Boundary Value Problems for Elliptic Pseudodifferential Equations

G. I. ÈSKIN

Volume 52

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

American Mathematical Society

10.1090/mmono/052

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VOLUME 52

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American Mathematical Society · Providence · Rhode Island

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КРАЕВЫЕ ЗАДАЧИ ДЛЯ ЭЛЛИПТИЧЕСКИХ ПСЕВДОДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

Г. П. ЭСКИН

ИЗДАТЕЛЬСТВО «НАУКА»
МОСКВА 1973

Translated from the Russian by S. Smith

1980 *Mathematics Subject Classification.* Primary 35S15;
Secondary 35S05, 47G05, 58G15.

ABSTRACT. The present monograph is devoted to the class of equations known as pseudodifferential equations, which comprises differential and multidimensional singular integral equations as well as integral equations of the first kind and integrodifferential equations whose kernels have a weak singularity. Mixed boundary value problems for elliptic equations are investigated with the use of the Wiener-Hopf method, the asymptotic behavior of their solutions is obtained and various examples are analyzed. It is intended for mathematicians and theoreticians in continuum mechanics.

Library of Congress Cataloging in Publication Data

Ėskin, Grigoriĭ Il'ich.

Boundary value problems for elliptic pseudodifferential equations.

Translation of Kraevye zadachi dlя ėllipticheskikh psevdodifferent͡sial'nykh uravneniĭ.

Bibliography: p.

Includes index.

1. Boundary value problems. 2. Pseudodifferential operators. I. Title.

QA379.E8413 515.3'5 80-39789

ISBN 0-8218-4503-9

10 9 8 7 6 5 4 3 94 93 92 91 90 89

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PREFACE TO THE ENGLISH EDITION

The English edition differs from the Russian in that an Introduction and three new sections (§§25–27) have been added. Moreover, various corrections, improvements and remarks have been made throughout the book, especially in Chapter VI.

The author wishes to express his thanks to the American Mathematical Society, for its decision to publish an English edition of his book, and to Dr. S. F. Smith, whose competent translation brought to light a number of inaccuracies in the Russian edition.

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PREFACE

The theory of boundary value problems for elliptic pseudodifferential equations has its roots in (i) the theory of multidimensional singular integral equations on a closed manifold (due to Giraud, Mihlin, Calderón and Zygmund, and others; see Mihlin's monograph [65] and the literature cited there), (ii) the theory of boundary value problems for elliptic differential equations with differential and pseudodifferential boundary conditions (due to Agmon, Douglis, Nirenberg, Browder, Lopatinskiĭ, Slobodeckiĭ, Peetre, Hörmander, Šapiro, Schechter, Agranovič, A. S. Dynin and others; see the survey by Agranovič [4]), and (iii) the theory of integral equations of convolution type on a halfline and singular integral equations on an open contour (see the works of Kreĭn, Gohberg and their students [50], [51], [60] and the monographs of Mushelišvili [67], Gahov [45] and Noble [70]).

In 1962 the author, starting from a paper [73] of Peetre and his own paper [35], considered mixed boundary value problems for multidimensional elliptic differential equations of chiefly second order. By reducing such a mixed boundary value problem to a pair of pseudodifferential equations and applying the Wiener-Hopf method, the author proved its solvability in a Sobolev-Slobodeckiĭ space determined by the factorization index of the corresponding symbol; in addition, he found the smoothness and asymptotic behavior of its solution.

These investigations were jointly continued by M. I. Višik and the author. Together they (i) constructed a theory of elliptic pseudodifferential equations and systems on a manifold with boundary that included a theory of multidimensional singular integral equations in domains with a smooth boundary, (ii) found a statement of boundary value problems for elliptic pseudodifferential equations and systems and proved their normal solvability in weighted spaces of functions of piecewise constant and variable order of smoothness,

and (iii) distinguished a subclass of pseudodifferential equations whose solutions for smooth right sides are smooth up to the boundary.

The normal solvability in L_2 of a singular integral operator was independently investigated by Simonenko [85].

The theory of pseudodifferential operators in $\mathbf{R}^n$ and on a manifold without boundary is due to Kohn and Nirenberg [59] and Hörmander [55] (see also the works of Friedrichs, Seeley, Bokobza-Haggiag and Unterberger, M. I. Višik and the author, and others).

Boundary value problems for elliptic pseudodifferential equations in domains with a smooth boundary have also been studied by Shamir [79], Dynin [33], Boutet de Monvel [16], [17], Šubin [95], Dikanskiĭ [28], Šnirel'man, the author and others.

In the course of writing the book most of these investigations underwent an essential revision: sharper and more complete results have been obtained and new, simpler proofs have been found. The book also contains new results that have been obtained by the author and are being published for the first time.

The author wishes to take this opportunity to express his gratitude to his teacher G. E. Šilov for his help and support. The author also sincerely thanks V. B. Lidskiĭ for his constant interest in the work.

M. S. Agranovič, A. S. Dikanskiĭ and B. P. Panejah have read the manuscript and made many useful remarks, for which the author is deeply grateful.

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