

Translations of
**MATHEMATICAL
MONOGRAPHS**


Volume 51

Equations of Mixed Type

M. M. Smirnov



American Mathematical Society



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Equations of Mixed Type

M. M. Smirnov



American Mathematical Society
Providence, Rhode Island

УРАВНЕНИЯ СМЕШАННОГО ТИПА

М. М. СМОРНОВ

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ABSTRACT. This book is devoted to the theory of partial differential equations of mixed type. The author introduces the reader to the current state of the mathematical problems closely connected with transonic gas dynamics.

Fundamental boundary value problems are considered: the Tricomi problem, the generalized Tricomi problem for Čaplygin's equation, the Frankel' problem and the modified Tricomi problem.

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