Translations of

MATHEMATICAL MONOGRAPHS

Volume 45

Matrix Groups

D. A. Suprunenko

American Mathematical Society

10.1090/mmono/045

TRANSLATIONS OF
MATHEMATICAL
MONOGRAPHS
VOLUME 45

D. A. Suprunenko

Matrix Groups

American Mathematical Society

ГРУППЫ

МАТРИЦ

Д. А. СУПРУНЕНКО

Издательство "Наука"
Главная редакция
физико-математической литературы
Москва 1972

Translated from the Russian by
Israel Program for Scientific Translations

Translation edited by K. A. Hirsch

AMS (MOS) subject classifications (1970). Primary 20G15;
Secondary 20B05, 15A15, 20E15, 20F50, 20E25.

Library of Congress Cataloging in Publication Data

Suprunenko, Dmitriĭ Alekseevich.
Matrix groups.

(Translations of mathematical monographs ; v. 45)
Translation of Gruppy matrits.
Bibliography: p.
Includes index.
1. Matrix groups. I. Title. II. Series.
QA171.S77813 512'.22 75-45115
ISBN 0-8218-1595-4

10 9 8 7 6 5 4 3 2 02 01 00 99

PREFACE

In this book we give an account of the classical resutls on the structure of normal subgroups of the general linear group over a division ring, of Burnside's and Schur's theorems on periodic linear groups, and of the theorem on the normal structure of $SL(n, Z)$ for $n > 2$. We also discuss the theory of solvable, nilpotent, and locally nilpotent linear groups. To give a better idea of the content of the book, here is a brief chapter-by-chapter survey.

In the first chapter we deal with permutation groups (finite and infinite). After an account of the elementary facts we establish a connection between the theory of primitive solvable permutation groups (not necessarily of finite degree) and the theory of solvable linear groups over prime fields. Here we also investigate nilpotent and locally nilpotent permutation groups. For example, we give a complete classification of the maximal nilpotent subgroups of the symmetric groups of finite degree. We prove, in particular, that the maximal transitive nilpotent subgroups of a finite symmetric group are conjugate.

A substantial part of the second chapter is devoted to the theory of determinants of matrices over a division ring and the structure of normal subgroups of the general linear group over a division ring; the exposition is based on Dieudonné [1] and Artin [1]. We also consider properties of endomorphisms of a module over an associative ring. In particular, we establish the permutability properties of transvections of a module.

In the third chapter we describe the normal subgroups of the stable general linear group over an arbitrary associative ring (Bass's theorem [1]) and the normal subgroups of $SL(n, Z)$, $n > 2$ (Bass, Lazard and Serre [1], Mennicke [1]). The stable (or limiting) general linear group $GL(\Delta)$ over a ring Δ is defined as the union of the following infinite ascending chain of groups:

$$GL(1, \Delta) \subset GL(2, \Delta) \subset \dots \subset GL(n, \Delta) \subset \dots$$

with a standard embedding of $GL(n, \Delta)$ in $GL(n + 1, \Delta)$.

In the fourth chapter we prove Mal′cev's local theorem on conditions for a faithful linear representation of a group and Clifford's theorem on the normal subgroups of an irreducible linear group. We also investigate properties of imprimitive linear groups and conditions for linear groups to be completely reducible.

In the fifth chapter we consider solvable matrix groups over a field. Attention is concentrated mainly on the structure of maximal primitive solvable subgroups of $GL(n, \Delta)$ for an arbitrary field Δ. We examine in detail the invariant series

$$G \supset V \supset A \supset F \supset (E_n), \tag{α}$$

where G is a maximal primitive solvable subgroup of $GL(n, \Delta)$, F a maximal abelian normal subgroup of G, V the centralizer of F in G, and A/F a subgroup of G/F that is maximal among the normal abelian subgroups of G/F contained in V/F. The series (α) uniquely determines G. In certain cases the study of (α) enables us to classify the maximal irreducible solvable subgroups of $GL(n, \Delta)$. From the properties of (α) it is easy to derive the theorems of Zassenhaus and Mal′cev on solvable linear groups. The construction of maximal solvable subgroups of the general linear group over an algebraically closed field is reduced to the construction of solvable subgroups of the symplectic group over a finite prime field. We list the maximal irreducible solvable groups of prime degree over an algebraically closed field and over a finite field. We establish conditions for complete reducibility of a solvable linear group. In particular, a solvable linear group over a field of characteristic zero is completely reducible if and only if all its cyclic subgroups are completely reducible.

The sixth chapter is devoted mainly to periodic linear groups over a field. We prove Burnside's criterion for the finiteness of a linear group, and Schur's theorems on the local finiteness of a periodic linear group over an arbitrary field and on the existence of an abelian normal subgroup of finite index in a complex periodic linear group. We study Sylow p-subgroups and Sylow Π-subgroups of $GL(n, \Delta)$. We present Platonov's structure theorems for periodic linear groups: the theorem that the Sylow p-subgroups of a periodic linear group are conjugate and the analog of the Schur-Zassenhaus theorem for periodic linear groups. We consider properties of a linear group, not necessarily periodic, that are connected with properties of the characteristic polynomials of its matrices.

In the seventh chapter we study nilpotent and locally nilpotent matrix groups over a field. Here we give a complete classification of all maximal locally nilpotent subgroups of the general linear group over an algebraically closed field. In particular, we prove that if Δ is an algebraically closed field such that char Δ divides n, then $GL(n, \Delta)$ contains no irreducible locally nilpotent subgroups; while if char $\Delta = 0$, or char $\Delta = p$ but n is not divisible by p, then $GL(n, \Delta)$ contains (up to conjugacy) a unique maximal irreducible locally nilpotent subgroup. For an algebraically closed ground field we also give a complete classification of the maximal irreducible nilpotent subgroups of class 2 of the general linear group. For an arbitrary ground field the classification of the maximal irreducible locally nilpotent subgroups of the general linear group is reduced to a complete description of the Sylow p-subgroups of the projective group $PGL(p^{\alpha}, \Delta)$. For a perfect field we give a canonical form of locally nilpotent reducible indecomposable linear groups.

The numbering of theorems, propositions, lemmas and formulas is independent within each section. References to the number of a theorem (lemma, proposition) are prefaced by the section number. For example, if Theorem 5 of §7 is referred to outside §7, it is called Theorem 7.5. Corollary 1 of Theorem 7.5 is referred to as Corollary 7.5.1. If this reference is in §7 itself, we write "Corollary 5.1".

I remember with gratitude the late A. I. Mal′cev, who encouraged me to begin work on this book. I would also like to thank the editor, F. I. Kizner, and my research students G. F. Žavrid, V. S. Konjuh, L. P. Lugovskaja, N. N. Metel′skiĭ, O. M. Neroslavskiĭ and V. P. Juferev, who helped me with the preparation of the manuscript.

This page intentionally left blank

TABLE OF CONTENTS

Preface iii

Chapter I. Elements of the theory of permutation groups 1
§1. Properties of mappings 1
§2. Transitivity 4
§3. Imprimitivity 7
§4. Permutation groups having a regular normal subgroup. Primitive solvable groups 15
§5. Nilpotent and locally nilpotent permutation groups 27

Chapter II. The general linear group 35
§6. Definitions. Prelininary propositions 35
§7. Endomorphisms 38
§8. Matrix representations of an endomorphism 44
§9. The Dieudonné determinant 52
§10. Normal subgroups of $GL(M)$ 58

Chapter III. The normal structure of the groups $GL(\Delta)$ and $GL(n, Z)$, $n > 2$ 64
§11. Normal subgroups of the infinite general linear group 64
§12. Normal subgroups of $GL(n, Z)$, $n > 2$. Subgroups of finite index 70

Chapter IV. Reducibility and imprimitivity 83
§13. Abelian groups with operators. The structure of semisimple algebras 83
§14. Linear representations. Reducibility and irreducibility of linear groups 90
§15. Primitivity and imprimitivity 103
§16. On normal subgroups of completely reducible groups 112
§17. Some conditions for complete reducibility of a linear group over a field 119

Chapter V. Solvable matrix groups 125
§18. Reducible solvable groups 125
§19. Primitive solvable groups. Boundedness of the length of the derived series of a solvable linear group 129
§20. Maximal primitive solvable subgroups of the general linear group 137
§21. Solvable matrix groups over a finite field 157
§22. Miscellany 168

Chapter VI. Periodic linear groups 177
§23. Finiteness conditions for linear groups. Local finiteness of a matrix group over a field 177

§24. Existence of an abelian normal subgroup of finite index in a periodic linear group over the field of complex numbers......... 186
§25. Sylow subgroups of the general linear group............................. 191
§26. Structure theorems on periodic matrix groups over a field...... 202

Chapter VII. Nilpotent and locally nilpotent matrix groups........................ 205
§27. Irreducible nilpotent matrix groups.. 205
§28. Irreducible locally nilpotent matrix groups................................ 216
§29. Reducible locally nilpotent groups.. 222

Bibliography.. 242

Subject Index.. 251

BIBLIOGRAPHY

E. Artin

[1] *Geometric algebra*, Interscience, New York, 1957. MR 18, 553.

L. Auslander

[1] *On a problem of Philip Hall*, Ann. of Math. (2) 86 (1967), 112–116. MR 36 #1540.

R. Baer

[1] *Linear algebra and projective geometry*, Academic Press, New York, 1952. MR 14, 675.

H. Bass

[1] *K-theory and stable algebra*, Inst. Hautes Études Sci. Publ. Math. No. 22 (1964), 5–60. MR 30 #4805.

H. Bass, M. Lazard and J.-P. Serre

[1] *Sous-groupes d'indice fini dans SL(n, Z)*, Bull. Amer. Math. Soc. 70 (1964), 385–392. MR 28 #5117.

H. Bass, J. Milnor and J.-P. Serre

[1] *Solution of the congruence subgroup problem for* SL_n $(n \geq 3)$ *and* Sp_{2n} $(n \geq 2)$, Inst. Hautes Études Sci. Publ. Math. No. 33 (1967), 59–137. MR 39 #5574.

A. Borel

[1] *Groupes linéaires algébriques*, Ann. of Math. (2) 64 (1956), 20–82. MR 19, 1195.

N. Bourbaki

[1] *Algèbre*. Chaps. 1–3, 2nd ed., Actualités Sci. Indust., nos. 1144, 1236, 1044, Hermann, Paris, 1951, 1955, 1958; English transl. of 1970 ed., Hermann, Paris; Addison-Wesley, Reading, Mass. 1973. MR 6, 113; 9, 406; 30 #3104; 43 #2.

R. Brauer and W. Feit

[1] *An analogue of Jordan's theorem in characteristic p*,' Ann. of Math. (2) 84 (1966), 119–131. MR 34 #246.

J. L. Brenner

[1] *The linear homogeneous group*. III, Ann. of Math. (2) 71 (1960), 210–223. MR 22 #1622.

W. Burnside

[1] *On an unsettled question in the theory of discontinuous groups*, Quart. J. Pure Appl. Math. 33 (1902), 230–238.

[2] *On criteria for the finiteness of the order of a group of linear substitutions*, Proc. London Math. Soc. (2) 3 (1905), 345–440.

[3] *On the arithmetical nature of the coefficients in a group of linear substitutions*, Proc. London Math. Soc. (2) 7 (1908), 8–13.

A. H. Clifford

[1] *Representations induced in an invariant subgroup*, Ann. of Math. (2) 38 (1937), 533–550.

C. W. Curtis and I. Reiner

[1] *Representation theory of finite groups and associative algebras*, Pure and Appl. Math., vol. 11, Interscience, New York, 1962. MR 26 #2519.

L. E. Dickson

[1] *Linear groups*, Teubner, Leipzig, 1901.

J. Dieudonné

[1] *Les déterminants sur un corps non commutatif*, Bull. Soc. Math. France 71 (1943), 27–45. MR 7, 3.

[2] *La géométrie des groupes classiques*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Heft 5, Springer-Verlag, Berlin, 1955. MR 17, 236.

[3] *Sur les groupes classiques*, Actualités Sci. Indust., no. 1040, Hermann, Paris, 1948. MR 9, 494.

P. G. L. Dirichlet

[1] *Vorlesungen über Zahlentheorie*, 4th ed., Vieweg, Braunschweig, 1894.

J. D. Dixon

[1] *Complete reducibility of infinite groups*, Canad. J. Math. 16 (1964), 267–274. MR 28 #5120.

[2] *The solvable length of a solvable linear group*, Math. Z. 107 (1968), 151–158. MR 38 #4569.

W. Feit and J. Thompson

[1] *Solvability of groups of odd order*, Pacific J. Math. 13 (1963), 775–1029. MR 29 #3538.

M. S. Garaščuk

[1] *On the theory of generalized nilpotent linear groups*, Dokl. Akad. Nauk BSSR 4 (1960), 276–277. (Russian) MR 24 #A1954.

M. S. Garaščuk and D. A. Suprunenko

[1] *Linear nil-groups*, Dokl. Akad. Nauk BSSR 4 (1960), 407–408. (Russian) MR 24 #A1955.

E. S. Golod

[1] *On nil-algebras and finitely approximable p-groups*, Izv. Akad. Nauk SSSR Ser. Mat. 28 (1964), 273–276; English transl., Amer. Math. Soc. Transl. (2) 48 (1965), 103–106. MR 28 #5082.

O. Grün

[1] *Über eine Faktorgruppe freier Gruppen.* I, Deutsche Math. 1 (1936), 772–782.

M. Hall

[1] *The theory of groups*, Macmillan, New York, 1959. MR 21 #1996.

P. Hall

[1] *Nilpotent groups*, Canad. Math. Congress Summer Seminar, Univ. of Alberta, Edmonton, Alberta, 1957; republished as: *The Edmonton notes on nilpotent groups*, Queen Mary College Math. Notes, London, Ontario, 1969. MR 44 #316.

B. Huppert

[1] *Lineare auflösbare Gruppen*, Math. Z. 67 (1957), 479–518. MR 19, 729.

I. D. Ivanjuta

[1] *Sylow p-subgroups of the countable symmetric group*, Ukrain. Mat. Ž. 15 (1963), 240–249. (Russian) MR 28 #2153.

[2] *On complete sets of Sylow subgroups of the countable symmetric group*, Ukrain. Mat. Ž. 18 (1966), no. 3, 112–115. (Russian) MR 34 #2673.

C. Jordan

[1] *Sur la résolution algébrique des équations primitives de degré p^2 (p étant premier impair)*, J. Math. Pures Appl. (2) 13 (1868), 124–126.

[2] *Traité des substitutions et des équations algébriques*, Gauthier-Villars, Paris, 1870.

[3] *Mémoire sur les équations différentielles linéaires à intégral algébrique*, J. Reine Angew. Math. 84 (1878), 89–215.

L. Kaloujnine [L. A. Kalužnin]

[1] *Sur les p-groupes de Sylow du groupe symétrique de degré p^m*, C. R. Acad. Sci. Paris 221 (1945), 222–224. MR 7, 239.

[2] *Sur les p-groupes de Sylow du groupe symétrique de degré p^m (Suite centrale ascendante et descendante)*, C. R. Acad. Sci. Paris 223 (1946), 703–705. MR 8, 251.

[3] *La structure des p-groupes de Sylow des groupes symétriques finis*, Ann. Sci. École Norm. Sup. (3) 65 (1948), 239–276. MR 10, 505.

I. Kaplansky

[1] *An introduction to differential algebra*, Actualités Sci. Indust., no. 1251, Hermann, Paris, 1957. MR 20 #177.

M. I. Kargapolov

[1] *On periodic matrix groups*, Sibirsk. Mat. Ž. 3 (1962), 834–838. (Russian) MR 26 #2517.

[2] *Finitely generated linear groups*, Algebra i Logika 6 (1967), no. 5, 17–20. (Russian) MR 38 #5942.

O. H. Kegel

[1] *Lectures on locally finite groups*, Mathematical Institute, Oxford, 1969.

O. H. Kegel and B. A. F. Wehrfritz

[1] *Locally finite groups*, North-Holland Math. Library, North-Holland, Amsterdam, 1973.

W. Klingenberg

[1] *Die Struktur der linearen Gruppen über einem nichtkommutativen lokalen Ring*, Arch. Math. 13 (1962), 73–81. MR 26 #1367.

E. R. Kolchin

[1] *On certain concepts in the theory of algebraic matric groups*, Ann. of Math. (2) 49 (1948), 774–789. MR 10, 232.

A. G. Kuroš

[1] *Theory of groups*, 3rd ed., "Nauka", Moscow, 1967; English transl. of 2nd ed., Chelsea, New York, 1960. MR 22 #727; 40 #2740.

[2] *Lectures on general algebra*, Fizmatgiz, Moscow, 1962; English transl., Chelsea, New York, 1963; Internat. Ser. Monographs Pure Appl. Math., vol. 70, Pergamon Press, Oxford, 1965. MR 25 #5097; 28 #1228; 31 #3483.

S. Lang

[1] *Algebra*, Addison-Wesley, Reading, Mass., 1965. MR 33 #5416.

A. I. Mal'cev

[1] *On faithful representation of infinite groups by matrices*, Mat. Sb. 8 (50) (1940), 405–422; English transl., Amer. Math. Soc. Transl. (2) 45 (1965), 1–18. MR 2, 216.

[2] *On a general method for obtaining local theorems in group theory*, Ivanov. Gos. Ped. Inst. Učen. Zap. Fiz.-Mat. Fak. 1 (1941), 3–9; English Transl., Chapter II in A. I. Mal'cev, *The metamathematics of algebraic systems. Collected papers*: 1936–1967, Studies in Logic and Foundations of Math., vol. 66, North-Holland, Amsterdam, 1971. MR 17, 823.

[3] *On certain classes of infinite solvable groups*, Mat. Sb. 28 (70) (1951), 567–588; English transl., Amer. Math. Soc. Transl. (2) 2 (1956), 1–21. MR 13, 203; 17, 824.

[4] *Foundations of linear algebra*, 3rd ed., "Nauka", Moscow, 1970; English transl. of 2nd ed., Freeman, San Francisco, Calif.,1963. MR 29 #3477.

[5] *Algebraic systems*, "Nauka", Moscow, 1970; English transl., Die Grundlehren der math. Wissenschaften, Band 192, Springer-Verlag, Berlin and New York, 1973. MR 44 #142.

G. Maxwell

[1] *Infinite general linear groups over rings*, Trans. Amer. Math. Soc. 151 (1970), 371–375. MR 41 #8531.

J. L. Mennicke

[1] *Finite factor groups of the unimodular group*, Ann. of Math. (2) 81 (1965), 31–37. MR 30 #2083.

Ju. I. Merzljakov

[1] *Central series and commutator series of matrix groups*, Algebra i Logika 3 (1964), no. 4, 49–59. (Russian) MR 30 #170.

[2] *On linear groups with bounded cyclic subgroups*, Sibirsk. Mat. Ž. 7 (1966), 318–322 = Siberian Math. J. 7 (1966), 257–259. MR 33 #5742.

[3] *Integer representation of holomorphs of polycyclic groups*, Algebra i Logika 9 (1970), 539–558 = Algebra and Logic 9 (1970), 326–337. MR 43 #6298.

J. Milnor

[1] *Whitehead torsion*, Bull. Amer. Math. Soc. 72 (1966), 358–426. MR 33 #4922.

V. L. Nisnevič

[1] *Über Gruppen, die durch Matrizen über einem kommutativen Feld isomorph darstellbar sind*, Mat. Sb. 8 (50) (1940), 395–404. (Russian; German summary) MR 2, 309.

P. S. Novikov and S. I. Adjan

[1] *On finite periodic groups*. I, II, III, Izv. Akad. Nauk SSSR Ser. Mat. 32 (1968), 212–244, 251–524, 709–731 = Math. USSR Izv. 2 (1968), 209–237, 241–479, 665–685. MR 39 #1532a,b,c.

V. P. Platonov

[1] *On the theory of algebraic linear groups*, Dokl. Akad. Nauk SSSR 146 (1962), 1025–1026 = Soviet Math. Dokl. 3 (1962), 1448–1449. MR 26 #1370.

[2] *Solvable algebraic groups*, Dokl. Akad. Nauk SSSR 151 (1963), 48–51 = Soviet Math. Dokl. 4 (1963), 924–927. MR 27 #2508.

[3] *Splittable linear groups*, Dokl. Akad. Nauk SSSR 151 (1963), 286–289 = Soviet Math. Dokl. 4 (1963), 1002–1005. MR 29 #4806.

[4] *Periodic subgroups of algebraic groups*, Dokl. Akad. Nauk SSSR 153 (1963), 270–272 = Soviet Math. Dokl. 4 (1963), 1653–1655. MR 28 #2152.

[5] *Solvable algebraic groups*, Dokl. Akad. Nauk BSSR 7 (1963), 298–300. (Russian) MR 27 #3639.

[6] *On solvable algebraic linear groups*, Dokl. Akad. Nauk BSSR 7 (1963), 439–442. (Russian) MR 27 #5825.

[7] *On complete reducibility of linear groups*, Vescī Akad. Navuk BSSR Ser. Fīz.-Tèhn. Navuk 1964, no. 3, 126–128. (Russian) MR 30 #171.

[8] *Structure of periodic linear groups and algebraic groups*, Dokl. Akad. Nauk SSSR 160 (1965), 541–544 = Soviet Math. Dokl. 6 (1965), 144–147. MR 30 #3924.

[9] *Algebraic d-groups*, Dokl. Akad. Nauk BSSR 7 (1963), 729–731. (Russian) MR 28 #1260.

[10] *Automorphisms of algebraic groups*, Dokl. Akad. Nauk SSSR 168 (1966), 1257–1260 = Soviet Math. Dokl. 7 (1966), 825–828. MR 35 #4230.

[11] *Theory of algebraic linear groups and periodic groups*, Izv. Akad. Nauk SSSR Ser. Mat. 30 (1966), 573–620; English transl., Amer. Math. Soc. Transl. (2) 69 (1968), 61–110. MR 33 #7428.

[12] *The Frattini subgroup of linear groups and finite approximability*, Dokl. Akad. Nauk SSSR 171 (1966), 798–801 = Soviet Math. Dokl. 7 (1966), 1557–1560. MR 35 #2971.

[13] *Linear groups with identical relations*, Dokl. Akad. Nauk BSSR 11 (1967), 581–582. (Russian) MR 35 #4314.

[14] *Proof of the finiteness hypothesis for solvable subgroups of algebraic groups*, Sibirsk. Mat. Ž. 10 (1969), 1084–1090 = Siberian Math. J. 10 (1968), 800–804. MR 40 #7355.

V. P. Platonov and A. E. Zalesskiĭ

[1] *The Auerbach problem*, Dokl. Akad. Nauk BSSR 10 (1966), 5–6. (Russian) MR 33 #224.

B. I. Plotkin

[1] *On some criteria for locally nilpotent groups*, Uspehi Mat. Nauk 9 (1954), no. 3 (61), 181–186; English transl., Amer. Math. Soc. Transl. (2) 17 (1961), 1–7. MR 16, 444; 23 #A1711.

[2] *On nil-groups*, Dokl. Akad. Nauk SSSR 94 (1954), 999–1001. (Russian) MR 15, 682.

[3] *Infinite-dimensional linear groups*, Dokl. Akad. Nauk SSSR 153 (1963), 42–45 = Soviet Math. Dokl. 4 (1963), 1617–1620. MR 28 #141.

[4] *Groups of automorphisms of algebraic systems*, "Nauka", Moscow, 1966; English transl., Wolters-Noordhoff, Groningen, 1972. MR 40 #217.

C. Procesi

[1] *The Burnside problem*, J. Algebra 4 (1966), 421–425. MR 35 #2956.

I. Reiner

[1] *Normal subgroups of the unimodular group*, Illinois J. Math. 2 (1958), 142–144. MR 20 #2379.

A. Rosenberg

[1] *The structure of the infinite general linear group*, Ann. of Math. (2) 68 (1958), 278–294. MR 21 #1319.

I. Schur

[1] *Über Gruppen periodischer linearer Substitutionen*, S.-B. Preuss. Akad. Wiss. 1911, 619–627.

A. Selberg

[1] *On discontinuous groups in higher-dimensional symmetric spaces*, Contributions to Function Theory (Internat. Colloq. Function Theory, Bombay, 1960), Tata Institute of Fundamental Research, Bombay, 1960, pp. 147–164. MR 24 #A188.

J.-P. Serre

[1] *Représentations linéaires des groupes finis*, Hermann, Paris, 1967. MR 38 #1190.

O. Ju. Šmidt

[1] *Selected works. Mathematics*, Izdat. Akad. Nauk SSSR, Moscow, 1959. (Russian) MR 21 #1259.

D. A. Suprunenko

[1] *Primitive solvable substitution groups*, Mat. Sb. 20 (62) (1947), 331–350. (Russian) MR 8, 562; 14, 1277.

[2] *Solvable matrix groups*, Učen. Zap. Leningrad. Gos. Univ. No. 12 (1951), 74–112. (Russian)

[3] *Irreducible nilpotent matrix groups of prime degree*, Mat. Sb. 31 (73) (1952), 353–358. (Russian) MR 14, 447.

[4] *On nilpotent groups of matrices*, Belorussk. Gos. Univ. Učen. Zap. Ser. Fiz.-Mat. 1953, no. 15, 3–6. (Russian) MR 17, 456.

[5] *On nilpotent transitive subgroups of the symmetric group*, Dokl. Akad. Nauk SSSR 99 (1954), 23–25. (Russian) MR 16, 793.

[6] *On irreducible nilpotent matrix groups*, Mat. Sb. 35 (77) (1954), 501–512. (Russian) MR 16, 793.

[7] *Locally nilpotent irreducible subgroups of the full linear group*, Dokl. Akad. Nauk SSSR 102 (1955), 41–44. (Russian) MR 17, 456.

[8] *Soluble and nilpotent linear groups*, Ministerstvo Vysšego Obrazovanija SSSR, Izdat. Belgosuniv. im V. I. Lenina, Minsk, 1958; English transl., Transl. Math. Monographs, vol. 9, Amer. Math. Soc., Providence, R. I., 1963. MR 27 #4861a,b.

[9] *Real linear locally nilpotent groups*, Mat. Sb. 50 (92) (1960), 59–66; English transl., Amer. Math. Soc. Transl. (2) 45 (1965), 23–31. MR 22 #2653.

[10] *On periodic linear groups*, Sibirsk. Mat. Ž. 3 (1962), 87–94. (Russian) MR 27 #2560.

[11] *Periodic subgroups of solvable matrix groups*, Dokl. Akad. Nauk SSSR 147 (1962), 310–312 = Soviet Math. Dokl. 3 (1962), 1612–1614. MR 26 #220.

[12] *On the Minkowski homomorphism*, In Memory of N. G. Čebotarev, Izdat. Kazan. Univ., Kazan, 1964, pp. 89–92. (Russian) MR 33 #4151.

[13] *The kernel of a homomorphism*, Sibirsk. Mat. Ž. 6 (1965), 199–206. (Russian) MR 30 #3893.

[14] *Locally nilpotent matrix groups over an arbitrary field*, Mat. Sb. 68 (110) (1965), 614–622. (Russian) MR 33 #198.

[15] *Locally nilpotent subgroups of the infinite symmetric group*, Dokl. Akad. Nauk SSSR 167 (1966), 302–304 = Soviet Math. Dokl. 7 (1966), 392–394. MR 33 #2709.

[16] *A condition for complete reducibility of a matrix group*, Izv. Akad. Nauk SSSR Ser. Mat. 27 (1963), 435–438. (Russian) MR 26 #6268.

[17] *On solvable subgroups of the multiplicative group of a field*, Izv. Akad. Nauk SSSR Ser. Mat. 26 (1962), 631–638; English transl., Amer. Math. Soc. Transl. (2) 46 (1965), 153–161. MR 25 #5111.

[18] *Two theorems on matrix groups*, Dokl. Akad. Nauk BSSR 8 (1964), 491–494. (Russian) MR 30 #172.

[19] *On maximal nilpotent subgroups of the full linear group over a factor ring* $Z/(p^m)$, Mat. Sb. 66 (108) (1965), 598–607. (Russian) MR 30 #4839.

[20] *Irreducible components of a reducible maximal locally nilpotent matrix group*, Izv. Akad. Nauk SSSR Ser. Mat. 32 (1968), 527–533 = Math. USSR Izv. 2 (1968), 481–486. MR 37 #1487.

[21] *The theory of commutative matrix algebras over an imperfect field*, Vescī Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk 1967, no. 3, 9–19. (Russian) MR 36 #208.

[22] *On the theory of solvable linear groups*, Sibirsk. Mat. Ž. 10 (1969), 1161–1172 = Siberian Math. J. 10 (1969), 859–867. MR 40 #4381.

[23] *Subgroups of the full linear group over a sfield D that contain the group U(n, D) of all special triangular matrices*, Dokl. Akad. Nauk BSSR 14 (1970), 305–308. (Russian) MR 43 #3354.

D. A. Suprunenko and R. F. Apatenok

[1] *Nilpotent irreducible groups over a finite field*, Dokl. Akad. Nauk BSSR 3 (1959), 475–478. (Russian)

[2] *Nilpotent irreducible matrix groups over a finite field*, Dokl. Akad. Nauk BSSR 5 (1961), 535–537. (Russian) MR 27 #5834.

D. A. Suprunenko and M. S. Garaščuk

[1] *Linear groups with a category*, Dokl. Akad. Nauk BSSR 6 (1962), 411–414. (Russian) MR 26 #1369.

[2] *Linear groups with Engel's condition*, Dokl. Akad. Nauk BSSR 6 (1962), 277–280. (Russian) MR 26 #1368.

D. A. Suprunenko and V. P. Platonov

[1] *On a theorem of Schur*, Dokl. Akad. Nauk BSSR 7 (1963), 510–512. (Russian) MR 27 #4860.

D. A. Suprunenko and R. I. Tyškevič

[1] *Commutative matrices*, Nauka i Tehnika, Minsk, 1966; English transl., Academic Press, New York, 1968. MR 34 #1356.

[2] *Reducible locally nilpotent linear groups*, Izv. Akad. Nauk SSSR Ser. Mat. 24 (1960), 787–806. (Russian) MR 25 #124.

D. A. Suprunenko and G. P. Žavrid

[1] *Intransitive nilpotent subgroups of the symmetric group*, Dokl. Akad. Nauk BSSR 2 (1958), 320–322. (Russian) MR 41 #313.

R. G. Swan

[1] *Representation of polycyclic groups*, Proc. Amer. Math. Soc. 18 (1967), 573–574. MR 35 #4306.

R. T. Vol'vačev

[1] *Sylow p-subgroups of the general linear group*, Izv. Akad. Nauk SSSR Ser. Mat. 27 (1963), 1031–1054; English transl., Amer. Math. Soc. Transl. (2) 64 (1967), 216–243. MR 28 #4037.

[2] *Periodic nilpotent linear groups over the field of rational numbers*, Dokl. Akad. Nauk SSSR 167 (1966), 499–502 = Soviet Math. Dokl. 7 (1966), 403–406. MR 36 #2712.

B. L. van der Waerden

[1] *Moderne Algebra*. Vol. I, 2nd ed., Springer, Berlin, 1937; English transl., Ungar, New York, 1949. MR 10, 587.

[2] *Moderne Algebra*. Vol. II, 2nd ed., Springer, Berlin, 1940; English transl., Ungar, New York, 1950. MR 10, 587.

[3] *Gruppen von linearen Transformationen*, Springer, Berlin, 1935.

B. A. F. Wehrfritz

[1] *Sylow theorems for periodic linear groups*, Proc. London Math. Soc. (3) 18 (1968), 125–140. MR 36 #3893.

[2] *Soluble periodic linear groups*, Proc. London Math. Soc. (3) 18 (1968), 141–157. MR 36 #3894.

[3]* *Infinite linear groups. An account of the group-theoretic properties of infinite groups of matrices*, Ergebnisse der Math. und ihrer Grenzgebiete, Band 76, Springer-Verlag, Berlin and New York, 1973. MR 49 #436.

A. J. Weir

[1] *Sylow p-subgroups of the classical groups over finite fields with characteristic prime to p*, Proc. Amer. Math. Soc. 6 (1955), 529–533. MR 17, 235.

G. P. Žavrid

[1] *On the theory of solvable matrix groups*, Dokl. Akad. Nauk BSSR 11 (1967), 873–877. (Russian) MR 36 #2713.

A. E. Zalesskiĭ

[1] *Solvable subgroups of the multiplicative group of a locally finite algebra*, Mat. Sb. 61 (103) (1963), 408–417. (Russian) MR 28 #134.

[2] *Solvable subgroups of the multiplicative group of a simple algebra*, Dokl. Akad. Nauk BSSR 7 (1963), 80–82. (Russian) MR 26 #6254.

[3] *Sylow p-subgroups of the general linear group over a skew field*, Izv. Akad. Nauk SSSR Ser. Mat. 31 (1967), 1149–1158 = Math. USSR Izv. 1 (1967), 1099–1108. MR 36 #2710.

H. Zassenhaus

[1] *Beweis eines Satzes über diskrete Gruppen*, Abh. Math. Sem. Univ. Hamburg 12 (1938), 289–312.

*Added by the editor.

This page intentionally left blank

Subject Index

Algebra over a field, 87
associative, 87
semisimple, 87
Automorphism of a module, 38

Basis of a module, 37
Burnside's criterion for finiteness of a linear group
First, 178
Second, 179
Burnside's Theorem, 45

Central series
Lower, 205
Upper, 205
Clifford's Theorems
First, 113
Second, 114
Cycle, 5
Complete, 5
Length of, 5
of a permutation, 5
Type, 5

Decomposition of a space into systems of imprimitivity, 103
Nonrefinable, 104
Degree of a permutation group, 2
Determinant, 57
Dieudonné, 57
d-group, 119
Dimension of a module, 37
d-matrix, 119

Element
Free, 37
Invertible, 35
Unipotent, 35
End M, 38
Endomorphism of a module, 38
Equivalence of linear representations, 91

Filter, 93

Group
Affine Aff(n, Δ), 50
$CSL(n, Z, N)$, 76
$E(n, \Delta, N)$, 65
$\Gamma_k(F)$, 222
Hypersymplectic $HSp(2l, P)$, 147
Linear
Absolutely irreducible, 100
Complete
$GL(M)$, 39
$GL(n, \Delta)$, 44
stable $GL(\Delta)$, 64
Imprimitive, 103
Primitive, 103
Special $SL(n, \Delta)$, 47
Stable $SL(\Delta)$, 94
Locally nilpotent, 216
Monomial, 105
Operator-simple, 83
Decomposable, 84
Permutation, 2
Primitive solvable, 23
$PSL(M)$, 63
Regular, 7
Satisfying the normalizer condition (= N-group), 27
$SF(X)$, 33
Symmetric, 2
Finite S_n, 2
Symplectic $Sp(2l, p)$, 147
Unitriangular $T(n, \Delta)$, 125

Homothety, 40
Hypercenter, 205
Hyperplane of a module, 37

Imprimitivity, 7
Criterion for, 105
Intransitivity, 5
Irreducible components
of a linear representation, 92
of a set of endomorphisms, 92

Jordan's Theorem, 190

Kronecker product
of endomorphisms, 110
of matrices, 110

Linear Δ-hull of a completely reducible group, 98

Linearly independent system of elements of a module, 37
Local system of a group, 93

Mal'cev's local theorem, 93
Mapping, 9
  Bijective, 9
  Injective, 9
  Surjective, 9
Matrix
  Elementary $t_{ij}(\lambda)$, 46
  of an endomorphism, 44
    in various bases, 45
  Semisimple, 119
Minkowski homomorphism, 65
Minkowski's Theorem, 68
Module, 36
  Free, 37
Multiplication of mappings, 1
Multiplicative group of a ring, 35

N-congruence subgroup $CL(n, \Delta, N)$, 65
N-group, 27
Nilpotency class
  of a group, 205
  of a ring, 88
Normal subgroups of transitive groups, 13

Operator isomorphism, 83
Orbit of a permutation group, 5
Orbital type, 5

Partition into systems of imprimitivity, 7
  Refinement of, 10
Primitivity, 7

Radical of an algebra, 87
Representation
  by a mapping, 3
    Equivalent, 3
  Linear, 90
    Decomposable, 92
    Faithful, 92
    Indecomposable, 92
    Irreducible, 92
    Reducible, 92
  of an algebra
    Left regular, 97
    Right regular, 101
Restriction of mappings, 1

Schur's Lemma, 85
Schur's Theorems
  First, 181
  Second, 189
Semigroup
  Adjoint, of a ring, 89
  Symmetric $\Sigma(X)$, 2
Similarity of subsets, 2
s-irreducible subgroup, 155
s-reducible subgroup, 155
Stabilizer, 6
Subgroup
  Admissible, 83
  Maximal transitive nilpotent of S_n, 30
  of the symplectic group
    s-irreducible, 155
    s-reducible, 155
Submodule, 37
System of imprimitivity, 7, 103

Tensor product
  of endomorphisms, 110
  of matrices, 110
Theorem
  Burnside's, 45
  Clifford's First, 113
  Clifford's Second, 114
  Jordan's, 190
  Mal'cev's local, 93
  Minkowski's, 68
  on the holomorph, 18
  Schur's First, 181
  Schur's Second, 189
Transitivity, 5
  Multiple, 6
Transvection of a module, 40

Ultrafilter, 93
Ultraproduct, 94

Vector space, 38

Whitehead's Lemma, 64
Wreath product
  of a linear group and a permutation group, 106
  of permutation groups, 11

ISBN 978-0-8218-1341-6

9 780821 813416

MMONO/45.S