

Systems of Quasilinear Equations and Their Applications to Gas Dynamics

by **B. L. ROŽDESTVENSKIĬ**
N. N. JANENKO

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СИСТЕМЫ
КВАЗИЛИНЕЙНЫХ УРАВНЕНИЙ
И ИХ ПРИЛОЖЕНИЯ
К ГАЗОВОЙ ДИНАМИКЕ

ИЗДАНИЕ ВТОРОЕ

Б. Л. РОЖДЕСТВЕНСКИЙ
И Н. Н. ЯНЕНКО

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ABSTRACT. This book is essentially a new edition, revised and augmented by results of the last decade, of the work of the same title published in 1968 by "Nauka". It is devoted to mathematical questions of gas dynamics.

In Chapter 1 the theory of systems of quasilinear equations, the basic mathematical apparatus of gas dynamics, is presented. Chapter 2 contains a consideration of the main problems of one-dimensional gas dynamics, while Chapter 3 is an account of difference methods. The last, fourth chapter is devoted to the theory of discontinuous solutions of systems of quasilinear equations.

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PREFACE TO THE SECOND EDITION

Ten years have passed since the appearance of the first edition of our book. In this time many new and interesting results on the questions considered in the book have been obtained, especially on difference methods of solving problems of mathematical physics and gas dynamics.

Some corrections of the text have been inserted in the second edition, and also a considerable number of additions to reflect the progress that has taken place. But as in the first edition, the general plan and style of the exposition affects the choice of material. This applies particularly to the new material in Chapter 3, which is devoted to difference methods. The great number of papers on difference methods, together with the necessarily small size of Chapter 3, have compelled us to omit a number of theoretical questions already covered in sufficient detail in accessible monographs and textbooks.

In preparing the second edition we have again made use of the help of our friends and colleagues, as well as our students. We express our profound gratitude to all of them.

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FROM THE PREFACE TO THE FIRST EDITION

In writing the book the authors interacted with various collectives of Soviet mathematicians. Among them we mention those headed by M. V. Keldyš, A. N. Tihonov and A. A. Samarskii, and I. M. Gel'fand. The interaction with friends and professional colleagues inevitably affected our opinions and points of view; a number of results became known to us through conversations with them.

During the course of a number of years each of us gave special courses for students on the theme of this book. As a result of work on the book, several new results were obtained which are published here for the first time.

The present book arose from many years of work during which time we constantly enjoyed the help of many of our friends and professional colleagues and also of many of our students.

We are grateful to A. N. Tihonov, whose advice we constantly followed.

The help of L. V. Ovsjannikov was especially valuable to us; he not only looked through the manuscript of the entire book and made a number of valuable remarks, but also placed at our disposal materials which we used in writing §13 of Chapter 1.

A. A. Samarskii read the manuscript for Chapter 3 and made a number of valuable suggestions.

N. N. Kuznecov helped us a great deal; he read the entire manuscript, made a number of valuable suggestions, and as editor of this book greatly contributed to its improvement.

We express our gratitude to all of them.

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INTRODUCTION

To describe the behavior of a continuous medium (a gas, liquid, or solid) theoretical physics uses various models which in most cases lead to nonlinear partial differential and integro-differential equations.

The mechanics of continuous media is the main area of practical application of systems of nonlinear partial differential equations, but it is not the only one. In describing the majority of real physical processes we arrive at nonlinear equations, and only essential additional assumptions regarding the smallness of the amplitudes of waves of the field or the amplitudes of oscillations of the medium, the amplitudes of the deviation for the equilibrium state, etc. lead to linear equations which have been studied more extensively. In Chapter 4 of the book a number of examples are presented of problems of physics, chemistry, and mathematics which are connected with nonlinear equations.

The study of general properties of nonlinear equations and methods of solving them is a rapidly developing area of modern mathematics.

For all the interesting facts and the variety of original and clever methods for investigating and solving nonlinear equations, this area of mathematics does not yet have as solid a theoretical foundation as the theory of linear equations. This is related primarily to the fact that the principle of superposition of solutions is not applicable to nonlinear differential equations, so that the manifold of solutions is not linear.

Systems of quasilinear equations are the simplest hyperbolic systems of nonlinear partial differential equations. Systems of equations with two independent variables have been most studied; these systems describe, in particular, nonstationary one-dimensional and supersonic two-dimensional stationary flows of compressible gases and liquids. However, even for these systems there is presently no sufficiently complete theory; there are no general existence and uniqueness theorems for a solution of the initial value problem.

This is due to the fact that for hyperbolic systems of nonlinear equations the solution of the Cauchy problem involves considerable complication both in the very formulation of the problem and the methods of solving it. Moreover,

almost all the basic difficulties that arise here are present already for the case of two independent variables, and it may be expected that solutions of multi-dimensional equations of gas dynamics locally have essentially the same features as solutions of one-dimensional equations.

The study of hyperbolic systems of nonlinear equations in two independent variables thus constitutes an altogether necessary and so far incomplete stage in the investigation of more general nonlinear equations.

On the basis of these considerations the authors decided to restrict attention mainly to the theory of hyperbolic systems in two independent variables and the study of one-dimensional, nonstationary flows of compressible liquids and gases. Therefore, as a rule, we speak of one of the variables as the time and denote it by the letter t .

Here we shall briefly describe the present state of the question of the solvability of the Cauchy problem for hyperbolic systems of quasilinear equations and the difficulties arising in the attempt to construct a global solution of this problem. The basic method of solving hyperbolic systems of quasilinear equations is the method of characteristics, which is expounded in detail in Chapter 1. The existence, uniqueness, and the continuous dependence on the initial data of a classical solution of the Cauchy problem are proved by means of this method. The results obtained are highly satisfactory in the sense that the classical solution is constructed in the entire domain of the variables t and x where it exists. We observe that the domain of existence of a classical solution is, in general, limited, since solutions of nonlinear equations, in contrast to solutions of linear equations, possess the property of unbounded growth of the magnitude of derivatives which is called the "gradient catastrophe".

The meaning of this property is that even for arbitrarily smooth initial data the first derivatives of a solution become unbounded, in general, in finite time. For some $t_0 > 0$ they become unbounded, and for $t > t_0$ a classical solution of the Cauchy problem does not exist.

From the point of view of gas dynamics this corresponds to the formation of a shock wave (a condensation jump) from the compression wave. Thus, if we wish to determine a solution of the Cauchy problem for any $t \geq 0$, i.e. globally (and this is precisely the problem, for example, in gas dynamics), then we must first find a definition of the solution, since, as already mentioned, a solution of the system of equations in the usual sense—a classical solution—does not exist for $t > t_0$.

In the majority of physical problems and, in particular, in gas dynamics the definition of a generalized solution is dictated by the very formulation of the problem. Thus, for example, in gas dynamics the basic physical laws from which we derive all consequences are the laws of conservation of mass, momentum,

and energy. These conservation laws have the character of integral relations, and they are applicable not only to smooth (differentiable) flows. On the other hand, the differential equations of gas dynamics are obtained from these conservation laws under the assumption that the flow is smooth.

We thus define a generalized solution of the equations of gas dynamics as a flow (possibly even with discontinuous parameters) satisfying the basic conservation laws of mass, momentum, and energy. To this we add the thermodynamic requirement that the entropy increase in each thermodynamically closed system. There is a broadly held opinion, which has so far not been contradicted by a single example, that a solution thus defined exists, is unique, and satisfies all reasonable requirements.

The requirement of thermodynamics regarding the increase of entropy is very essential: it indicates the possible direction of the process of rapid variation of the state of the gas. This requirement does not enter in considering classical solutions of the equations of gas dynamics for a gas without viscosity or thermal conductivity, since in smooth flows the entropy of the system is conserved by virtue of the same basic conservation laws.

In gas dynamics another approach to generalized (discontinuous) flows of an ideal gas without viscosity or thermal conductivity is well known. Since a gas without dissipation is an idealization of a gas possessing dissipative processes, it is natural to consider its discontinuous flow as a "limit flow" of a viscous, conducting gas as the coefficients of viscosity and thermal conductivity tend to zero. Here it is assumed that the viscous flows are always described by classical solutions of the differential equations, while the limit as the dissipative coefficients tend to zero exists and is unique in a reasonable sense. This assumption has, in fact, so far not been contradicted by a single example, although precise proofs have presently been obtained only for the very special case of a stationary shock wave.

Here it should be observed that in many cases real gases possess rather small dissipation, so that they can be "approximated" by nondissipative gases. However, the presence of dissipative processes, even small ones, leads to an increase of the entropy of the system. Thus, the requirement of the increase of entropy in the discontinuous flow of an ideal gas is related to the representation of this flow as a "limit" flow of a viscous, thermally conducting gas.

We note that from a mathematical point of view the requirement of the increase of entropy is the condition guaranteeing the uniqueness of a generalized solution and its stability with respect to perturbations.

Although this formulation of the problem of the flow of compressible gases has been known for more than a century (Riemann studied the simplest discontinuous flows), the progress in studying general properties of generalized

solutions of the equations of gas dynamics has been relatively minor. Thus, as we have already mentioned, there are still no satisfactory existence and uniqueness theorems.

On the other hand, practical requirements occasioned by the pressing necessity of the practical study of discontinuous flows and also new computing capabilities related to the application of rapid computing technology have led to the situation that, in spite of our inadequate information regarding general properties of discontinuous flows, various numerical algorithms have been created and used which make it possible to satisfactorily compute flows with shock waves. It should be mentioned that in creating these numerical algorithms the majority of the conjectures that we mentioned above were accepted as true.

In view of the fact that a direct and rigorous justification of various assumptions regarding generalized solutions in gas dynamics is a difficult problem, it is natural to hope to verify our views for model equations and systems of equations which to some extent imitate the equations of gas dynamics.

A consequence of this desire was the appearance in the last decade of the so-called *theory of generalized solutions of systems of quasilinear equations* or, more briefly, *the theory of systems of quasilinear equations* (here systems of hyperbolic type are usually meant). This theory poses the problem of introducing in analogy with gas dynamics the concept of a generalized solution for an "arbitrary" system of quasilinear partial differential equations of hyperbolic type, proving its existence, uniqueness, and continuous dependence on the initial data of the problem, and studying the properties of such solutions. At least formally, this theory is more general than one-dimensional gas dynamics and includes the latter as a special case.

It has attracted the attention of many mathematicians, and the number of results obtained through the efforts of Soviet and foreign scientists make it possible to expect further development of the theory.

On the basis of this view of the development of the theory of generalized (discontinuous) solutions of systems of quasilinear equations, the authors limited their attention to the case of only two independent variables and included in the book the following basic questions:

1. Methods of constructing classical solutions of systems of quasilinear equations; proofs of existence and uniqueness theorems and continuous dependence of classical solutions; analytic methods of constructing solutions of systems of nonlinear equations; and conditions for the formation of discontinuities in solutions of arbitrary systems of quasilinear equations. These questions are discussed in Chapter 1. Results obtained for classical solutions of systems of quasilinear equations during recent years are presented there.

2. Classical and generalized solutions of the equations of gas dynamics for

one-dimensional, nonstationary flows. This question is discussed in Chapter 2. The authors considered it expedient to consider in detail certain questions of gas dynamics which are discussed in many textbooks. The foundations of thermodynamics, a derivation of the equations of gas dynamics for various symmetries of a one-dimensional flow, the Hugoniot conditions, generalized properties of flows, the theory of the shock transition, and self-similar and analytic solutions of gas dynamics are presented. The inclusion in the book of these traditional questions of gas dynamics makes it possible to expound from a unified point of view certain mathematical problems which arise in gas dynamics; moreover, the majority of numerical methods in gas dynamics are actually based on this material. The basic problem of the theory of discontinuous solutions of the equations of gas dynamics—the problem of the decay of an arbitrary discontinuity—and also the interaction of shock waves with one another, with travelling waves, and with a contact boundary are considered in detail.

3. Chapter 3 is devoted to difference methods for solving the equations of gas dynamics. In our time these methods have become a basic means of studying problems of gas dynamics, and therefore progress in the study of discontinuous flows is to a considerable extent connected with difference methods.

In this chapter we have had to present the basic concepts of the theory of difference methods. Unfortunately, the majority of assertions of this theory pertain only to the case of linear equations.

The present situation regarding the justification of difference methods applied to the numerical solution of problems of gas dynamics is briefly as follows. Classical solutions (smooth flows) can be computed with almost arbitrary accuracy. The basic method—the numerical method of characteristics—for classical solutions is sufficiently justified. On the other hand, numerical methods applied to compute discontinuous flows have not been rigorously justified, and in the majority of cases certain conjectures regarding the behavior of solutions, the approximation of certain equations by others, etc. are used. Simple equations for which the behavior of the discontinuous solution is well known are most frequently used to verify the various assumptions. It is no accident that in this chapter in most cases each scheme is subject to verification on a simple quasi-linear equation with a solution which can be written out explicitly.

This situation regarding the justification of difference methods indicates that progress in this area is to a considerable extent connected with progress in the study of general properties of generalized solutions of systems of quasi-linear equations and, in particular, the equations of gas dynamics. On the other hand, difference methods provide experimental material and strongly stimulate the development of the theory of generalized solutions.

4. Chapter 4 is devoted to the theory of generalized solutions of systems of

quasilinear equations of hyperbolic type, and contains the basic results obtained in this area in recent years. A major success here is the construction of a theory of a generalized solution of a single quasilinear equation, which may be considered near completion. For this equation existence, uniqueness, and continuous dependence of the generalized solution of the initial data are proved, and the equivalence of the definitions of a generalized solution from the point of view of the conservation law on the one hand and as a limit of "viscous solutions" on the other is demonstrated.

On the other hand, as in gas dynamics, the study of generalized solutions of systems of equations encounters major difficulties, and very meager results have so far been obtained here. The basic problem, which is currently being subjected to thorough investigation, is the problem of the decay of an arbitrary discontinuity. By means of this simple problem it is possible to study the structure of the generalized solution, and on the basis of this structure it is even possible to construct generalized solutions for the case of a system of two equations.

It should be noted that in recent years more general problems for systems of quasilinear equations have also been studied intensively.

In Chapter 4 the basic results obtained for a single quasilinear equation are presented, the problem of the decay of a discontinuity for an arbitrary hyperbolic system of quasilinear equations is considered, and some results pertaining to more general cases are presented. To conclude this chapter a number of problems are described in various areas of science which are related to the theory of systems of quasilinear equations and, in particular, to discontinuous solutions of such equations.

From what has been said above it should be clear that the mathematical theory of discontinuous solutions of systems of quasilinear equations and, in particular, of the equations of gas dynamics, although it contains many remarkable results and achievements, is far from complete. We hope that our book will afford the reader an idea of the modern methods of solving and studying systems of quasilinear equations and will at the same time spur him to further investigations in this interesting and rapidly developing area of applied mathematics.

The book is subdivided into chapters, sections, and subsections. Formulas are numbered independently in each subsection, and hence references contain the number of the section and subsection in addition to the number of the formula, so that formula (2.7.18) signifies formula (18) in Subsection 7 of Section 2 (i.e., §2.7) of the given chapter. The number of the formula alone is given only in the case where the reference does not go beyond the confines of the given subsection.

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BIBLIOGRAPHY

- S. Abarbanel and D. Gottlieb
(1973) *Higher order accuracy finite difference algorithms for quasi-linear, conservation law hyperbolic systems*, Math. Comp. **27**, 505–523.
- V. B. Adamskii
(1956) *Integration of a system of self-similar equations in the problem of a shock of short duration in a cold gas*, Akust. Ž. **2**, 3–9; English transl. in Soviet Phys. Acoustics **2**.
- D. N. de G. Allen and R. V. Southwell
(1955) *Relaxation methods applied to determine the motion, in two dimensions, of a viscous fluid past a fixed cylinder*, Quart. J. Mech. Appl. Math. **8**, 129–145.
- L. V. Al'tšuler
(1965) *Use of shock waves in high pressure physics*, Uspehi Fiz. Nauk **85**, 197–258; English transl. in Soviet Phys. Uspekhi **8**.
- D. A. Anderson
(1974) *A comparison of numerical solutions to the inviscid equations of fluid motion*, J. Comput. Phys. **15**, 1–20.
- P. A. Andrejanov
(1975) *The stability of the solutions of the Cauchy problem for first order quasilinear equations*, Mat. Zametki **17**, 79–89; English transl. in Math. Notes **17**.
- V. Ja. Arsenin and N. N. Janenko
(1956) *On the interaction of shock waves with simple waves*, Dokl. Akad. Nauk SSSR **109**, 713–716. (Russian)
- K. I. Babenko and I. M. Gel'fand
(1958) *Some remarks on hyperbolic systems*, Naučn. Dokl. Vysš. Školy Fiz.-Mat. Nauk, no. 1, 12–18. (Russian)
- N. S. Bahvalov
(1961) *Error estimates for numerical integration of quasilinear first-order equations*, Ž. Vyčisl. Mat. i Mat. Fiz. **1**, 771–783; English transl. in USSR Comput. Math. and Math. Phys. **1**.
(1967) *Parabolic systems with small parameters for the leading derivatives*, Dokl. Akad. Nauk SSSR **174**, 263–266; English transl. in Soviet Math. Dokl. **8**.
(1970) *The existence in the large of a regular solution of a quasilinear hyperbolic system*, Ž. Vyčisl. Mat. i Mat. Fiz. **10**, 969–980; English transl. in USSR Comput. Math. and Math. Phys. **10**.
- V. B. Balakin
(1970) *Methods of the Runge-Kutta type for gas dynamics*, Ž. Vyčisl. Mat. i Mat. Fiz. **10**, 1512–1519; English transl. in USSR Comput. Math. and Math. Phys. **10**.
- Donald P. Ballou
(1970) *Solutions to nonlinear hyperbolic Cauchy problems without convexity conditions*, Trans. Amer. Math. Soc. **152**, 441–460.

- N. K. Bari
(1961) *Trigonometric series*, Fizmatgiz, Moscow; English transl., Vols. 1, 2, Pergamon Press, Oxford, and Macmillan, New York, 1964.
- Karl Bechert
(1940) *Zur Theorie ebener Störungen in reibungsfreien Gasen*. I, II, Ann. Physik (5) **37(429)**, 89–123; (5) **38(430)**, 1–25.
(1941) *Über die Ausbreitung von Zylinder- und Kugelwellen in reibungsfreien Gasen und Flüssigkeiten*, Ann. Physik (5) **39(431)**, 169–202.
- R. Becker
(1921) *Stosswelle und Detonation*, Z. Phys. **8**, 321–362.
- Wilhelm Blaschke
(1924) *Vorlesungen über Differentialgeometrie und geometrische Grundlagen von Einsteins Relativitätstheorie*, Band I, 2nd ed., Springer-Verlag, 1924; reprint, Dover, New York, 1945. [Also 5th rev. ed., W. Blaschke and K. Leichtweiss, *Elementare Differentialgeometrie*, Springer-Verlag, 1973.]
- Ju. E. Bojarincev
(1966) *On convergence of difference schemas for equations with variable coefficients*, Trudy Mat. Inst. Steklov. **74**, 16–37; English transl. in Proc. Steklov Inst. Math. **74**.
- V. A. Borovikov
(1969) *On the problem of discontinuity decay for a system of two quasilinear equations*, Dokl. Akad. Nauk SSSR **185**, 250–252; English transl. in Soviet Math. Dokl. **10**.
(1972) *On the decay of a discontinuity for a system of two quasilinear equations*, Trudy Moskov. Mat. Obšč. **27**, 53–92; English transl. in Trans. Moscow Math. Soc. **27**.
- K. V. Brušlinskii and Ja. M. Každan
(1963) *Self-similar solutions of certain problems of gas dynamics*, Uspehi Mat. Nauk **18**, no. 2(110), 3–24; English transl. in Russian Math. Surveys **18**.
- J. M. Burgess
(1940) *Application of a model system to illustrate some points of the statistical theory of free turbulence*, Nederl. Akad. Wetensch. Proc. **43**, 2–12.
(1948) *A mathematical model illustrating the theory of turbulence*, Advances in Appl. Mech., Academic Press, pp. 171–199.
- Samuel Z. Burstein and Arthur A. Mirin
(1970) *Third order difference methods for hyperbolic equations*, J. Comput. Phys. **5**, 547–571.
- È. B. Byhovskii
(1962a) *Nonadmissible viscosity matrices for the equations of isothermal gas motion*, Dokl. Akad. Nauk SSSR **146**, 751–753; English transl. in Soviet Math. Dokl. **3**.
(1962b) *The small-parameter method (“vanishing viscosity”) for a system of equations of gas dynamics*, Ž. Vychisl. Mat. i Mat. Fiz. **2**, 1128–1131; English transl. in USSR Comput. Math. and Math. Phys. **2**.
(1972) *An initial boundary value problem for the equation $u_t + a_x(u) = 0$* , Dokl. Akad. Nauk SSSR **202**, 511–514; English transl. in Soviet Math. Dokl. **13**.
(1974) *Boundary and initial-boundary value problems “in the large” for a quasilinear conservation law*, Dokl. Akad. Nauk SSSR **215**, 17–20; English transl. in Soviet Math. Dokl. **15**.
- Élie Cartan
(1945) *Les systèmes différentiels extérieurs et leurs applications géométriques*, Actualités Sci. Indust., No. 994, Hermann, Paris, 1945.
- Sydney Chapman and T. G. Cowling
(1952) *The mathematical theory of non-uniform gases*, 2nd ed., Cambridge Univ. Press.
- W. Chester
(1950) *The propagation of sound waves in an open-ended channel*, Philos. Mag. (7) **4**, 11–33.

- R. F. Chisnell
 (1955) *The normal motion of a shock wave through a non-uniform one-dimensional medium*, Proc. Roy. Soc. London Ser. A **232**, 350–370.
 (1957) *The motion of a shock wave in a channel, with applications to cylindrical and spherical shock waves*, J. Fluid Mech. **2**, 286–298.
- Julian D. Cole
 (1951) *On a quasi-linear parabolic equation occurring in aerodynamics*, Quart. Appl. Math. **9**, 225–236.
- Lothar Collatz
 (1951) *Numerische Behandlung von Differentialgleichungen*, Springer-Verlag.
- Charles C. Conley and Joel A. Smoller
 (1970) *Viscosity matrices for two-dimensional nonlinear hyperbolic systems*, Comm. Pure Appl. Math. **23**, 867–884.
 (1971) *Shock waves as limits of progressive wave solutions of higher order equations*, Comm. Pure Appl. Math. **24**, 459–472.
- E. D. Conway and J. A. Smoller
 (1973) *Shocks violating Lax's condition are unstable*, Proc. Amer. Math. Soc. **39**, 353–356.
- E. T. Copson
 (1953) *On sound waves of finite amplitude*, Proc. Roy. Soc. London Ser. A **216**, 539–547.
- R. Courant
 (1962) *Methods of mathematical physics. Vol. II: Partial differential equations*, Interscience.
- R. Courant and K. O. Friedrichs
 (1948) *Supersonic flow and shock waves*, Interscience.
- R. Courant, K. Friedrichs and H. Lewy
 (1928) *Über die partiellen Differentialgleichungen der mathematischen Physik*, Math. Ann. **100**, 32–74; English transl., IBM J. Res. Develop. **11** (1967), 215–234.
- Richard Courant, Eugene Isaacson and Mina Rees
 (1952) *On the solutions of nonlinear hyperbolic differential equations by finite differences*, Comm. Pure Appl. Math. **5**, 243–255.
- Richard Courant and Peter Lax
 (1949) *On nonlinear partial differential equations with two independent variables*, Comm. Pure Appl. Math. **2**, 255–273.
- Constantine M. Dafermos
 (1973a) *Solution of the Riemann problem for a class of hyperbolic systems of conservation laws by the viscosity method*, Arch. Rational Mech. Anal. **52**, 1–9.
 (1973b) *The entropy rate admissibility criterion for solutions of hyperbolic conservation laws*, J. Differential Equations **14**, 202–212.
 (1974) *Structure of solutions of the Riemann problem for hyperbolic systems of conservation laws*, Arch. Rational Mech. Anal. **53**, 203–217.
- C. M. Dafermos and R. J. DiPerna
 (1976) *The Riemann problem for certain classes of hyperbolic systems of conservation laws*, J. Differential Equations **20**, 90–114.
- Ronald J. DiPerna
 (1973) *Global solutions to a class of nonlinear hyperbolic systems of equations*, Comm. Pure Appl. Math. **26**, 1–28.
 (1975) *Decay and asymptotic behavior of solutions to nonlinear hyperbolic systems of conservation laws*, Indiana Univ. Math. J. **24**, 1047–1071.
 (1976a) *Singularities of solutions of nonlinear hyperbolic systems of conservation laws*, Arch. Rational Mech. Anal. **60**, 75–100.
 (1976b) *Global existence of solutions to nonlinear hyperbolic systems of conservation laws*, J. Differential Equations **20**, 187–212.

V. F. D'jačenko

- (1961a) *Cauchy's problem for quasilinear systems*, Dokl. Akad. Nauk SSSR **136**, 16–17; English transl. in Soviet Math. Dokl. **2**.
- (1961b) *On the numerical calculation of discontinuous solutions of quasilinear systems*, Ž. Vyčisl. Mat. i Mat. Fiz. **1**, 1127–1129; English transl. in USSR Comput. Math. and Math. Phys. **1**.
- (1963) *Conditions for the uniqueness of a continuous solution of the problem of the decay of the discontinuity for a set of three equations*, Dokl. Akad. Nauk SSSR **153**, 1245–1248; English transl. in Soviet Math. Dokl. **4**.

Alexander Doktor

- (1977) *Global solution of mixed problem for a certain system of nonlinear conservation laws*, Czechoslovak Math. J. **27(102)**, 69–95.

G. A. Dombrovskii

- (1964) *A method of approximation to an adiabatic curve in the theory of plane gas flows*, "Nauka", Moscow. (Russian)

Jim Douglas, Jr.

- (1956) *On the relation between stability and convergence in the numerical solution of linear parabolic and hyperbolic differential equations*, J. Soc. Indust. Appl. Math. **4**, 20–37.
- (1958) *The application of stability analysis in the numerical solution of quasilinear parabolic differential equations*, Trans. Amer. Math. Soc. **89**, 484–518.

Avron Douglis

- (1952) *Some existence theorems for hyperbolic systems of partial differential equations in two independent variables*, Comm. Pure Appl. Math. **5**, 119–154.
- (1959) *An ordering principle and generalized solutions of certain quasi-linear partial differential equations*, Comm. Pure Appl. Math. **12**, 87–112.
- (1961) *The continuous dependence of generalized solutions of non-linear partial differential equations upon initial data*, Comm. Pure Appl. Math. **14**, 267–284.
- (1972) *Layering methods for nonlinear partial differential equations of first order*, Ann. Inst. Fourier (Grenoble) **22**, fasc. 3, 141–227.

V. G. Dulov

- (1958) *Decay of an arbitrary initial discontinuity of the gas parameters across a jump-discontinuity in the cross-sectional area*, Vestnik Leningrad. Univ. No. 19 (Ser. Mat. Met. Astr. vyp. 4), 76–99. (Russian)

K. V. Emel'janov

- (1970) *The difference scheme for a differential equation with a small parameter multiplying the highest derivatives*, Čisl. Metody Meh. Splošnoi Sredy **1**, no. 5, 20–30. (Russian)
- (1973) *The difference scheme for a three-dimensional elliptic equation with a small parameter multiplying the highest derivatives*, Trudy Inst. Mat. i Meh. Ural. Naučn. Centr Akad. Nauk SSSR Vyp. 11, 30–42. (Russian)

Z. I. Fedotova and Ju. I. Šokin

- (1975) *Invariant difference schemes with a polynomial viscosity matrix*, Dokl. Akad. Nauk SSSR **222**, 51–53; English transl. in Soviet Math. Dokl. **16**.

S. P. Finikov

- (1948) *Cartan's method of exterior forms in differential geometry*, OGIZ, Moscow. (Russian)

Linus Richard Foy

- (1964) *Steady state solutions of hyperbolic systems of conservation laws with viscosity terms*, Comm. Pure Appl. Math. **17**, 177–188.

K. O. Friedrichs

- (1948) *Nonlinear hyperbolic differential equations for functions of two independent variables*, Amer. J. Math. **70**, 555–589.
- (1954) *Symmetric hyperbolic linear differential equations*, Comm. Pure Appl. Math. **7**, 345–392.

G. Ja. Galin

(1958) *Shock waves in media with arbitrary equations of state*, Dokl. Akad. Nauk SSSR **119**, 1106–1109; English transl. in Soviet Phys. Dokl. **3**.

(1959) *A theory of shock waves*, Dokl. Akad. Nauk SSSR **127**, 55–58; English transl. in Soviet Phys. Dokl. **4**.

F. R. Gantmaher

(1966) *The theory of matrices*, 2nd ed., “Nauka”, Moscow; English transl. of 1st ed., Vols. 1, 2, Chelsea, New York, 1959.

John Gary and Richard Helgason

(1970) *A matrix method for ordinary differential eigenvalue problems*, J. Comput. Phys. **5**, 169–187.

C. W. Gear

(1967) *The numerical integration of ordinary equations*, Math. Comp. **21**, 146–156.

(1969) *The automatic integration of stiff ordinary equations*, Information Processing 68 (Proc. IFIP Congr., Edinburgh), Vol. I: Math., Software, North-Holland, pp. 187–193.

I. M. Gel'fand

(1959) *Some problems in the theory of quasilinear equations*, Uspehi Mat. Nauk **14**, no. 2(86), 87–158; English transl. in Amer. Math. Soc. Transl. (2) **29** (1963).

I. M. Gel'fand and G. E. Šilov

(1958) *Generalized functions*. Vols. I, II, III, Fizmatgiz, Moscow; English transl., Academic Press, 1964, 1968, 1967.

P. Germain and R. Bader

(1953) *Unicité des écoulements avec chocs dans la mécanique de Bergers*, Office Nat. d'Études et de Recherches Aeronautiques (ONÉRA), Paris.

David Gilbarg

(1951) *The existence and limit behavior of the one-dimensional shock layer*, Amer. J. Math. **73**, 256–274.

James Glimm

(1965) *Solutions in the large for nonlinear hyperbolic systems of equations*, Comm. Pure Appl. Math. **18**, 697–715.

James Glimm and Peter D. Lax

(1970) *Decay of solutions of systems of nonlinear hyperbolic conservation laws*, Mem. Amer. Math. Soc., No. 101, Amer. Math. Soc., Providence, R. I.

S. K. Godunov

(1956) *On uniqueness of the solution of hydrodynamic equations*, Mat. Sb. **40(82)**, 467–478. (Russian)

(1959) *A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics*, Mat. Sb. **47(89)**, 271–306.

(1960) *On the concept of generalized solution*, Dokl. Akad. Nauk SSSR **134**, 1279–1282; English transl. in Soviet Math. Dokl. **1**.

(1961a) *An interesting class of quasilinear systems*, Dokl. Akad. Nauk SSSR **139**, 521–523; English transl. in Soviet Math. Dokl. **2**.

(1961b) *On nonunique “smearing out” of discontinuities in solutions of quasilinear systems*, Dokl. Akad. Nauk SSSR **136**, 272–273; English transl. in Soviet Math. Dokl. **2**.

(1962) *The problem of a generalized solution in the theory of quasilinear equations and in gas dynamics*, Uspehi Mat. Nauk **17**, no. 3(105), 147–158; English transl. in Russian Math. Surveys **17**.

S. K. Godunov and V. S. Rjaben'kii

(1973) *Difference schemes (introduction to the theory)*, “Nauka”, Moscow, French transl., “Mir”, Moscow, 1977.

- S. K. Godunov and K. A. Semendjaev
(1962) *Difference methods for numerical solution of the problems of gas dynamics*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **2**, 3–14; English transl. in *USSR Comput. Math. and Math. Phys.* **2**.
- V. Ja. Gol'din, N. I. Ionkin and N. N. Kalitkin
(1969) *The entropy scheme of computation in gas dynamics*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **9**, 1411–1413; English transl. in *USSR Comput. Math. and Math. Phys.* **9**.
- Édouard Goursat
(1929) *Cours d'analyse mathématique*. Vol. II, Part 2, 5th ed., Gauthier-Villars, Paris; English transl. of 2nd (1915) ed., Ginn, Boston, Mass., 1917; reprint, Dover, New York, 1959.
- J. M. Greenberg
(1973) *Estimates for fully developed shock solutions to the equation $\partial u/\partial t - \partial v/\partial x = 0$ and $\partial v/\partial t - \partial \sigma(u)/\partial x = 0$* , *Indiana Univ. Math. J.* **22**, 989–1003.
- Ku [Gu] Chao-hao, Li Ta-chien [Li Da-qian], Houtsung-yi [Hou Zong-yi] and others
(1961/62) *Discontinuous initial value problems for systems of quasilinear hyperbolic equations*. I, II, III, *Acta Math. Sinica* **11**, 314–323, 324–327; **12**, 132–143; English transl. in *Chinese Math. Acta* **2**, **3**.
- G. Guderley
(1942) *Starke kugelige und zylindrische Verdichtungsstöße in der Nähe des Kugelmittelpunktes bzw. der Zylinderachse*, *Luftfahrtforschung* **19**, 302–311.
- A. R. Hačaturov
(1977) *Spectral characteristics of a family of difference schemes for Navier-Stokes equations*, *Čisl. Metody Meh. Splošnoi Sredy* **8**, no. 6, 108–119. (Russian)
- Wolf Häfele
(1954) *Z. Naturforsch.* **9a**, 269.*
- G. H. Hardy and W. W. Rogosinski
(1950) *Fourier series*, 2nd ed., Cambridge Univ. Press.
- Francis H. Harlow
(1963) *The particle-in-cell method for numerical solution of problems in fluid dynamics*, *Proc. Sympos. Appl. Math.*, vol. XV, Amer. Math. Soc., Providence, R. I., pp. 269–288.
- Philip Hartman
(1964) *Ordinary differential equations*, Wiley.
- Philip Hartman and Aurel Wintner
(1952) *On hyperbolic partial differential equations*, *Amer. J. Math.* **74**, 834–864.
- Einar Hille
(1948) *Functional analysis and semi-groups*, 1st ed., Amer. Math. Soc. Colloq. Publ., vol. 31, Amer. Math. Soc., Providence, R. I., Chapter XX.
- Sebastian v. Hoerner
(1955) *Lösungen der hydrodynamischen Gleichungen mit linearem Verlauf der Geschwindigkeit*, *Z. Naturforsch.* **10a**, 687–692.
-
- * Editor's note. This item is reproduced exactly as it appears in the original. On the basis of V. Horner's paper cited below it seems likely that the authors intended one of the following three articles:
- Wolf Häfele
(1955a) *Zur analytischen Behandlung ebener, starker, instationärer Stosswellen*, *Z. Naturforsch.* **10a**, 1006–1016.
(1955b) *Über die Stabilität des Stosswellentypus aus der Klasse der Homologie-Lösungen*, *Z. Naturforsch.* **10a**, 1017–1027.
- C. F. v. Weizsäcker
(1954) *Genäherte Darstellung starker instationärer Stosswellen durch Homologie-Lösungen*, *Z. Naturforsch.* **9a**, 269–275.

Eberhard Hopf

- (1950) *The partial differential equation $u_t + uu_x = \mu u_{xx}$* , *Comm. Pure Appl. Math.* **3**, 201–230.
 (1969) *On the right weak solution of the Cauchy problem for a quasilinear equation of first order*, *J. Math. Mech.* **19**, 483–487.

Lars Hörmander

- (1963) *Linear partial differential operators*, Springer-Verlag, Berlin, and Academic Press, New York.

S. A. Hristianovič

- (1937) *Le problème de Cauchy pour les équations non linéaires hyperboliques*, *Mat. Sb.* **2(44)**, 871–899. (Russian; French summary)

A. M. Il'in

- (1965) *Stability of difference schemes for the Cauchy problem for systems of partial differential equations*, *Dokl. Akad. Nauk SSSR* **164**, 491–494; English transl. in *Soviet Math. Dokl.* **6**.
 (1969) *A difference scheme for a differential equation with a small parameter multiplying the highest derivative*, *Mat. Zametki* **6**, 237–248; English transl. in *Math. Notes* **6**.

A. M. Il'in and O. A. Oleinik

- (1958) *Behavior of the solutions of the Cauchy problem for certain quasilinear equations for unbounded increase of the time*, *Dokl. Akad. Nauk SSSR* **120**, 25–28; English transl. in *Amer. Math. Soc. Transl. (2)* **42** (1964).
 (1960) *Asymptotic behavior of solutions of the Cauchy problem for some quasilinear equations for large values of the time*, *Mat. Sb.* **51(93)**, 191–216. (Russian)

Z. A. Iskander-Zade

- (1966) *On the question of the stability of the trivial solutions of a parabolic system of partial differential equations*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **6**, 921–927; English transl. in *USSR Comput. Math. and Math. Phys.* **6**.

Nobutoshi Itaya

- (1970) *The existence and uniqueness of the solution of the equations describing compressible viscous fluid flow*, *Proc. Japan Acad.* **46**, 379–382.
 (1974) *On the temporally global problem of the generalized Burgess' equation*, *J. Math. Kyoto Univ.* **14**, 129–177.

N. N. Janenko

- (1955a) *Reduction of a system of quasilinear equations to a quasilinear equation*, *Uspehi Mat. Nauk* **10**, no. 3 (65), 173–178. (Russian)
 (1955b) *On discontinuities of solutions of quasilinear equations*, *Uspehi Mat. Nauk* **10**, no. 2(64), 195–202. (Russian)
 (1964) *Compatibility theory and integration methods for systems of nonlinear partial differential equations*, *Proc. Fourth All-Union Math. Congr. (Leningrad, 1961)*, "Nauka", Leningrad, pp. 247–252. (Russian)

N. N. Janenko and Ju. E. Bojarincev

- (1961) *The convergence of difference schemes for the heat conduction equation with variable coefficients*, *Dokl. Akad. Nauk SSSR* **139**, 1322–1324; English transl. in *Soviet Math. Dokl.* **2**.

N. N. Janenko and G. V. Demidov

- (1971) *On the structure of absolutely approximating and absolutely correct difference schemes*, *Problems in Appl. Math. and Mech. (A. A. Dorodnicyn Sixtieth Birthday Volume)*, "Nauka", Moscow, pp. 137–144. (Russian)

N. N. Janenko, G. V. Demidov and S. A. Kantor

- (1972) *Evolutionary double-layer difference schemes*, *Čisl. Metody Meh. Splošnoi Sredy* **3**, no. 5, 95–114. (Russian)

- N. N. Janenko and I. K. Jaušev
(1966) *On an absolutely stable schema for integration of the equations of hydrodynamics*, Trudy Mat. Inst. Steklov. **74**, 141–146; English transl. in Proc. Steklov Inst. Math. **74**.
- N. N. Janenko and V. E. Neuvažev
(1966) *A method of computing gas-dynamic motions with nonlinear heat conduction*, Trudy Mat. Inst. Steklov. **74**, 138–140; English transl. in Proc. Steklov Inst. Math. **74**.
- N. N. Janenko and Ju. I. Šokin
(1968) *Correctness of first differential approximations of difference schemes*, Dokl. Akad. Nauk SSSR **182**, 776–778; English transl. in Soviet Math. Dokl. **9**.
(1973) *On a group classification of difference schemes for the system of equations of gas dynamics*, Trudy Mat. Inst. Steklov. **122**, 85–97; English transl. in Proc. Steklov Inst. Math. **122**.
- N. N. Janenko, E. V. Vorozhcov and V. M. Fomin*
(1976a) *Differential analyzers of shock waves*, Dokl. Akad. Nauk SSSR **227**, 50–53; English transl. in Soviet Math. Dokl. **17**.
(1976b) *Differential analyzers of shock waves. Applications of the theory*, Čisl. Metody Meh. Splošnoi Sredy **7**, no. 6, 8–23. (Russian)
(1976c) *Differential analyzers of shock waves: Theory*, Computers & Fluids **4**, 171–183.
- I. K. Jaušev
(1967) *Decay of an arbitrary discontinuity in a channel with a jump in the cross-sectional area*, Izv. Sibirsk. Otdel. Akad. Nauk SSSR **1967**, no. 8 (Ser. Tehn. Nauk, Vyp. 2), 109–120. (Russian)
- Alan Jeffrey
(1973) *Quasi-linear hyperbolic systems and continuum mechanics*, Math. Balkanica **3**, 166–183.
(1975) *Smooth fronted waves in the shallow water approximation*, Proc. Roy Soc. Edinburgh Sect. A **73**, 107–116.
(1976) *Quasilinear hyperbolic systems and waves*, Res. Notes in Math., No. 5, Pitman.
- Fritz John
(1974) *Formation of singularities in one-dimensional nonlinear wave propagation*, Comm. Pure Appl. Math. **27**, 377–405.
- A. S. Kalašnikov
(1959a) *Construction of generalized solutions of quasilinear equations of first order without convexity conditions as limits of solutions of parabolic equations with a small parameter*, Dokl. Akad. Nauk SSSR **127**, 27–30. (Russian)
(1959b) *Uniqueness of the solution of the Cauchy problem for a class of quasilinear hyperbolic systems*, Uspehi Mat. Nauk **14**, no. 2(86), 195–202. (Russian)
- L. V. Kantorovič and G. P. Akilov
(1959) *Functional analysis in normed spaces*, Fizmatgiz, Moscow; English transl., Macmillan, 1964.
- A. V. Kažihov
(1975) *Correctness “in the large” of mixed boundary value problems for a model system of equations of a viscous gas*, Dinamika Splošnoi Sredy Vyp. **21**, 18–47. (Russian)
(1976) *The global solvability of one-dimensional boundary value problems for the equations of a viscous heat-carrying gas*, Dinamika Splošnoi Sredy Vyp. **24**, 44–61. (Russian)
- J. B. Keller
(1956) *Spherical, cylindrical and one-dimensional gas flows*, Quart. Appl. Math. **14**, 171–184.
- N. E. Kotchine [Kočin]
(1926) *Sur la théorie des ondes de choc dans une fluide*, Rend. Circ. Mat. Palermo **50**, 305–344.

*Editor's note. For b) the actual order of the authors' names is Vorozhcov, Fomin and Janenko, and for c) it is Fomin, Vorozhtsov and Yanenko.

I. V. Konoval'cev

(1968) *Stability in C and L_p of double layer difference schemes for parabolic equations with variable coefficients*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **8**, 894–899; English transl. in *USSR Comput. Math. and Math. Phys.* **8**.

V. P. Korobeinikov, N. S. Mel'nikova and E. V. Rjazonov

(1961) *Theory of point explosions*, Fizmatgiz, Moscow. (Russian)

Heinz-Otto Kreiss

(1958) *Über Sachsgemässe Cauchy probleme für Systeme von linearen partiellen Differentialgleichungen*, *Kungl. Tekn. Högskol. Handlingar* (Stockholm), No. 127.

(1962) *Über die Stabilitätsdefinition für Differenzgleichungen die partielle Differentialgleichungen approximieren*, *Nordisk Tidskr. Informationsbehandling (BIT)* **2**, 153–181.

(1964) *On difference approximations of the dissipative type for hyperbolic differential equations*, *Comm. Pure Appl. Math.* **17**, 335–353.

S. N. Kružkov

(1960) *The Cauchy problem in the large for certain nonlinear first-order differential equations*, *Dokl. Akad. Nauk SSSR* **132**, 36–39; English transl. in *Soviet Math. Dokl.* **1**.

(1964) *Generalized solutions of nonlinear equations of the first order and certain problems for quasilinear parabolic equations*, *Vestnik Moskov. Univ. Ser. I Mat. Meh.* No. 6, 65–74. (Russian)

(1965) *Methods for constructing generalized solutions for the Cauchy problem for a quasilinear equation of the first order*, *Uspehi Mat. Nauk* **20**, no. 6(126), 112–118. (Russian)

(1970) *First order quasilinear equations in several independent variables*, *Mat. Sb.* **81(123)**, 228–255; English transl. in *Math. USSR Sb.* **10**.

A. G. Kulikovskii

(1962) *The structure of shock waves*, *Prikl. Mat. Meh.* **26**, 631–641; English transl. in *J. Appl. Math. Mech.* **26**.

V. F. Kuropatenko

(1962) *A method of constructing difference schemes for the numerical integration of the equations of gas dynamics*, *Izv. Vysš. Učebn. Zaved. Matematika* no. 3(28), 75–83. (Russian)

(1966) *On difference methods for the equations of hydrodynamics*, *Trudy Mat. Inst. Steklov.* **74**, 107–137; English transl. in *Proc. Steklov Inst. Math.* **74**.

Paul Kutler and Harvard Lomax

(1971) *The computation of supersonic flow fields about wing-body combinations by "shock-capturing" finite difference techniques*, *Proc. Second Internat. Conf. Numer. Methods Fluid Dynamics* (Berkeley, Calif., 1970), *Lecture Notes in Physics*, Vol. 8, Springer-Verlag, 1971, pp. 24–29.

Paul Kutler, Leonidas Sakell and Gene Aiello

(1975) *Two-dimensional shock-on-shock interaction problem*, *AIAA J.* **13**, 361–367.

N. N. Kuznecov

(1959) *Some asymptotic properties of the generalized solution of the Cauchy problem for a quasilinear equation of first order*, *Uspehi Mat. Nauk* **14**, no. 2(86), 203–209. (Russian)

(1960) *Problem of the decay of an arbitrary discontinuity for a system of quasilinear equations of the first order*, *Dokl. Akad. Nauk SSSR* **131**, 503–506; English transl. in *Soviet Math. Dokl.* **1**.

(1967a) *The weak solutions of the Cauchy problem for a multidimensional quasilinear equation*, *Mat. Zametki* **2**, 401–410; English transl. in *Math. Notes* **2**.

(1967b) *Some mathematical problems of chromatography*, *Vyčisl. Metody i Programmovanie Vyp.* **6**, 242–258. (Russian)

(1972) *The asymptotic behavior of the solutions of the finite difference Cauchy problem*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **12**, 334–351; English transl. in *USSR Comput. Math. and Math. Phys.* **12**.

- (1973) *Application of the smoothing method to some systems of hyperbolic quasilinear equations*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **13**, 92–102; English transl. in *USSR Comput. Math. and Math. Phys.* **13**.
- (1975) *On stable methods of solving a quasilinear equation of first order in a class of discontinuous functions*, *Dokl. Akad. Nauk SSSR* **225**, 1009–1012; English transl. in *Soviet Math. Dokl.* **16**.
- (1977) *On stable methods for solving non-linear first order partial differential equations in the class of discontinuous functions*, *Topics in Numer. Anal. III (Proc. Roy. Irish Acad. Conf., Dublin, 1976)*, Academic Press, pp. 183–197.
- N. N. Kuznecov and Ci Čžun-tao [Chi Chung-tao]
- (1964a) *A uniqueness theorem in the theory of quasilinear hyperbolic equations*, *Vestnik Moskov. Univ. Ser. I Mat. Meh. No. 3*, 25–30. (Russian)
- (1964b) *On the uniqueness of the generalized solution of the Cauchy problem for a hyperbolic system of two quasilinear equations*, *Vestnik Moskov. Univ. Ser. I Mat. Meh. No. 4*, 3–6. (Russian)
- N. N. Kuznecov and B. L. Roždestvenskii
- (1959a) *Construction of the generalized solution of the Cauchy problem for a quasilinear equation*, *Uspehi Mat. Nauk* **14**, no. 2(86), 211–215. (Russian)
- (1959b) *Existence and uniqueness of the generalized solution of the Cauchy problem for a nonhomogeneous law of conservation*, *Dokl. Akad. Nauk SSSR* **126**, 486–489. (Russian)
- (1965) *On the construction of a generalized solution of the Cauchy problem for a quasilinear equation*, *Uspehi Mat. Nauk* **20**, no. 1(121), 209–212. (Russian)
- N. N. Kuznecov and V. A. Tupčiev
- (1975) *On an extension of a theorem due to Glimm*, *Dokl. Akad. Nauk SSSR* **221**, 287–290; English transl. in *Soviet Math. Dokl.* **16**.
- N. N. Kuznecov and S. A. Vološin
- (1976) *On monotone difference approximations for a first-order quasilinear equation*, *Dokl. Akad. Nauk SSSR* **229**, 1317–1320; English transl. in *Soviet Math. Dokl.* **17**.
- O. A. Ladyženskaja
- (1952) *Solution of Cauchy's problem for hyperbolic systems by the method of finite differences*, *Leningrad. Gos. Univ. Učen. Zap.* **144** (Ser. Mat. No. 23), 192–246. (Russian)
- (1956) *On the construction of discontinuous solutions of quasilinear hyperbolic equations as limits of solutions of the corresponding parabolic equations when the "coefficient of viscosity tends toward zero*, *Dokl. Akad. Nauk SSSR* **111**, 291–294. (Russian)
- L. D. Landau and E. M. Lifšic
- (1953) *Mechanics of continuous media*, 2nd ed., GITTL, Moscow; English transl., Parts 1, 2, *Course of theoretical physics*, Vols. 6, 7, Pergamon Press, Oxford, and Addison-Wesley, Reading, Mass., 1959.
- L. D. Landau, N. N. Meiman and I. M. Halatnikov
- (1958) *Numerical methods for the integration of partial differential equations by the method of finite differences*, *Proc. Third All-Union Math. Congr. (Moscow, 1956)*, Vol. III, Izdat. Akad. Nauk SSSR, Moscow, pp. 92–100. (Russian)
- Richard Latter
- (1955) *Similarity solution for a spherical shock wave*, *J. Appl. Phys.* **26**, 954–960.
- Peter D. Lax
- (1953) *Nonlinear hyperbolic equations*, *Comm. Pure Appl. Math.* **6**, 231–258.
- (1954a) *The initial value problem for nonlinear hyperbolic equations in two independent variables*, *Contributions to the Theory of Partial Differential Equations*, *Ann. of Math. Studies*, No. 33, Princeton Univ. Press, Princeton, N. J., pp. 211–229.

- (1954b) *Weak solutions of nonlinear hyperbolic equations and their numerical computation*, Comm. Pure Appl. Math. **7**, 159–193.
- (1957) *Hyperbolic systems of conservation laws. II*, Comm. Pure Appl. Math. **10**, 537–566.
- (1960) *The scope of the energy method*, Bull. Amer. Math. Soc. **66**, 32–35.
- (1964) *Development of singularities of solutions of nonlinear hyperbolic partial differential equations*, J. Mathematical Phys. **5**, 611–613.
- (1971) *Shock waves and entropy*, Contributions to Nonlinear Functional Analysis (Proc. Sympos., Madison, Wisc.), Academic Press, pp. 603–634.
- P. D. Lax and R. D. Richtmyer
 (1956) *Survey of the stability of linear finite difference equations*, Comm. Pure Appl. Math. **9**, 267–293.
- Peter Lax and Burton Wendroff
 (1960) *Systems of conservation laws*, Comm. Pure Appl. Math. **13**, 217–237.
- Lewis Leibovich
 (1974) *Solutions of the Riemann problem for hyperbolic systems of quasilinear equations without convexity conditions*, J. Math. Anal. Appl. **45**, 81–90.
- M. A. Leontovič
 (1950) *Introduction to thermodynamics*, GITTL, Moscow; German transl., VEB Deutscher Verlag Wiss., Berlin, 1953.
- Hans Lewy
 (1927) *Über das Anfangswertproblem einer hyperbolischen nichtlinearen partiellen Differentialgleichung zweiter Ordnung mit zwei unabhängigen Veränderlichen*, Math. Ann. **98**, 179–191.
- C. C. Lin
 (1955) *The theory of hydrodynamic stability*, Cambridge Univ. Press.
- Tai-Ping Liu
 (1974) *The Riemann problem for general 2×2 conservation laws*, Trans. Amer. Math. Soc. **199**, 89–112.
- (1975a) *The Riemann problem for general systems of conservation laws*, J. Differential Equations **18**, 218–234.
- (1975b) *Existence and uniqueness theorems for Riemann problems*, Trans. Amer. Math. Soc. **212**, 375–382.
- (1976) *Uniqueness of weak solutions of the Cauchy problem for general 2×2 conservation laws*, J. Differential Equations **20**, 369–388.
- V. Ju. Ljapidevskii
 (1973) *The global well-posedness of the Cauchy problem for a certain class of nonlinear hyperbolic systems of equations*, Dinamika Splošnoi Sredy Vyp. **15**, 74–88. (Russian)
- (1974a) *The continuous dependence on the initial conditions of generalized solutions of the system of equations of gas dynamics*, Ž. Vyčisl. Mat. i Mat. Fiz. **14**, 982–991; English transl. in USSR Comput. Math. and Math. Phys. **14**.
- (1974b) *On the uniqueness of the generalized solution of a system of equations of gas dynamics*, Dokl. Akad. Nauk SSSR **215**, 535–538; English transl. in Soviet Math. Dokl. **15**.
- (1975) *On correctness classes for nonlinear hyperbolic systems*, Dokl. Akad. Nauk SSSR **225**, 507–510; English transl. in Soviet Math. Dokl. **16**.
- L. A. Ljusternik and V. I. Sobolev
 (1951) *Elements of functional analysis*, GITTL, Moscow; English transl., Ungar, New York, 1961.
- G. S. S. Ludford
 (1955) *Generalised Riemann invariants*, Pacific J. Math. **5**, 441–450.
- R. W. MacCormack
 (1969) *The effect of viscosity on hypervelocity impact cratering*, AIAA Paper No. 69–354.

G. I. Maičuk

- (1961) *Methods of calculation for nuclear reactors*, Gosatomizdat, Moscow. (Russian)
 (1967) *Numerical methods in weather forecasting*, Gidrometeoizdat, Leningrad; English transl., Academic Press, 1974.
 (1973) *Methods of numerical mathematics*, "Nauka", Sibirsk. Otdel., Novosibirsk; English transl., Springer-Verlag, 1975.

M. H. Martin

- (1953a) *The propagation of a plane shock into a quiet atmosphere*, Canad. J. Math. **5**, 37–39.
 (1953b) *The Monge-Ampère partial differential equation $rt - s^2 + \lambda^2 = 0$* , Pacific J. Math. **3**, 165–187.

N. N. Meiman

- (1954) *On the theory of partial differential equations*, Dokl. Akad. Nauk SSSR **97**, 593–596. (Russian)

John Nash

- (1962) *Le problème de Cauchy pour les équations différentielles d'un fluide général*, Bull. Soc. Math. France **90**, 487–497.

J. von Neumann and R. D. Richtmyer

- (1950) *A method for the numerical calculation of hydrodynamic shocks*, J. Appl. Phys. **21**, 232–237.

V. E. Neuvažev

- (1962) *The propagation of a spherical blast wave in a heat-conducting gas*, Prikl. Mat. Meh. **26**, 1094–1099; English transl. in J. Appl. Math. Mech. **26**.

Takaaki Nishida

- (1968) *Global solution for an initial boundary value problem of a quasilinear hyperbolic system*, Proc. Japan Acad. **44**, 642–646.

Takaaki Nishida and Joel A. Smoller

- (1973) *Solutions in the large for some nonlinear hyperbolic conservation laws*, Comm. Pure Appl. Math. **26**, 183–200.

Johannes Nitsche

- (1953) *Über Unstetigkeit in den Ableitungen von Lösungen quasilinearer hyperbolischer Differentialgleichungssysteme*, J. Rational Mech. Anal. **2**, 291–297.

Arnold Nordsieck

- (1962) *On numerical integration of ordinary differential equations*, Math. Comp. **16**, 22–49.

D. E. Ohocimskii et al.

- (1957) *Computation of point explosion taking into account counterpressure*, Trudy Mat. Inst. Steklov. **50**. (Russian)

O. A. Oleinik

- (1954a) *The Cauchy problem for nonlinear equations in a class of discontinuous functions*, Dokl. Akad. Nauk SSSR **95**, 451–454; English transl. in Amer. Math. Soc. Transl. (2) **42** (1964).
 (1954b) *The Cauchy problem for nonlinear equations in a class of discontinuous functions*, Uspehi Mat. Nauk **9**, no. 3(61), 231–233. (Russian)
 (1955) *Boundary value problems for partial differential equations with a small parameter multiplying the highest derivatives, and the Cauchy problem for nonlinear equations in the large*, Uspehi Mat. Nauk **10**, no. 3(65), 229–234. (Russian)
 (1956a) *The Cauchy problem for nonlinear differential equations of the first order with discontinuous initial conditions*, Trudy Moskov. Mat. Obšč. **5**, 433–454. (Russian)
 (1956b) *On discontinuous solutions of nonlinear differential equations*, Dokl. Akad. Nauk SSSR **109**, 1098–1101. (Russian)
 (1957a) *On the uniqueness of the generalized solution of the Cauchy problem for a nonlinear system of equations occurring in mechanics*, Uspehi Mat. Nauk **12**, no. 6(78), 169–176. (Russian)

- (1957b) *Discontinuous solutions of nonlinear differential equations*, Uspehi Mat. Nauk **12**, no. 3(75), 3–73; English transl. in Amer. Math. Soc. Transl. (2) **26** (1963).
- (1958) *On a class of discontinuous solutions of quasilinear equations of the first order*, Naučn. Dokl. Vysš. Školy Fiz.-Mat. Nauki No. 3, 91–98. (Russian)
- (1959a) *Construction of a generalized solution of the Cauchy problem for a quasilinear equation of first order by the introduction of “vanishing velocity”*, Uspehi Mat. Nauk **14**, no. 2(86), 159–164; English transl. in Amer. Math. Soc. Transl. (2) **33** (1963).
- (1959b) *Uniqueness and stability of the generalized solution of the Cauchy problem for a quasilinear equation*, Uspehi Mat. Nauk **14**, no. 2(86), 165–170; English transl. in Amer. Math. Soc. Transl. (2) **33** (1963).
- O. A. Oleĭnik and N. D. Vvedenskaja
 (1957) *The solution of the Cauchy problem and a boundary value problem for nonlinear equations in a class of discontinuous functions*, Dokl. Akad. Nauk SSSR **113**, 503–506. (Russian)
- L. V. Ovsjannikov
 (1960) *Group properties of Čaplygin’s equation*, Ž. Prikl. Meh. i Tehn. Fiz. (PMTF) no. 3, 126–145. (Russian)
- (1962) *Group properties of differential equations*, Izdat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk. (Russian)
- (1973) *On the foundations of the theory of shallow water*, Dinamika Splošnoi Sredy Vyp. 15, 104–125. (Russian)
- D. Ju. Panov
 (1957) *Numerical solution of quasilinear hyperbolic systems of partial differential equations*, GITTL, Moscow. (Russian)
- G. N. Patterson
 (1956) *Molecular flow of gases*, Wiley, New York, and Chapman & Hall, London.
- B. V. Pavlov and A. Ja. Povzner
 (1973) *A method for the numerical integration of systems of ordinary differential equations*, Ž. Vyčisl. Mat. i Mat. Fiz. **13**, 1056–1059; English transl. in USSR Comput. Math. and Math. Phys. **13**.
- I. Petrowsky [I. G. Petrovskii]
 (1937) *Über das Cauchy’sche Problem für Systeme von partiellen Differentialgleichungen*, Mat. Sb. **2(44)**, 815–870.
- (1961) *Lectures on partial differential equations*, 3rd ed., Fizmatgiz, Moscow; English transl., Saunders, Philadelphia, Pa., 1967.
- A. R. Pinskiĭ and A. I. Ruzanov
 (1976) *Application of Picard’s method to the solution of nonstationary elasto-plastic problems*, Prikl. Probl. Pročnosti i Plastičnosti Vyp. 5, 53–58. (Russian)
- G. N. Položii
 (1962) *Numerical solution of two- and three-dimensional boundary-value problems of mathematical physics and functions of a discrete argument*, Izdat. Kiev. Univ., Kiev, 1962; English transl., *The method of summary representation for numerical solution of problems of mathematical physics*, Pergamon Press, 1965.
- Ju. P. Popov and A. A. Samarskii
 (1969) *Totally conservative difference schemes*, Ž. Vyčisl. Mat. i Mat. Fiz. **9**, 953–958; English transl. in USSR Comput. Math. and Math. Phys. **9**.
- (1970) *Totally conservative difference schemes for the equations of gas dynamics in Euler’s variables*, Ž. Vyčisl. Mat. i Mat. Fiz. **10**, 773–779; English transl. in USSR Comput. Math. and Math. Phys. **10**.
- P. K. Raševskii
 (1947) *Geometrical theory of partial differential equations*, OGIZ, Moscow. (Russian)

- V. E. Raspopov, V. P. Šapeev and N. N. Janenko
 (1974) *An application of the method of differential relations to the one-dimensional equations of gas dynamics*, Izv. Vysš. Učebn. Zaved. Math. no. 11(150), 69–74; English transl. in Soviet Math. (Iz. VUZ) **18**.
 (1977) *The method of differential relations for the one-dimensional equations of gas dynamics*, Čisl. Metody Meh. Splošnoi Sredy **8**, no. 2, 100–105. (Russian)
- R. V. Razumeiko
 (1973) *Error estimation for the numerical integration of a first order quasilinear equation*, Mat. Zametki **13**, 207–215; English transl. in Math Notes **13**.
- Robert D. Richtmyer and K. W. Morton
 (1967) *Difference methods for initial-value problems*, 2nd ed., Interscience.
- Bernhard Riemann
 (1860) *Über die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite*, Abh. Königl. Gess. Wiss. Göttingen **8**; reprinted in his *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*, Teubner, Leipzig, 1876 (reprint, Dover, New York, 1953), pp. 145–164.
 (1876) *Versuch einer allgemeinen Auffassung der Integration und Differentiation* (manuscript, 1847), first published in his *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*, Teubner, Leipzig, 1876 (reprint, Dover, New York, 1953), pp 331–344.
- Charles Riquier
 (1910) *Les systèmes d'équations aux dérivées partielles*, Gauthier-Villars, Paris.
- V. S. Rjaben'kii
 (1952) *On the application of the method of finite differences to the solution of the Cauchy problem*, Dokl. Akad. Nauk SSSR **86**, 1071–1074. (Russian)
- V. S. Rjaben'kii and A. F. Filippov
 (1956) *On the stability of difference equations*, GITTL, Moscow; German transl., VEB Deutscher Verlag Wiss., Berlin, 1960.
- B. L. Roždestvenskii
 (1957) *Systems of quasilinear equations*, Dokl. Akad. Nauk SSSR **115**, 454–457; English transl. in Amer. Math. Soc. Transl. (2)**42** (1964).
 (1958a) *The Cauchy problem for quasilinear equations*, Dokl. Akad. Nauk SSSR **122**, 551–554; English transl. in Amer. Math. Soc. Transl. (2)**42** (1964).
 (1958b) *Uniqueness of the generalized solution of the Cauchy problem for hyperbolic systems of quasilinear equations*, Dokl. Akad. Nauk SSSR **122**, 762–765; English transl. in Amer. Math. Soc. Transl. (2)**42** (1964).
 (1959a) *Conservativeness of systems of quasilinear equations*, Uspehi Mat. Nauk **14**, no. 2(86), 217–218; English transl. in Amer. Math. Soc. Transl. (2)**42** (1964).
 (1959b) *On the discontinuity of solutions of quasilinear equations*, Mat. Sb. **47(89)**, 485–494; English transl. in Amer. Math. Soc. Transl. (2) **101** (1973).
 (1960) *Discontinuous solutions of hyperbolic systems of quasilinear equations*, Uspehi Mat. Nauk **15**, no. 6(96), 59–117; English transl. in Russian Math. Surveys **15**.
 (1961) *A new method of solving the Cauchy problem in the large for quasilinear equations*, Dokl. Akad. Nauk SSSR **138**, 309–312; English transl. in Soviet Math. Dokl. **2**.
 (1962a) *A system of quasilinear equations in the theory of surfaces*, Dokl. Akad. Nauk SSSR **143**, 50–52; English transl. in Soviet Math. Dokl. **3**.
 (1962b) *Construction of discontinuous solutions of systems of two quasilinear equations*, Dokl. Akad. Nauk SSSR **144**, 58–61; English transl. in Soviet Math. Dokl. **3**.
 (1962c) *Construction of discontinuous solutions to systems of quasilinear equations*. I, Ž. Vyčisl. Mat. i Mat. Fiz. **2**, 1019–1043; English transl. in USSR Comput. Math. and Math. Phys. **2**.

- (1963) *Construction of discontinuous solutions to systems of quasilinear equations*. II, *Ž. Vyčisl. Mat. i Mat. Fiz.* **3**, 79–98; English transl. in *USSR Comput. Math. and Math. Phys.* **3**.
- (1974) *The Picard method as a method for the numerical solution of problems of mathematical physics*, *Čisl. Metody Meh. Splošnoi Sredy* **5**, no. 2, 96–107. (Russian)
- B. L. Roždestvenskiĭ, E. I. Ermakova and V. G. Priimak
 (1977) *A study of the stability of difference schemes of higher order accuracy*, Preprint No. 14 (1977), Keldyš Inst. Appl. Math. Acad. Sci. USSR, Moscow. (Russian) MR **58** #19207.
- B. L. Roždestvenskiĭ and N. N. Janenko
 (1968) *Systems of quasilinear equations and their applications to the dynamics of gases*, 1st ed., “Nauka”, Moscow. (Russian)
- B. L. Roždestvenskiĭ and A. D. Sidorenko
 (1967) *On the impossibility of “gradient catastrophe” for weakly nonlinear systems*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **7**, 1176–1179; English transl. in *USSR Comput. Math. and Math. Phys.* **7**.
- V. V. Rusanov
 (1968) *Difference schemes of the third order of accuracy for a continuous computation of discontinuous solutions*, *Dokl. Akad. Nauk SSSR* **180**, 1303–1305; English transl. in *Soviet Math. Dokl.* **9**.
- O. S. Ryžov and S. A. Hristianovič
 (1958) *On nonlinear reflection of weak shock waves*, *Prikl. Mat. Meh.* **22**, 586–599; English transl. in *J. Appl. Math. Mech.* **22**.
- A. A. Samarskiĭ
 (1971) *Introduction to the theory of difference schemes*, “Nauka”, Moscow. (Russian)
 (1977) *Theory of difference schemes*, “Nauka”, Moscow. (Russian)
- A. A. Samarskiĭ and V. Ja. Arsenin
 (1961) *On the numerical solution of the equations of gas dynamics with various types of viscosity*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **1**, 357–360; English transl. in *USSR Comput. Math. and Math. Phys.* **1**.
- A. A. Samarskiĭ and A. V. Gulin
 (1973) *Stability of difference schemes*, “Nauka”, Moscow. (Russian)
- A. A. Samarskiĭ and Ju. P. Popov
 (1975) *Difference schemes for gas dynamics*, “Nauka”, Moscow. (Russian)
- L. Schwartz
 (1950) *Théorie des distributions*. Vols. I, II, *Actualités Sci. Indust.*, Nos. 1091, 1122, Hermann, Paris.
- L. I. Sedov
 (1946) *Le mouvement d'air en cas d'une forte explosion*, *C. R. (Dokl.) Acad. Sci. URSS* **52**, 17–20.
 (1957) *Similarity and dimensional methods in mechanics*, 4th ed., GITTL, Moscow; English transl., Academic Press, 1959.
- A. D. Sidorenko
 (1968) *Wave adiabats for media with an arbitrary equation of state*, *Dokl. Akad. Nauk SSSR* **178**, 818–821; English transl. in *Soviet Phys. Dokl.* **13**.
 (1973) *A variant of the problem with a contact discontinuity for systems of three quasilinear equations*, *Differencial'nye Uravnenija* **9**, 774–777; English transl. in *Differential Equations* **9**.
- A. D. Sidorenko and B. L. Roždestvenskiĭ
 (1968) *A problem with contact discontinuity*, *Ž. Vyčisl. Mat. i Mat. Fiz.* **8**, 1352–1359; English transl. in *USSR Comput. Math. and Math. Phys.* **8**.

- V. I. Smirnov
 (1958) *A course in higher mathematics*, Vol. IV, 5th ed., Fizmatgiz, Moscow; English transl. of 3rd ed., Pergamon Press, Oxford, and Addison-Wesley, Reading, Mass., 1964.
 (1959) *A course in higher mathematics*, Vol. V, 2nd rev. ed., Fizmatgiz, Moscow; English transl., Pergamon Press, Oxford, and Addison-Wesley, Reading, Mass., 1964.
- V. Smirnov and S. Sobolev
 (1932) *Sur une méthode nouvelle dans le problème plan des vibrations élastiques*, Trudy Seismolg. Inst. Akad. Nauk SSSR No. 20.
- J. A. Smoller
 (1969a) *On the solution of the Riemann problem with general step data for an extended class of hyperbolic systems*, Michigan Math. J. **16**, 201–210.
 (1969b) *A uniqueness theorem for Riemann problems*, Arch. Rational Mech. Anal. **33**, 110–115.
 (1970) *Contact discontinuities in quasi-linear hyperbolic systems*, Comm. Pure Appl. Math. **23**, 791–801.
- Joel A. Smoller and Charles C. Conley
 (1972) *Viscosity matrices for two-dimensional non-linear hyperbolic systems*. II, Amer. J. Math. **94**, 631–650.
- J. A. Smoller and J. L. Johnson
 (1969) *Global solutions for an extended class of hyperbolic systems of conservation laws*, Arch. Rational Mech. Anal. **32**, 169–189; erratum, **37** (1970), 399–400.
- S. L. Sobolev
 (1962) *Applications of functional analysis in mathematical physics*, reprint, Izdat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk; English transl., Amer. Math. Soc., Providence, R. I., 1963.
- Ju. I. Šokin
 (1973) *On the first differential approximation method in the theory of difference schemes for hyperbolic systems of equations*, Trudy Mat. Inst. Steklov. **122**, 66–84; English transl. in Proc. Steklov Inst. Math. **122**.
 (1976) *Analysis of the properties of approximation viscosity of difference schemes by means of the method of differential approximation*, Proc. Fifth Internat. Conf. Numer. Methods Fluid Dynamics (Enschede, 1976), Lecture Notes in Physics, Vol. 59, Springer-Verlag, pp. 410–414.
- V. A. Solonnikov
 (1976) *The solvability of the initial-boundary value problem for the equations of motion of a viscous incompressible fluid*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **56**, 128–142; English transl. in J. Soviet Math. **14** (1980), no. 2.
- K. P. Stanjukovič
 (1955) *Unsteady motion of continuous media*, GITTL, Moscow; English transl., Pergamon Press, 1960.
- V. V. Stepanov
 (1958) *Course in differential equations*, Fizmatgiz, Moscow; German transl., VEB Deutscher Verlag Wiss., Berlin, 1963.
- Gilbert Strang
 (1963) *Accurate partial difference methods*. I, Arch Rational Mech. Anal. **12**, 392–402.
 (1964) *Accurate partial difference methods*. II, Numer. Math. **6**, 37–46.
 (1966) *Necessary and sufficient conditions for well-posed Cauchy problems*, J. Differential Equations **2**, 107–114.
- I. E. Tamm
 (1946) *Foundations of the theory of electricity*, 3rd ed., GITTL, Moscow. (Russian)
- A. H. Taub
 (1946) *Interaction of progressive rarefaction waves*, Ann. of Math. (2) **47**, 811–828.

A. N. Tihonov and A. A. Samarskiĭ

- (1954) *Discontinuous solutions of quasilinear equations of the first order*, Dokl. Akad. Nauk SSSR **99**, 27–30; English transl. in Amer. Math. Soc. Transl. (2) **42** (1964).
 (1959) *Convergence of difference schemes in the class of discontinuous coefficients*, Dokl. Akad. Nauk SSSR **124**, 529–532. (Russian)
 (1961) *Homogeneous difference schemes*, Ž. Vyčisl. Mat. i Mat. Fiz. **1**, 5–63; English transl. in USSR Comput. Math. and Math. Phys. **1**.
 (1972) *The equations of mathematical physics*, 4th ed., “Nauka”, Moscow; English transl. of 2nd ed., Pergamon Press, Oxford, and Macmillan, New York, 1963; Vols. 1, 2, Holden-Day, San Francisco, Calif., 1964, 1967.

V. A. Titov and G. I. Šiškin

- (1976) *A numerical solution of a parabolic equation with small parameters multiplying the derivatives with respect to the space variables*, Trudy Inst. Mat. i Meh. Ural. Naučn. Centr Akad. Nauk SSSR Vyp. 21, 38–43. (Russian)

V. E. Troščiev

- (1970) *The divergence of the “cross” scheme for the equations of gas dynamics*, Čisl. Metody Meh. Splošnoi Sredy **1**, no. 5, 87–93. (Russian)

H. S. Tsien

- (1956) *The Poincaré-Lighthill-Kuo method*, Adv. in Appl. Mech., Vol. IV, Academic Press, pp. 281–349.

V. A. Tupčiev

- (1964) *On the decay of an arbitrary discontinuity for a system of two first-order quasilinear equations*, Ž. Vyčisl. Mat. i Mat. Fiz. **4**, 817–825; English transl. in USSR Comput. Math. and Math. Phys. **4**.
 (1966) *The problem of decay of an arbitrary discontinuity for a system of quasilinear equations without the convexity condition*, Ž. Vyčisl. Mat. i Mat. Fiz. **6**, 527–547; English transl. in USSR Comput. Math. and Math. Phys. **6**.
 (1972a) *On the isolatedness of a solution of the problem of decay of an arbitrary discontinuity*, Inform. Bjull. Čisl. Metody Meh. Splošnoi Sredy **1**, no. 2, 82–93. (Russian)
 (1972b) *The asymptotic behavior of the solution of the Cauchy problem for the equation $\epsilon^2 u_{xx} = u_t + [\varphi(u)]_x$ that degenerates for $\epsilon = 0$ into the problem of the decay of an arbitrary discontinuity for the case of a rarefaction wave*, Ž. Vyčisl. Mat. i Mat. Fiz. **12**, 770–775; English transl. in USSR Comput. Math. and Math. Phys. **12**.
 (1973a) *The uniqueness of the continuous solution of the problem of the decay of an arbitrary discontinuity for a gradient system*, Mat. Zametki **13**, 251–258; English transl. in Math. Notes **13**.
 (1973b) *On the method of introducing viscosity in the study of problems involving the decay of a discontinuity*, Dokl. Akad. Nauk SSSR **211**, 55–58; English transl. in Soviet Math. Dokl. **14**.

T. D. Ventcel'

- (1957) *Certain quasilinear parabolic systems*, Dokl. Akad. Nauk SSSR **117**, 21–24. (Russian)
 (1963) *Quasilinear parabolic systems with increasing coefficients*, Vestnik Moskov. Univ. Ser. I Mat. Meh. no. 6, 34–44. (Russian)

A. I. Vol'pert

- (1967) *The spaces BV and quasilinear equations*, Mat. Sb. **73(115)**, 255–302; English transl. in Math. USSR Sb. **2**.

A. I. Vol'pert and S. I. Hudjaev

- (1972) *On the Cauchy problem for composite systems of nonlinear differential equations*, Mat. Sb. **87(129)**, 504–528; English transl. in Math. USSR Sb. **16**.

- E. V. Vorožcov
(1977) *Numerical tests of differential analyzers of shock waves*, Čisl. Metody Meh. Splošnoi Sredy **8**, no. 2, 12–27. (Russian)
- N. D. Vvedenskaja
(1956) *Solution of the Cauchy problem for a nonlinear equation with discontinuous initial conditions by the method of finite differences*, Dokl. Akad. Nauk SSSR **111**, 517–520. (Russian)
(1961) *An example of nonuniqueness of a generalized solution of a quasilinear system of equations*, Dokl. Akad. Nauk SSSR **136**, 532–533; English transl. in Soviet Math. Dokl. **2**.
- R. F. Warming, Paul Kutler and Harvard Lomax
(1973) *Second- and third-order noncentered difference schemes for nonlinear hyperbolic equations*, AIAA J. **11**, 189–196.
- Burton Wendroff
(1972) *The Riemann problem for materials with nonconvex equations of state*. I, II, J. Math. Anal. Appl. **38**, 454–466, 640–658.
- Hermann Weyl
(1949) *Shock waves in arbitrary fluids*, Comm. Pure Appl. Math. **2**, 103–122.
- G. B. Whitham
(1958) *On the propagation of shock waves through regions of non-uniform area or flow*, J. Fluid Mech. **4**, 337–360.
- Masaya Yamaguti and Takaaki Nishida
(1968) *On some global solutions for quasilinear hyperbolic equations*, Funkcial. Ekvac. **11**, 51–57.
- Ju. S. Zav'jalov
(1955) *On some integrals of one-dimensional flow of a gas*, Dokl. Akad. Nauk SSSR **103**, 781–782. (Russian)
(1956) *On the integration of certain equations for the nonisentropic motion of a gas*, Dissertation, Tomsk State Univ., Tomsk. (Russian)
- Ja. B. Zel'dovič
(1956) *Motion of a gas under the action of a momentary pressure (shock)*, Akust. Ž. **2**, 28–38; English transl. in Soviet Phys. Acoustics **2**.
- Ja. B. Zel'dovič and Ju. P. Raizer
(1966) *Physics of shock waves and high-temperature hydrodynamics phenomena*, 2nd ed., "Nauka", Moscow; English transl., Vols. I, II, Academic Press, 1966, 1967.
- Zhang Tong and Guo Yu-fa
(1965) *A class of initial-value problems for systems of aerodynamic equations*, Acta Math. Sinica **15**, 386–396; English transl. in Chinese Math. Acta **7**, no. 1.
- A. I. Žukov
(1959) *A limit theorem for difference operators*, Uspehi Mat. Nauk **14**, no. 3(87), 129–136; English transl. in Amer. Math. Soc. Transl. (2) **102** (1973).
(1960) *Application of the method of characteristics to the numerical solution of one-dimensional problems in gas dynamics*, Trudy Mat. Inst. Steklov. **58**; English transl., Clearinghouse Federal Sci. Tech. Information, Springfield, Va., 1967.
- A. I. Žukov and Ja. M. Každan
(1956) *On the motion of gas under the influence of a short impulse*, Akust. Ž. **2**, 352–357; English transl. in Soviet Phys. Acoustics **2**.
- A. Zygmund
(1959) *Trigonometric series*, 2nd rev. ed., Vols. I, II, Cambridge Univ. Press.

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