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Volume 45

Matrix Groups

D. A. Suprunenko



American Mathematical Society

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PREFACE

In this book we give an account of the classical results on the structure of normal subgroups of the general linear group over a division ring, of Burnside's and Schur's theorems on periodic linear groups, and of the theorem on the normal structure of $SL(n, Z)$ for $n > 2$. We also discuss the theory of solvable, nilpotent, and locally nilpotent linear groups. To give a better idea of the content of the book, here is a brief chapter-by-chapter survey.

In the first chapter we deal with permutation groups (finite and infinite). After an account of the elementary facts we establish a connection between the theory of primitive solvable permutation groups (not necessarily of finite degree) and the theory of solvable linear groups over prime fields. Here we also investigate nilpotent and locally nilpotent permutation groups. For example, we give a complete classification of the maximal nilpotent subgroups of the symmetric groups of finite degree. We prove, in particular, that the maximal transitive nilpotent subgroups of a finite symmetric group are conjugate.

A substantial part of the second chapter is devoted to the theory of determinants of matrices over a division ring and the structure of normal subgroups of the general linear group over a division ring; the exposition is based on Dieudonné [1] and Artin [1]. We also consider properties of endomorphisms of a module over an associative ring. In particular, we establish the permutability properties of transvections of a module.

In the third chapter we describe the normal subgroups of the stable general linear group over an arbitrary associative ring (Bass's theorem [1]) and the normal subgroups of $SL(n, Z)$, $n > 2$ (Bass, Lazard and Serre [1], Mennicke [1]). The stable (or limiting) general linear group $GL(\Delta)$ over a ring Δ is defined as the union of the following infinite ascending chain of groups:

$$GL(1, \Delta) \subset GL(2, \Delta) \subset \dots \subset GL(n, \Delta) \subset \dots$$

with a standard embedding of $GL(n, \Delta)$ in $GL(n + 1, \Delta)$.

In the fourth chapter we prove Mal'cev's local theorem on conditions for a faithful linear representation of a group and Clifford's theorem on the normal subgroups of an irreducible linear group. We also investigate properties of imprimitive linear groups and conditions for linear groups to be completely reducible.

In the fifth chapter we consider solvable matrix groups over a field. Attention is concentrated mainly on the structure of maximal primitive solvable subgroups of $GL(n, \Delta)$ for an arbitrary field Δ . We examine in detail the invariant series

$$G \supset V \supset A \supset F \supset (E_n), \quad (\alpha)$$

where G is a maximal primitive solvable subgroup of $GL(n, \Delta)$, F a maximal abelian normal subgroup of G , V the centralizer of F in G , and A/F a subgroup of G/F that is maximal among the normal abelian subgroups of G/F contained in V/F . The series (α) uniquely determines G . In certain cases the study of (α) enables us to classify the maximal irreducible solvable subgroups of $GL(n, \Delta)$. From the properties of (α) it is easy to derive the theorems of Zassenhaus and Mal'cev on solvable linear groups. The construction of maximal solvable subgroups of the general linear group over an algebraically closed field is reduced to the construction of solvable subgroups of the symplectic group over a finite prime field. We list the maximal irreducible solvable groups of prime degree over an algebraically closed field and over a finite field. We establish conditions for complete reducibility of a solvable linear group. In particular, a solvable linear group over a field of characteristic zero is completely reducible if and only if all its cyclic subgroups are completely reducible.

The sixth chapter is devoted mainly to periodic linear groups over a field. We prove Burnside's criterion for the finiteness of a linear group, and Schur's theorems on the local finiteness of a periodic linear group over an arbitrary field and on the existence of an abelian normal subgroup of finite index in a complex periodic linear group. We study Sylow p -subgroups and Sylow Π -subgroups of $GL(n, \Delta)$. We present Platonov's structure theorems for periodic linear groups: the theorem that the Sylow p -subgroups of a periodic linear group are conjugate and the analog of the Schur-Zassenhaus theorem for periodic linear groups. We consider properties of a linear group, not necessarily periodic, that are connected with properties of the characteristic polynomials of its matrices.

In the seventh chapter we study nilpotent and locally nilpotent matrix groups over a field. Here we give a complete classification of all maximal locally nilpotent subgroups of the general linear group over an algebraically closed field. In particular, we prove that if Δ is an algebraically closed field such that $\text{char } \Delta$ divides n , then $GL(n, \Delta)$ contains no irreducible locally nilpotent subgroups; while if $\text{char } \Delta = 0$, or $\text{char } \Delta = p$ but n is not divisible by p , then $GL(n, \Delta)$ contains (up to conjugacy) a unique maximal irreducible locally nilpotent subgroup. For an algebraically closed ground field we also give a complete classification of the maximal irreducible nilpotent subgroups of class 2 of the general linear group. For an arbitrary ground field the classification of the maximal irreducible locally

nilpotent subgroups of the general linear group is reduced to a complete description of the Sylow p -subgroups of the projective group $PGL(p^\alpha, \Delta)$. For a perfect field we give a canonical form of locally nilpotent reducible indecomposable linear groups.

The numbering of theorems, propositions, lemmas and formulas is independent within each section. References to the number of a theorem (lemma, proposition) are prefaced by the section number. For example, if Theorem 5 of §7 is referred to outside §7, it is called Theorem 7.5. Corollary 1 of Theorem 7.5 is referred to as Corollary 7.5.1. If this reference is in §7 itself, we write "Corollary 5.1".

I remember with gratitude the late A. I. Ma'cev, who encouraged me to begin work on this book. I would also like to thank the editor, F. I. Kizner, and my research students G. F. Žavrid, V. S. Konjuh, L. P. Lugovskaja, N. N. Metel'skii, O. M. Neroslavskii and V. P. Juferev, who helped me with the preparation of the manuscript.

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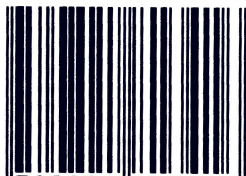
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