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Е. Г. ГОЛЬШТЕЙН

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FOREWORD

The lectures on convex programming theory which the author presented in the summer of 1968 to the students of the All-Union School on Mathematical Programming in the city of Alma-Ata were used as the basis for this book.

It is well known that duality theory and the corresponding dual approach to the analysis of linear problems play a significant role in linear programming. On the one hand, this approach allows one to give sufficiently precise mathematical descriptions of a number of economic mechanisms. On the other hand, it leads toward effective computational methods of linear programming. A significant place in the book is given to the construction of a duality theory for convex programming problems. A pair of dual convex programming problems is closely connected with the problem of finding a saddle point of a Lagrangian function. Therefore, the approach to the analysis of convex programming problems which we develop here is closely related to the well-known approach of Kuhn and Tucker [5], [11]. However, as we shall show, the manner of presentation adopted in the book allows us to construct a more general theory of convex programming.

The method of constructing a theory of dual convex programming problems adopted in the book is not entirely traditional (see [1] and [12]). Here, the basic theorem on antagonistic games due to John von Neumann was taken as the starting point (usually, the connection between convex programming theorems and von Neumann's theorem is not noted).

Besides the basic facts of convex programming theory, the book includes a number of results on the marginal values of convex programming problems.

Although the presentation is carried out for the case of finite-dimensional problems, many of the theorems remain in force for more general infinite-dimensional problems. Moreover, the transition to functional spaces does not, as a rule, involve any change in the structure of the corresponding arguments.

The author

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Thus Theorem 17 has been proved under extremely weak assumptions, and it contains, as a particular case, the theorem on the marginal values of a linear programming problem.

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