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Differential Forms Orthogonal to Holomorphic Functions or Forms, and Their Properties

L. A. Aĭzenberg
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ДИФФЕРЕНЦИАЛЬНЫЕ ФОРМЫ,
ОРТОГОНАЛЬНЫЕ ГОЛОМОРФНЫМ
ФУНКЦИЯМ ИЛИ ФОРМАМ,
И ИХ СВОЙСТВА

Л. А. АЙЗЕНБЕРГ И Ш. А. ДАУТОВ

ИЗДАТЕЛЬСТВО «НАУКА»
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ABSTRACT. This book is devoted to the description of exterior differential forms orthogonal to holomorphic forms of degree $n-p, 0 \leq p \leq n$ (in particular, to holomorphic functions if $p=n$) with respect to integration over the boundary of a bounded domain D in $\mathbf{C}^n$. The Martinelli-Bochner-Koppelman formula, which is an integral representation of exterior differential forms, is given, and the characteristic properties of the trace of a holomorphic function on the boundary are studied. The question of representation and multiplication of distributions lying in $\mathcal{D}'(\mathbf{R}^{2n-1})$ is discussed with the aid of $\bar{\partial}$-closed forms of type $(n, n-1)$ with harmonic coefficients.

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PREFACE TO THE AMERICAN EDITION

In the six years that have elapsed since this book appeared in the USSR, many new results have been obtained in this field of multidimensional complex analysis. These results are presented in a supplement (Chapter V–VII), written by the authors especially for the American edition. The results of A. M. Kytmanov have made the greatest impact on the contents of the supplement. He has also written Chapter VII of the supplement at the request of the authors. We take this opportunity to thank him for this work; we also thank him and S. G. Myslivech for help in preparing the manuscript of the supplement.

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PREFACE

In this book we consider the problem of characterizing the exterior differential forms which are orthogonal to holomorphic functions (or forms) in a domain $D \subset \mathbf{C}^n$ with respect to integration over the boundary, and some related questions. We give a detailed account of the derivation of the Bochner-Martinelli-Koppelman integral representation of exterior differential forms, which was obtained recently (1967) but has already found many important applications. A complete proof of this representation has not previously been available in our* literature. We study the properties of $\bar{\partial}$-closed forms of type $(p, n-1)$, $0 \leqslant p \leqslant n-1$, which turn out to be the duals (with respect to the orthogonality mentioned above) to holomorphic functions (or forms) in several complex variables, and resemble holomorphic functions of one complex variable in their properties. At the end of the book, we give some applications, in particular to the problem of multiplying distributions, and also a brief historical survey and a discussion of open problems.

We hope that this little book will be useful to mathematicians and theoretical physicists interested in several complex variables.

The greater part of the results expounded below were obtained by us during the years 1970–73. They were reported in seminars at Krasnoyarsk, Moscow State University, Urals State University (Sverdlovsk), and the Institute of Mathematics in the Siberian Division of the Academy of Sciences of the USSR. We thank the participants of these seminars for useful discussions. Above all we are grateful to G. M. Khenkin and V. P. Palamodov for valuable remarks.

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* *Editor's note*. The authors mean "Russian".

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SUPPLEMENT TO THE BRIEF HISTORICAL SURVEY

§16 contains the solution to Problem 3 (see "Brief historical survey and open problems" which follows Chapter IV). All the results of this section are due to Kytmanov [2]–[4]. A special case of Problem 3 (viz., Problem 2) is discussed in §17. Theorem 17.1 was proved for the ball in $\mathbf{C}^n$ by Aronov [2], and in the general case by Aronov and Kytmanov [1]. Theorem 17.2 and Corollaries 17.3 and 17.4 were obtained by Aĭzenberg and Kytmanov [1], [2]. Corollary 17.7 was obtained for the ball by Agranovskiĭ and Val'skiĭ [1], and for arbitrary domains by Stout [1]. Theorems 17.9 and 17.10 were proved by Kytmanov [1]. The results of §18 are due to Aĭzenberg and Dautov [1]. All the assertions of §19 except for Theorems 19.4 and 19.5 are due to Kytmanov [8]. Theorems 19.4 and 19.5 were proved by Romanov [1]; Theorem 19.5 is in fact true for arbitrary domains with smooth boundary in $\mathbf{C}^n$ (Romanov [2]). In §20 differential criteria for the existence of holomorphic extensions of functions are presented, which are criteria different from the tangential Cauchy-Riemann equations. Theorem 20.1 was proved for domains D in Kähler manifolds with $\partial D \in C^\infty$ and $f \in C^\infty(\overline{D})$ by Folland and Kohn [1]; in the form stated here, it is contained in Aronov and Kytmanov [1]. Theorem 20.4 is a reformulation of a result of Forelli [1]. The remaining results of §20 are due to Kytmanov [7].

The strengthening of Theorem 6.1 (Theorem 21.5) was obtained by Dautov (see Aĭzenberg and Yuzhakov [1], Theorem 26.1). The results of §22 are due to Dautov. Weighted formulas for solving the $\bar{\partial}$-problem and weighted estimates (Theorem 23.1) were proved by Dautov and Khenkin [1], [2]. Theorem 21.6 is due to Khenkin [5]. For $\alpha = 0$, Theorem 23.2 (for functions of the Nevanlinna class) was proved by Khenkin [4]–[6] and Skoda [1], [2]; for $\alpha > 0$ (functions of the Nevanlinna-Dzhrbashy class) it was proved by Dautov and Khenkin [1], [2]. In these papers, there is also a generalization of Theorem 23.2 to strictly pseudoconvex domains in manifolds.

Chapter VII is devoted to the solution of Problem 13. The results of this chapter are due to Kytmanov [5], [6]. We note that, for $n = 1$, Theorem 24.1 is proved in Bremermann [1] (p. 49). That part of Theorem 24.2 which proves that the boundary value of a harmonic function of class H is a distribution can be deduced from a theorem of Roĭtberg [1] which states that, if L is an elliptic operator of order $2m$ in a bounded domain $\Omega \subset \mathbf{R}^n$ with sufficiently smooth boundary, then every solution f of the equation $Lf = 0$ with finite-order singularity on $\partial\Omega$ defines a certain distribution on $\partial\Omega$. Theorem 24.2 gives a method for constructing this distribution. A different method of multiplying higher-dimensional distribution by means of analytic representations has been proposed by Ivanov and Verzhbalovich [1]. We note that, with this method as with ours, it would not be possible to have counterexamples like those of Itano [1].

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