Operators, Oscillations, Waves. Open Systems

by
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PREFACE*

The last fifteen years have seen the birth and significant growth of the theory of nonselfadjoint operators. But unlike the spectral analysis of selfadjoint operators, the new theory has not had its physical counterpart. For a long time physics provided a field of activity only for the problem of expansion in eigenfunctions, closely related to the Fourier method in the theory of oscillations. Invariant subspaces, triangular models and representations, characteristic functions and other important concepts and results had no interpretation in physics, a state of affairs which is the more regrettable because the tie between mathematics and physics has always been especially fruitful in the theory of operators.

The theory of open systems developed in this book originated in an attempt to remove this deficiency, but it also has independent interest. The theory of open systems tries to uncover the structure of a given physical system and to develop methods for the solution of concrete problems. Of course, this theory will be successful at present only for certain classes of physical systems and for the time being one cannot foresee how far it will advance in the future.

The questions discussed in the book are on the borderline between mathematics and physics. As a result, the mathematical definitions are often motivated by preliminary arguments and examples from physics. Although the basic

* This translation incorporates corrections and additions furnished by the author.

results are formulated as theorems and are rigorously proved, a desirable degree of generality and logical completeness is not always maintained, particularly in connection with resolution.

It is assumed that the reader is familiar with the basic theory of operators as presented in the book by N. I. Ahiezer and I. M. Glazman [2], (see also [38, 41]). In the present book the necessary information on nonselfadjoint operators is either presented in full as needed or is stated without proof, with a reference to the relevant literature. For the important class of systems with a finite number of degrees of freedom the corresponding operators are defined in finite-dimensional spaces and in this case require only linear algebra. Further information on the theory of open systems and operator clusters is given in the book by M. S. Livšic and A. A. Jancevič, "Theory of operator clusters in Hilbert spaces", Kharkov University, 1971.

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