 - $\tilde{D}_A(\mu)$, 168
 - bounds for the modulus of, 168
 - rules for calculation of, 169
 - $D_A^{(p)}(\mu) = \det^{(p)}(I - \mu A)$ (regularized), 167
 - $D_A(\mu) = \det(I - \mu A)$ of an operator $A \in \mathfrak{S}_1$, 157
 - property of being an entire function of genus zero, 157
- Characteristic number of an operator, 265
 - of finite algebraic multiplicity, 266
- Characteristic operator-function of a dissipative operator, 180
- Characteristic regularized determinant of the real component of a dissipative Volterra operator, 183
- Characterization of the operators from \mathfrak{S}_{II} and $\mathfrak{S}_{II}^{(0)}$ for a regular sequence II, 143
- Classes of functions (\mathfrak{A}_+) and (\mathfrak{A}_-) , 188, 204ff
- Classes of operators \mathfrak{S}_p ($0 < p < 1$), 65
- Closed linear hull \mathfrak{E}_A of all root subspaces, 16
- Closedness of the set of Volterra operators, 17
- Complete normal operator, 252
- Condition for the invariance, with respect to a given operator A , of the subspace of values of a projector, 1-2
- Condition for the real component of a Volterra operator with nuclear imaginary component to belong to the ideal \mathfrak{S}_{II} , 192
- Cone in a Banach space, 152
 - adjoint, 152-153
 - pointed, 152
 - reproducing, 152
- Cone
 - $\hat{\mathfrak{K}}$, 74
 - $\hat{\mathfrak{K}}_n$, 147
 - \mathcal{H} of nonnegative selfadjoint operators from the s.n. ideal \mathfrak{S} , 153
 - \mathcal{H}^* adjoint to \mathcal{H} , 154
- Conical norms
 - $\|\cdot\|_{\mathfrak{K}; p}$, 153 $\|\cdot\|_{\mathcal{H}; p}$, 154 $\|\cdot\|_{\mathfrak{S}; \mathcal{H}}$ and $\|\cdot\|_{II; \mathcal{H}}$, 155
- Conjecture of Krein-Macaev, 221
- Construction of regular binormalizing sequences with the help of slowly varying functions, 144
- Continuous dependence upon the operator A from \mathfrak{S}_∞
 - of the eigenvalues, 18
 - of the s -numbers, 30

- Contraction, 178, 231
 - simple, 235
 - spectrum of, 234
- Convergence of the series $\sum_{n=1}^{\infty} n^{2l} s_n^2(A)$ for an operator having an l th derivative, 121-122
- Convergence of the series $\sum_{n=1}^{\infty} n^2 s_n^2(A)$ for an operator with a kernel having a first derivative in the mean, 120
- Criterion for an operator to belong to an ideal \mathfrak{S}_ϕ (first), 87; (second), 132-133
- Cross-norm, x
- Cross-space, 65

- Decomposition of a space into a trivial subspace and a subspace on which an operator is simple, 223
- Definite characteristic numbers of the first and second kind, 301
- Determinant (see also Characteristic determinant)
 - $\det(I - A)$, 157
 - $\det^{(p)}(I - A)$ (regularized), 166
 - $\widetilde{\det}(I - A)$, 168
 - von Koch, 170
- Diagonal-cell operator, 51
 - belonging to an ideal \mathfrak{S}_ϕ , 82
- Dimension of an operator, 7
- Discreteness of the set of eigenvalues of an operator bundle, 266-267
- Dissipative operator, 175
 - localization of the spectrum and estimation of the norm of the resolvent, 176
- Dissipative Volterra operator
 - simple, 186
 - with one-dimensional imaginary component, 187
- Distance from a continuous operator to the ideal of completely continuous operators, 62
- Distance from an operator A
 - in \mathfrak{S}_{11} to $\mathfrak{S}_{11}^{(0)}$, 141
 - in \mathfrak{S}_r to \mathfrak{R}_n , 147
 - in \mathfrak{S}_ϕ to \mathfrak{R}_n and \mathfrak{R} , 87
- Dolph, C. L., xi
- Domain $\mathfrak{D}(A)$, 1
- Duffin, R., 301
- Dunford, N., ix, 95, 244

- Eigenspace, 5
- Eigenvalue, 5
- Eigenvector, 5
 - of an operator bundle, 265
 - of finite rank, 266
- J -orthonormal system, 264
- Equivalence of two s.n. functions, 76
- Example by Babenko, 319
- Example by Carleman, 118
- Example by Levin-Macaev, 250
- Expansion of the determinant $D_{A^*, A}(\lambda)$ into an infinite product
 - in the case of a dissipative operator A with nuclear imaginary component, 182
 - in the case of a nondissipative operator A with nuclear imaginary component, 189

- F*-perturbation determinant, 174
F-regular point of the operator *A*, 158
 Fage, M. K., 334
 Fan, Ky, x, 29, 46, 49, 72, 93
 Finite-dimensional operator, 7
 Foiaş, C., 231, 234, 235
 Formula for calculating the trace of a nuclear integral operator
 with a continuous Hermitian nonnegative kernel, 114
 with a Hermitian nonnegative continuous matrix kernel, 124
 with a Hermitian nonnegative Hilbert-Schmidt kernel, 115
 with a Hermitian nonnegative Hilbert-Schmidt matrix kernel, 124
 with a Hilbert-Schmidt kernel, 117
 Formula for the differentiation of the trace of a complicated function of an operator-
 function, 164
 Formula for the logarithmic derivative
 of the perturbation determinant $D_{B,A}(\mu)$, 174
 of the determinant $\det(I - A(\mu))$, 163
 Fredholm, I., vii, 118-119
 Function adjoint to a symmetric norming function, 125
 Function $M_A(r) = \max_{|\lambda| < r} |(I - \lambda A)^{-1}|$, 243
 Function $N(r; A)$ for a completely continuous operator *A*, 200
- Gantmaher, F. R., 46, 321
 Gapoškin, V. F., 319
 Gel'fand, I. M., 95, 313, 314, 316, 335
 Gel'fond, A. O., 122
 General form of a continuous linear functional
 on a separable space $\mathfrak{E}_\Phi^{(0)}$, 130
 on the space \mathfrak{E}_1 , 129
 on the space \mathfrak{E}_p , 132
 on the space \mathfrak{E}_Φ , 134
 Geometric interpretation of *s*-numbers, 30-31
 Georgiu, S. A., 94
 Ginzburg, Ju. P., 237
 Glazman, I. M., xi, 328
 Gohberg, I. C., vii, ix, 21, 95, 149, 155, 187, 193, 216, 257, 318
 Gram matrix, 311
- Hadamard, J., 171, 321
 Hadamard's inequality, 171, 321
 Hamburger, H., 259
 Hayman, W. K., 199
 Hermitian nonnegative kernel
 continuous, 112
 Hilbert-Schmidt, 115
 matrix, 123-124
 Hilbert, D., vii
 Hilbert identity, 4
 Hilbert-Schmidt matrix, 324
 Hilbert-Schmidt operator, test for, 107

- Hille, Einar, ix, 94, 123, 156, 171
- Holomorphy of an operator-function
 strong, 3-4
 weak, 137
- Horn, Alfred, x, 37, 46, 48, 49
- Ideal \mathfrak{K} of finite-dimensional operators in the ring \mathfrak{R} , 66
- Ideal \mathfrak{S}_c of completely continuous operators, 15, 66
- Ideals in the ring \mathfrak{R} , 15
 closed, 15
 selfadjoint (symmetric), 15
 two-sided, 15, 66
- Ideal, symmetrically-normed, see Symmetrically-normed ideal
- Identity of the Fredholm determinant $D(\lambda)$ of a continuous kernel \mathcal{K} with the characteristic determinant of the operator K determined by the kernel \mathcal{K} , 158
- Indefinite scalar product, 263-264
- Inequalities
 of Horn for sums of values of a function (which becomes convex following an exponential change of variable) at the s -numbers of a product of completely continuous operators, 49
 of Horn-Fan for the s -numbers of products and sums
 of completely continuous operators, 48
 of continuous operators, 63
 of Ky Fan
 for s -numbers, 29-30
 for sums of values of a nondecreasing convex function of the s -numbers of sums of operators, 49
 for the s -numbers under a perturbation of a finite-dimensional operator, 29
 for the sums of the s -numbers of a diagonal-cell operator, 52
 for sums of values of a convex function, 37
 Ostrowski's generalization of, 43
 for sums of values of a nondecreasing convex function
 at the arguments $t_j = |(A\phi_j, \phi_j)|$, 50
 generalization of, 51ff
 at the s -numbers of a diagonal-cell operator, 55
- Inequality $|A_1 \cdots A_n|_p \leq |A_1|_{p_1} \cdots |A_n|_{p_n}$, 92-93
- Inessential extension of an operator, 193
- Infinite von Koch determinant, 170
- Integral of F. Riesz, 5
- Integral operator J , 187
- Interpolation theorems for wandering operator-functions, 137, 150
- Invariant norm, 68
- Invariant subspace, 1
- Iohvidov, I. S., 264, 265
- J -orthonormal system of eigenvectors, 264
- Jensen's Theorem, 184
- Kac, I. S., 123
- Kato, T., 231, 248

- Keldyš, M. V., vii, ix, xi, 222, 244, 246, 252, 256, 257, 260, 263, 265, 269, 274, 278, 281, 282, 299
 von Koch, H., 169, 170
 Kolmogorov, A. N., 31
 Korenbljum, B. I., 281, 282
 Krasnosel'skii, M. A., 339
 Krein, M. G., viii, ix, xi, 46, 113, 121, 149, 152, 155, 156, 174, 175, 182, 187, 189, 193, 203, 216, 221, 237-242, 257, 264, 265, 285, 289, 291, 296, 319, 339
 Krein, S. G., 139, 303, 304
 k th associated operator, 46
 Kuroda, S. T., 77, 156, 175
- Lalesco, T., 93
 Langer, H. K., xi, 224, 231, 235, 291, 296
 Laptev, G. I., 304, 305
 Laptev's transformation, 305
 Lemma
- on $Q_A A Q_A$ being a Volterra operator on \mathfrak{E}_A^\perp , 17
 - stronger result for dissipative operators, 224
 - on the completeness of the systems of root vectors of dissipative operators having a group of properties which are invariant with respect to orthogonal projection, 225
 - on computing the determinant $\det(PA^*AP + Q)$, where P is the ortho-projector onto \mathfrak{E}_A , 232
 - of Gel'fand, 313
 - of Gohberg-Krein on the order of a Volterra operator, 257
 - on the independence upon the choice of basis of the matrix trace of a nonnegative operator, 96
 - of Ky Fan on the calculation of the maximum of the modulus of the sum $\sum_{j=1}^n (UA\phi_j, \phi_j)$, 47
 - generalization, 63
 - of Ky Fan on the monotonicity of a symmetric normalizing function, 72
 - of Mackey, 334
 - of Markus, 72-73
 - on the monotone dependence of the eigenvalues upon a completely continuous operator, 26
 - of Orlicz, (first), 314, (second), 315
 - of Schur on the reduction of the matrix of an operator to triangular form on \mathfrak{E}_A , 16
 - on the splitting up of the determinant $\det(AA^*)$ for an invertible operator A into the product of determinants corresponding to invariant subspaces, 233
 - of Sz.-Nagy on the equality of the dimension of subspaces, 13
 - on "the uniform approach to zero" inside a sector
 - of the Fredholm resolvent of a complete normal operator, 252
 - of the resolvent of a selfadjoint operator, 254
 - of Weyl-Horn on the estimation of a Gramian, 33
- Levin, B. Ja., ix, 182, 189, 203, 250
 Levinson, N., ix, 184, 189, 211
 Levinson-Sjeberg Theorem, ix
 Lindelöf-Valiron Theorem, 210, 216
 Lidskii, V. B., ix, xi, 101, 156, 208, 225, 231, 242, 244, 246, 248, 250, 278

Lineal

- \dot{c} , 71
- c_ϕ , 80
- Livsic, M. S., viii, 180, 187, 226, 227, 328
- Ljance, V. È., 339
- Ljubič, Ju. I., ix, 263
- Logarithmic length, 199
- Lorch, E. R., 315
- Lower bound for the sum $\sum_{j=1}^{\infty} |\psi_j - \phi_j|^2$, where $\{\psi_j\}$ is a Bari basis and $\{\phi_j\}$ is an orthonormal basis, 326

- Macaev, V. I., viii, ix, 149, 155, 156, 193, 210, 211, 215-217, 219-221, 242, 244, 248-251, 263, 278
- Mackey, G. W., 334
- Mal'cev, A. I., 265
- Markus, A. S., ix, xi, 72, 73, 95, 248, 259, 263, 267, 306, 318, 337, 341, 349
- Matrix criterion (necessary and sufficient) for the nuclearity of an operator, 97
- Matrix criterion (sufficient) for an operator to belong to an ideal \mathfrak{S}_p , 95
- Matrix trace of an operator, 97
- Matrix of van Koch, 170
- Matrix with absolutely convergent determinant, 169-170
- Maximal dissipative operators (unbounded), 231
- Mazurkiewicz, S., 122
- Mil'man, D. P., 339
- Minimal angle between subspaces, 339
- Minimax properties of the eigenvalues
 - of completely continuous operators, 25
 - of bounded operators, 59-60
- Mitjagin, B. S., 73, 143
- Mononormalizing s.n. function, 88
- Mukminov, B. R., xi, 227, 328

- n -fold completeness of the system of eigenvectors and associated vectors, 269
- n th width of a set in a linear metric space, 31
- Naïmark, M. A., ix, 278
- Necessary and sufficient conditions for the pointwise coincidence
 - of ideals \mathfrak{S}_ϕ and \mathfrak{S}_{ϕ_1} , 82
 - of ideals \mathfrak{S}_r and $\mathfrak{S}_{r'}$, and $\mathfrak{S}_{11'}$ and $\mathfrak{S}_{11''}$, 150
- Necessary and sufficient test for an operator to belong to the Hilbert-Schmidt class, 107
- Necessary and sufficient test for the nonseparability of an ideal \mathfrak{S}_ϕ , 90
- von Neumann, J., viii, x, 65, 197
- Neutral vector of a bundle, 301
- Nevanlinna, R., 188
- Nonnegative operator, 24-25
- Norm
 - $\| \cdot \|_{11,p}$, 150 $\| \cdot \|_{r,p}$, 150 $\| \cdot \|_{k,p}$, 153 $\| \cdot \|_{\mathcal{L},p}$, 154 $\| \cdot \|_{11;\mathcal{L}}$, 155
 - $\| \cdot \|_{r;\mathcal{L}}$, 155
 - $\| \cdot \|_\phi$, 80
 - dominance property of, 82
- Norm-ideals, x, 65
- Norm, invariant, 68

- Normal eigenvalue, 8
 necessary and sufficient condition, (first) 9, (second) 10
 Normal point, 11
 Norming function, 71
 symmetric, 71
 Nuclear operators, x , 65, 95-96 (def.)
 Nuclearity of an integral operator with a kernel of class $Lip\ \alpha$ for $\alpha > 1/2$, 118
 Nuclearity of an integral operator, tests for, 112ff, 124
- ω -linear independence of a sequence
 of vectors, 316-317
 of subspaces, 336
- Operator
 contraction, 231
 with finite matrix trace, 96
 of finite order, 256
 Hilbert-Schmidt, x , 65, 106-107 (def.), 109
 smoothing, S_h of Steklov and its properties, 114ff
- Operator bundle, 265
 characteristic number of, 265
 eigenvector of, 265
- Order
 $p(A)$ of an operator A , 256
 of an analytic function at a point, 173
 of an eigenvalue, 346
 of a root vector, 345-346
 of a sequence, 256
- Orlicz, W., 314, 315
- Orthogonal direct sum of operators, 228
- Orthoprojector, 2
- Oscillation of a viscous fluid, 303
- Ostrowski, A., 43
- Palant, Ju. A., 43, 248, 272, 274, 299
- Paraska, V. I., 123
- Partial Schmidt series, 61
- Partially isometric operator, 6
- Perturbation determinant
 $D_{B/A}(\lambda)$, 174
 $D_{B/A}(\mu)$, 171ff
 connection with regularized determinants, 172
 order at a point, 173
- Phillips, R. S., 231, 248
- Phragmén-Lindelöf Theorem, ix
- Poincaré, H., vii
- Polar representation of a bounded operator, 6-7
- Poljackii, V. T., 237
- Pontrjagin, L. S., 264, 265, 296
- Potapov, V. P., viii, 237
- Power A^* of a dissipative operator A , 248

- "Principle value of the trace" of the real component of a Volterra operator with nuclear imaginary component, 196
 Problem of small oscillations of a viscous fluid, 303
 Projector, 1
 orthogonal, 2
 Proper multiplicity, 5
 Properties of normal eigenvalues, 10-11
 Properties of the trace, 99ff
 Property invariant with respect to orthogonal projection, 225

 Quadratic bundle, 291
 reduction of the problem of completeness to the problem of completeness for operators, 293
 Quadratically close sequences
 of subspaces, 336
 of vectors, 316

 Range $\mathfrak{R}(A)$, 1
 Rate of decrease like a power, of the eigenvalues, when the s -numbers decrease like a power, 41
 R -dissipative, 296
 Riesz, F., vii, 5
 Reflexiveness of the space \mathfrak{E}_p , $1 < p < \infty$, 132
 Regular point, 3
 in the sense of Fredholm, 158
 of a bundle, 293-294
 Relation of the spaces \mathfrak{E}_p with \mathfrak{E}_{11} and \mathfrak{E}_q with \mathfrak{E}_r for $\sum_j \pi_j^p < \infty$, 150
 Relation of the spaces \mathfrak{E}_ω and \mathfrak{E}_{11} with \mathfrak{E}_p , 149
 Resolvent, 3
 Fredholm $A(\lambda)$, 67
 its belonging, together with the operator A , to a two-sided ideal, 67
 Resolvent set $\rho(A)$, 3
 Riesz basis, see Basis, Riesz
 Riesz-Herglotz Theorem, 198
 Ring of operators \mathfrak{R} , 1
 Root lineal, 5
 Root multiplicity, 13
 stability of, 13ff
 Root number, 13
 Root subspace, 5
 Root vector, 5
 Rules for calculating the determinant $\det(I - A)$, 160ff
 Rutman, M. A., 152

 S.n. ideal, see symmetrically-normed ideal
 S.n. function, see symmetric norming function
 s -numbers of a bounded operator, 59ff
 first definition, 59
 second definition, 61
 s -numbers of a completely continuous operator, 24ff
 first definition, 26
 second definition, 29

- simplest properties, 27
- third definition, 31
- Sahnović, L. A., viii, ix, 197, 216, 251
- Scalar products, indefinite, 263-264
 - topologically equivalent, 310
- Schatten, R., viii, x, 65
- Schmidt, E., 24
- Schmidt expansion
 - of a bounded operator, 61
 - of a completely continuous operator, 28
- Schmidt series, 61
- Schur, I., viii, 16, 93, 107, 227
- Schur system, 16, 36, 57, 98, 107
- Schwartz, J. T., 95, 244
- Selfadjointness of every two-sided ideal of the ring \mathfrak{R} , 67
- Set \mathfrak{R}_n of operators of dimension $\leq n$, 87
- Simple operator (bounded), 233
- Slowly varying function, 42, 144
 - properties of, 144
- Small oscillation of a viscous fluid, 303
- Smirnov, V. I., 204
- Smith, K., viii, 197
- Space
 - c_0 , 71; ℓ_n , 126; l , 128; m , 128; \mathfrak{R}_* , 134; Π , (Pontrjagin), 264
- Spectrum
 - of a bundle, 294
 - condensed, of a selfadjoint operator, 59
 - of an operator, $\sigma(A)$, 4
- Stability of the root multiplicity, 13ff
- Steklov operator, 114ff
- Stinespring, W. F., 95, 119, 123
- Stone, M., 246
- Strongly damped bundle, 301
 - structure of the spectrum of, 302
- Structure of the spectrum of a quadratic bundle, 294
- Subspace of zeros $\mathfrak{Z}(A)$, 6
- Summation by Abel's method of an expansion in root vectors, 248
- Symmetric norm, 68
 - properties of, 68ff
 - in \mathfrak{R} , 133
- Symmetric norming function, 71ff
 - binormalizing, 88
 - continuity of, 76
 - first definition, 71
 - maximal, 76
 - minimal, 76
 - mononormalizing, 88
 - $n + s$ conditions for equivalence
 - to the maximal one (Kuroda's result), 77
 - to the minimal one, 76-77
 - natural domain of, 80

- Φ_{II} , 139
 - properties of, 139
 - characteristic property of, 140-141, 179
- $\Phi_{\mathfrak{r}}$, 145
 - properties of, 145-146
 - properties of, 71ff
 - second definition, 74-75
 - upper and lower bounds for, 76
- Symmetrically-normed ideal, \mathfrak{x} , 24, 65, 70 (def.)
 - \mathfrak{S}_1 of nuclear operators, 95, 96; \mathfrak{S}_2 of Hilbert-Schmidt operators, 106-107;
 - \mathfrak{S}_p , 92; \mathfrak{S}_Φ , 80; $\mathfrak{S}_\Phi^{(0)}$, 87; \mathfrak{S}_{II} (nonseparable), 141; $\mathfrak{S}_{II}^{(0)}$ (separable), 141;
 - \mathfrak{S}_{II} (intermediate between $\mathfrak{S}_{II}^{(0)}$ and \mathfrak{S}_{II}), 143; $\mathfrak{S}_{II;p}$, 150; $\tilde{\mathfrak{S}}_{\mathfrak{r};p}$, 150;
 - $\mathfrak{S}_{\mathfrak{r}}$ (separable), 147; \mathfrak{S}_ω , 149; \mathfrak{S}_{II} , 149
 - description of separable ones, 89
- Symmetry of an arbitrary invariant norm on \mathfrak{R} , 79
- Sz.-Nagy, B., 13, 224, 231, 235

- Tamarkin, J. D., vii, ix, 94, 123, 156, 171
- Tauberian theorem, ix
 - of Keldyš, 281, 282
 - of Korenbljum, 281
- Test for the adjointness of s.n. ideals, 127
- Test for the ideals \mathfrak{S}_Φ and \mathfrak{S}_ω to coincide, 85
- Test for the normality of the spectrum in a given region, 11
- Tests for the nuclearity of integral operators, 112ff
 - with a Hermitian nonnegative continuous kernel, 114
 - with a Hermitian nonnegative Hilbert-Schmidt matrix kernel, 124
 - with a Hilbert-Schmidt kernel, 115
- Tests (necessary and sufficient) for a Bari basis, 320-324
- Three lines theorem, 136ff, 150
- Theorem
 - of Allahverdiev on an approximation property of s -numbers, 28-29
 - on the asymptotic behavior of the number of positive characteristic numbers of the real component of a Volterra operator with finite-dimensional imaginary component, 206
 - on the asymptotic behavior of the s -numbers of the operator $A = H(I + S)$, 284-285
 - of Banach on the sequence biorthogonal to a basis, 307
 - of Bari on the characteristics of a Riesz basis, 310-311
 - of Bari on the ω -linear independence of a sequence which is quadratically close to a Riesz basis, 317
 - generalization by Markus to the case of subspaces, 337
 - of Bendixson, 176
 - of Bernštein, 118
 - of Binet-Cauchy, 33
 - on a bound for the determinant $\det(AA^*)$ for an invertible contraction A , 236
 - on a bound for the perturbation determinant $D_{A^*,A}(\lambda)$ for a completely continuous dissipative operator A , 180
 - of Calkin on the smallest and largest two-sided ideals of the ring \mathfrak{R} , 66
 - of Cartwright-Levinson, 189, 203, 211
 - on the completeness of the system of root vectors of a dissipative operator A with nuclear imaginary component under the condition $\lim_{n \rightarrow \infty} n s_n(A) = 0$, 239
 - on the completeness of the system of root vectors of an operator $A = L + T$, where L is a selfadjoint operator with discrete spectrum and $p(L^{-1}TL^{-1}) < \infty$, 276-277

- on the completeness of the system of root vectors of a completely continuous operator with the set of values of the quadratic form (Af, f) lying in a sector of angular measure π/p
 under the condition $s_n(A) = o(n^{-1/p})$, 246
 under the condition $s_n(\{e^{i\alpha}A\}_{\mathcal{F}}) = o(n^{-1/p})$, 249
 with nonnegative Hermitian components under the condition $s_n(\{e^{i\alpha}A\}_{\mathcal{F}}) = o(1/\sqrt{n})$, 250
- on the connection between bases of subspaces and vector bases, 344, 345
 on the "constancy" of the number of linearly independent solutions of the equation $(I - A(\lambda))\phi = 0$, 21
- on the continuous dependence of the characteristic determinant $D_A(\lambda)$ upon the operator $A \in \mathfrak{S}_1$, 159
 on the continuous dependence of the regularized characteristic determinant $D_A^{(p)}(\lambda)$ upon the operator $A \in \mathfrak{S}_p$, 167
- on the convergence of the sequences $\{X_n A\}$, $\{A X_n\}$ and $\{X_n A X_n\}$ in the norm of a separable ideal \mathfrak{S} if $\{X_n\}$ converges strongly, 90
 defining the separable ideal \mathfrak{S}_p , 92
- on forming a Riesz basis and a Bari basis from bases of the eigenspaces of a bounded dissipative operator, 329-330
 corollary of, 332
- of Fredholm on the convergence of the series of p th powers of the eigenvalues of an integral operator with a Lipschitz kernel, 118-119
- of Gel'fand on a necessary and sufficient condition for a sequence of subspaces to be a basis equivalent to an orthogonal one, 335
- of Glazman on a Riesz basis of eigenvectors of a dissipative operator, 328
- of Gohberg on holomorphic operator-functions with completely continuous values, 21
- of Hausdorff on the convexity of the set of values of the quadratic form (Af, f) on the unit sphere, 245
- of Hayman, 199
- on the holomorphy of the determinant $\det(I - A(\mu))$ for a holomorphic operator-function, 163
- of Horn on the exactness of Weyl's inequalities, 37
- on the invariance of the set of normal points under the perturbation by a completely continuous operator B
 of a selfadjoint operator H , 23
 generalization of, 275
 of a unitary operator U , 23
- of Iohvidov, 264
- of Jensen, 184
- of Keldyš on the asymptotic behavior of the distribution function of the spectrum of the operator $A = H(I + S)$, 282
 generalization of, 283
- of Keldyš on the completeness of the eigenvectors and associated vectors of a linear bundle, 260
 generalization of, 262
- of Keldyš on the n -fold completeness of the system of eigenvectors and associated vectors of an n th order operator bundle and the conjugate bundle, 269-270, 299
 generalization by Allahverdiev, 275
 generalization by Palant, 274, 275, 299
- of Keldyš on the completeness of the system of root vectors of the operator $A = H(I + S)$, $p(H) < \infty$, 257
 stronger version, 263

- of Krein on the asymptotic behavior of the eigenvalues of the operator $A = H(I + S)$, 289
- of Krein on the asymptotic behavior of the s -numbers of the operator $A = H(I + S)H$, 285
- generalization of, 288-289
- of Krein on a necessary and sufficient condition for the completeness of the system of root vectors of a dissipative operator with nuclear imaginary component, 242
- of Krein on the completeness of the system of root vectors of a completely continuous dissipative operator with nuclear imaginary component, 238
- analysis of the exactness of the condition, 239ff
- of Langer-Sz.-Nagy-Foiaş on the orthogonal decomposition of a contraction into a unitary operator and a simple contraction, 235
- of Levin, 189
- of Levinson, 189
- generalization of, 203
- of Levinson-Sjeberg, ix
- of Lidskii on the completeness of the system of root vectors
- of a Hilbert-Schmidt operator with nonnegative Hermitian components, 250
- of a nuclear dissipative operator, 231
- of Lidskii on the identity of the matrix trace and spectral trace of a nuclear operator, 101
- of Lindelöf-Valiron, 210, 216
- of Livsić on the completeness of the system of root vectors of a dissipative operator, 226
- application to integral operators, 227
- generalization of, 228 (cf. 231)
- on the localization of the spectrum of a quadratic bundle in vertical sectors, 297
- of Lorch on the characterization of a Riesz basis by its property of permutability, 315
- of Macaev on the asymptotic behavior of the eigenvalues of the nonnegative term C in the decomposition of a Volterra operator $A = C + T$, $T \in \mathfrak{S}_p$, $p < \frac{1}{2}$, 220
- of Macaev on the completeness of the system of root vectors of a completely continuous dissipative operator A for certain asymptotic behavior of the functions $n_{\pm}(\rho; A_{\varphi})$ and $n_{\pm}(\rho; A_{\mathcal{F}})$, 251.
- of Macaev on the completeness of the system of root vectors of an operator $A = L + T$, where L is a selfadjoint operator with discrete spectrum and $TL^{-1} \in \mathfrak{S}_w$, 278
- of Macaev on entire functions, 210-211
- of Macaev on the estimation of the growth of the resolvent of a Volterra operator A with $s_n(A) = o(n^{-1,p})$, 244
- of Macaev on Volterra operators with components of class \mathfrak{S}_p , $1 < p < \infty$, 215-216
- of Macaev on Volterra operators which are the sum of a nonnegative operator and an operator
- of class \mathfrak{S}_p , $\frac{1}{2} < p < 1$, 217
- with given order of growth of its s -numbers, 219
- of Macaev-Gohberg-Krein on the s -numbers of a Volterra operator, 216
- of Markus on the construction of a basis from the root subspaces (and their bases) of a dissipative operator, 349-350
- of Markus on a lower bound for the minimal angle between root subspaces of a dissipative operator, 349
- of Markus on a sufficient test for the quadratic closeness of a basis of subspaces to an orthogonal one, 341
- on the minimax properties of the eigenvalues of a completely continuous operator, 25
- generalization of, 25

- of Nevanlinna, 188
- on the one-one correspondence between s.n. functions on \mathfrak{k} and invariant norms $|\cdot|_{\mathfrak{e}}$ on \mathfrak{R} , 78
- on the perturbation determinant of a dissipative operator, 178
- of Phragmén-Lindelöf, ix
- of Riesz-Herglotz, 198
- of Sahnovič on the existence of a maximal eigenchain for a completely continuous operator, 197
- of Schur on a bound for the sum of the squares of the moduli of the eigenvalues of a Hilbert-Schmidt operator, 107
- of three lines, 136-137
 - for operator functions which wander
 - in the spaces \mathfrak{S}_p , 136ff
 - in the spaces $\mathfrak{S}_{11, p}$, 150
- on the topological equivalence of the norms on s.n. ideals which coincide elementwise, 70
- on the two-fold completeness of the eigenvectors and associated vectors of a quadratic bundle, 296
- Theory of Hilbert-Schmidt-Mercer, 113, 124
- Theory of von Neumann-Schatten, x, 65
- Tihomirov, V. M., 31
- Topologically equivalent scalar products, 310
- Trace
 - matrix, 97
 - finite, 96
 - properties of, 99ff
- Trivial subspace
 - of a completely continuous dissipative operator, 224
 - of a dissipative operator, 224
 - \mathfrak{L}_T of a nonselfadjoint operator, 223
- Uniform norm $|\cdot|$, 1
- Uniformly positive operator, 302
- Uniqueness of a closed two-sided ideal in the ring \mathfrak{R} , 67
- Vector
 - eigen-, 5
 - of the first kind, 301
 - neutral, 301
 - root, 5
 - of the second kind, 301
- Vilenkin, N. Ja., 95
- Volterra operator, 16
 - simple, 186
 - with two-dimensional imaginary component and one negative eigenvalue for its real component, 208
- Volterra-ity of the operator $Q_A A Q_A$ on \mathfrak{C}_A , 17
 - stronger result for dissipative operators, 224
- Von Koch determinant, 170
- Von Koch matrix, 170

- Weakly damped bundle, 300
 - structure of the spectrum of, 300-301
- Wermer, J., ix, 334
- Weyl, H., x, 33, 35, 39, 45, 122
- Weyl's inequalities connecting the eigenvalues and s -numbers of a completely continuous operator, 35-36
 - Weyl's proof of, 45
- Weyl's majorant theorem, 39-40
 - generalization by Ostrowski, 43
- Wolf, F., ix

Recent Titles in This Series

(Continued from the front of this publication)

- 101 **Zhang Zhi-fen, Ding Tong-ren, Huang Wen-zao, and Dong Zhen-xi**, Qualitative theory of differential equations, 1992
- 100 **V. L. Popov**, Groups, generators, syzygies, and orbits in invariant theory, 1992
- 99 **Norio Shimakura**, Partial differential operators of elliptic type, 1992
- 98 **V. A. Vassiliev**, Complements of discriminants of smooth maps: Topology and applications, 1992 (revised edition, 1994)
- 97 **Itiro Tamura**, Topology of foliations: An introduction, 1992
- 96 **A. I. Markushevich**, Introduction to the classical theory of Abelian functions, 1992
- 95 **Guangchang Dong**, Nonlinear partial differential equations of second order, 1991
- 94 **Yu. S. Il'yashenko**, Finiteness theorems for limit cycles, 1991
- 93 **A. T. Fomenko and A. A. Tuzhilin**, Elements of the geometry and topology of minimal surfaces in three-dimensional space, 1991
- 92 **E. M. Nikishin and V. N. Sorokin**, Rational approximations and orthogonality, 1991
- 91 **Mamoru Mimura and Hiroshi Toda**, Topology of Lie groups, I and II, 1991
- 90 **S. L. Sobolev**, Some applications of functional analysis in mathematical physics, third edition, 1991
- 89 **Valerii V. Kozlov and Dmitrii V. Treshchëv**, Billiards: A genetic introduction to the dynamics of systems with impacts, 1991
- 88 **A. G. Khovanskii**, Fewnomials, 1991
- 87 **Aleksandr Robertovich Kemer**, Ideals of identities of associative algebras, 1991
- 86 **V. M. Kadets and M. I. Kadets**, Rearrangements of series in Banach spaces, 1991
- 85 **Mikio Ise and Masaru Takeuchi**, Lie groups I, II, 1991
- 84 **Djáo Trông Thi and A. T. Fomenko**, Minimal surfaces, stratified multivarifolds, and the Plateau problem, 1991
- 83 **N. I. Portenko**, Generalized diffusion processes, 1990
- 82 **Yasutaka Sibuya**, Linear differential equations in the complex domain: Problems of analytic continuation, 1990
- 81 **I. M. Gelfand and S. G. Gindikin, Editors**, Mathematical problems of tomography, 1990
- 80 **Junjiro Noguchi and Takushiro Ochiai**, Geometric function theory in several complex variables, 1990
- 79 **N. I. Akhiezer**, Elements of the theory of elliptic functions, 1990
- 78 **A. V. Skorokhod**, Asymptotic methods of the theory of stochastic differential equations, 1989
- 77 **V. M. Filippov**, Variational principles for nonpotential operators, 1989
- 76 **Phillip A. Griffiths**, Introduction to algebraic curves, 1989
- 75 **B. S. Kashin and A. A. Saakyan**, Orthogonal series, 1989
- 74 **V. I. Yudovich**, The linearization method in hydrodynamical stability theory, 1989
- 73 **Yu. G. Reshetnyak**, Space mappings with bounded distortion, 1989
- 72 **A. V. Pogorelev**, Bendings of surfaces and stability of shells, 1988
- 71 **A. S. Markus**, Introduction to the spectral theory of polynomial operator pencils, 1988
- 70 **N. I. Akhiezer**, Lectures on integral transforms, 1988
- 69 **V. N. Salii**, Lattices with unique complements, 1988
- 68 **A. G. Postnikov**, Introduction to analytic number theory, 1988
- 67 **A. G. Dragalin**, Mathematical intuitionism: Introduction to proof theory, 1988
- 66 **Ye Yan-Qian**, Theory of limit cycles, 1986
- 65 **V. M. Zolotarev**, One-dimensional stable distributions, 1986
- 64 **M. M. Lavrent'ev, V. G. Romanov, and S. P. Shishat'skii**, Ill-posed problems of mathematical physics and analysis, 1986
- 63 **Yu. M. Berezanskii**, Selfadjoint operators in spaces of functions of infinitely many variables, 1986
- 62 **S. L. Krushkal', B. N. Apanasov, and N. A. Gusevskii**, Kleinian groups and uniformization in examples and problems, 1986

(See the AMS catalog for earlier titles)

ISBN 978-0-8218-1568-7



9 780821 815687

MMONO/18

AMS *on the Web*
www.ams.org