

Lectures on Constructive Mathematical Analysis

by **B. A. KUSHNER**



Volume 60

**TRANSLATIONS OF
MATHEMATICAL MONOGRAPHS**

American Mathematical Society

10.1090/mmono/060

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

VOLUME 60

**Lectures on
Constructive
Mathematical Analysis**

by B. A. KUSHNER

American Mathematical Society · Providence · Rhode Island

**ЛЕКЦИИ ПО КОНСТРУКТИВНОМУ
МАТЕМАТИЧЕСКОМУ АНАЛИЗУ**

Б. А. КУШНЕР

**ИЗДАТЕЛЬСТВО «НАУКА»
ГЛАВНАЯ РЕДАКЦИЯ
ФИЗИКО-МАТЕМАТИЧЕСКОЙ ЛИТЕРАТУРЫ
МОСКВА 1973**

Translated from the Russian by E. Mendelson

Translation edited by Lev J. Leifman

1980 *Mathematics Subject Classification*. Primary 03F50.

Library of Congress Cataloging in Publication Data

Kushner, B. A. (Boris Abramovich)

Lectures on constructive mathematical analysis.

(Translations of mathematical monographs; v. 60)

Translation of: *Lektsii po konstruktivnomu matematicheskomu analizu*.

Bibliography: p.

Includes indexes.

1. Mathematical analysis. 2. Constructive mathematics. I. Title.

II. Series.

QA300.K8713 1985

515

84-18459

ISBN 0-8218-4513-6

All rights reserved except those granted to the United States Government

This book may not be reproduced in any form without permission of the publisher.

The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Copyright © 1984 by the American Mathematical Society

Table of Contents

Preface	v
Introduction	1
CHAPTER 1. Normal Algorithms and Enumerable Sets	29
1. Normal algorithms	29
2. Some undecidable algorithmic problems of the theory of algorithms	62
3. Decidable and enumerable sets	66
CHAPTER 2. Constructive Real Numbers	79
1. Natural numbers, integers, and rational numbers	79
2. Constructive real numbers (CRNs). Basic definitions	87
3. Equality and order relations on the set of CRNs	90
4. Arithmetic operations on CRNs	105
5. Rational numbers in the constructive continuum	113
CHAPTER 3. Constructive Convergence. Effective Uncountability of the Constructive Continuum	117
1. Basic definitions. First theorems on limits	117
2. Completeness of the constructive continuum. Nested segments theorem	121
3. Example of a monotonic, bounded, nonconvergent sequence of rational numbers	129
4. Effective uncountability of the constructive continuum	134
CHAPTER 4. The Impossibility of Certain Algorithms Connected With Constructive Real Numbers	139
1. Some algorithmic problems connected with the equality and order relations on the constructive continuum. Applications to algebra	139
2. The impossibility of some algorithms connected with convergence	148
3. Constructive real numbers and systematic expansions	153

CHAPTER 5. Constructive Functions

1. Basic definitions. Some examples
2. Properties of continuity. Uniformly continuous functions
3. The structure of constructive functions
4. Mean value theorems for constructive functions

CHAPTER 6. Differentiation of Constructive Functions

1. Basic definitions
2. Mean value theorems of differential calculus
3. The impossibility of certain algorithms with differentiation

CHAPTER 7. Riemann Integration of Constructive Functions

1. Basic definitions. Theorems on boundedness of integrable functions
2. Some criteria of integrability. Integrability of uniformly continuous functions. Integrability of the absolute value and product of integrable functions
3. The integral as a function of the upper limit. The Newton-Leibn theorem. Theorem on substitution of variables

CHAPTER 8. Singular Coverings and Some of Their Applications

1. Basic definitions. Existence of singular coverings
2. Examples of constructive functions with unusual properties
3. Impossibility of certain algorithms connected with integration

CHAPTER 9. Constructive Metric Spaces

1. Constructive metric spaces. Basic definitions and some examples
Completion of constructive metric spaces
2. Consistent sets. Algorithmic operators. Continuity theorem (first formulation)
3. Theorem on the choice of an enumerable covering. Strong form of the continuity theorem. Some counterexamples

Bibliography

Index of Names

Subject Index

Index of Notation

*Dedicated to
Andrei Andreevich Markov
on his seventieth birthday*

Preface

The basis of this book was a special course given by the author at the Mechanics-Mathematics Faculty of Moscow University. The material presumes almost no previous knowledge and is completely understandable to a reader who is in command of a standard course of mathematical analysis. A more detailed description of the book is presented at the end of the Introduction.

The author is deeply grateful to his teachers A. A. Markov and N. M. Nagornyi. Without the fruitful contacts with them over many years, this book could not have been written.

The author considers it his pleasant duty to thank for their help with this book the Chairman of the Scientific Council on the Complex Problem "Cybernetics", Academician A. I. Berg, and Council Staff members B. V. Biryukov and E. S. Geller. The author is also very grateful to S. I. Adyan for his assistance and helpful advice.

The author wishes to apologize to his many colleagues whose names it is impossible to mention here and whose friendly support was of invaluable help in his work. To all of them the author is deeply grateful.

This page intentionally left blank

Bibliography

In this bibliography, in addition to sources that were directly cited in the book, the author has included works that are known to him to be primarily related to constructive (computable, recursive) analysis. (We have omitted only a series of papers of Goodstein that are mentioned in the bibliography of Goodstein [5].) The primary bibliographic information in the domain of intuitionistic analysis can be found in the monographs of Heyting [3] and Fraenkel and Bar-Hillel [1].

The compilation of this bibliography was concluded in January, 1972.

ABERTH, OLIVER

- [1] *Analysis in the computable number field*, J. Assoc. Comput. Mach. **15** (1968), 275–299.
- [2] *A chain of inclusion relations in computable analysis*, Proc. Amer. Math. Soc. **2** (1969), 539–548.

ADLER, ANDREW

- [1] *Some recursively unsolvable problems in analysis*, Proc. Amer. Math. Soc. **22** (1969), 523–526.

ALEKSANDROV, P. S.

- [1] *Introduction to the general theory of sets and functions*, GITTL, Moscow, 1948; German transl., VEB Deutscher Verlag Wiss., Berlin, 1956.

BANACH, S. AND MAZUR, S.

- [1] *Sur les fonctions calculables*, Ann. Soc. Polon. Math. **16** (1937), 223.*

BISHOP, ERRETT

- [1] *The constructive development of abstract analysis*, Abstracts of Reports on Invitation, Internat. Congr. Math., Moscow, 1966, p. 39.
- [2] *Foundations of constructive analysis*, McGraw-Hill, 1967.
- [3] *The constructivization of abstract mathematical analysis*, Proc. Internat. Congr. Math. (Moscow, 1966), “Mir”, Moscow, 1968, pp. 308–313.

*See footnote (6) in the Introduction.

- [4] *Mathematics as a numerical language*, Intuitionism and Proof Theory (Proc. Conf., Buffalo, N.Y., 1968; A. Kino et al., editors), North-Holland, 1970, pp. 53–71.

BOREL, ÉMILE

- [1] *Leçons sur la théorie des fonctions*, 3rd ed., Gauthier-Villars, Paris, 1928.

CLEAVE, JOHN

- [1] *The primitive recursive analysis of ordinary differential equations and the complexity of their solutions*, J. Comput. System Sci. **3** (1969), 447–455.

VAN DER CORPUT, J. G.

- [1] *On the fundamental theorem of algebra*. I, II, III, Nederl. Akad. Wetensch. Proc. **49** (1946), 722–732, 878–886, 985–994=Indag. Math. **8** (1946) 430–440, 549–557, 605–614.

CURRY, HASKELL B.

- [1] *Foundations of mathematical logic*, McGraw-Hill, 1963.

DEMUTH, OSVALD

- [1] *Lebesgue integration in constructive analysis*, Dokl. Akad. Nauk SSSR **160** (1965), 1239–1241; English transl. in Soviet Math. Dokl. **6** (1965).
- [2] *The Lebesgue integral in constructive analysis*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **4** (1967), 30–43; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **4** (1967).
- [3] *A necessary and sufficient condition for Riemann integrability of constructive functions*, Dokl. Akad. Nauk SSSR **176** (1967), 757–758; English transl. in Soviet Math. Dokl. **8** (1967).
- [4] *The Lebesgue integral and the concept of measurability of functions in constructive analysis*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 21–28; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).
- [5] *The connection between Riemann and Lebesgue integrability of constructive functions*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 29–31; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).
- [6] *The spaces \mathcal{L}_n and S in constructive mathematics*, Comment. Math. Univ. Carolinae **10** (1969), 261–284.
- [7] *The Lebesgue measurability of sets in constructive mathematics*, Comment. Math. Univ. Carolinae **10** (1969), 463–492. (Russian)

- [8] *The differentiability of constructive functions*, Comment. Math. Univ. Carolinae **10** (1969), 167–175. (Russian)
- [9] *Linear functionals in the constructive spaces \mathcal{L}_γ* , Comment. Math. Univ. Carolinae **10** (1969), 357–390. (Russian)
- [10] *Mean value theorems for the constructive Lebesgue integral*, Comment. Math. Univ. Carolinae **11** (1970), 249–269. (Russian)
- [11] *The representability of constructive functions of weakly bounded variation*, Comment. Math. Univ. Carolinae **11** (1970), 667–691. (Russian)
- [12] *The integrability of derivatives of constructive functions*, Comment. Math. Univ. Carolinae **11** (1970), 667–691. (Russian)
- [13] *A necessary and sufficient condition for the absolute continuity of constructive functions*, Comment. Math. Univ. Carolinae **11** (1970), 705–726. (Russian)
- [14] *The superpositions of absolutely continuous constructive functions*, Comment. Math. Univ. Carolinae **12** (1971), 423–451. (Russian)
- [15] *A necessary and sufficient condition for the representability of constructive functions in the form of the sum of a singular and an absolutely continuous function*, Comment. Math. Univ. Carolinae **12** (1971), 587–610. (Russian)
- [16] *A condition for the differentiability of constructive functions of bounded variation*, Comment. Math. Univ. Carolinae **12** (1971), 687–711. (Russian)

DETLOVS, V. K.

- [1] *The equivalence of normal algorithms and recursive functions*, Trudy Mat. Inst. Steklov. **52** (1958), 75–139; English transl. in Amer. Math. Soc. Transl. (2) **23** (1963).

FAN DIN' ZIEU. See PHAN DINH DIỆU

FEFERMAN, SOLOMON

- [1] *Systems of predicative analysis*, J. Symbolic Logic **29** (1964), 1–30.

FIKHTENGOL'TS, G. M.

- [1] *A course in differential and integral calculus*. Vol. I, 4th ed., Fizmatgiz, Moscow, 1958; German transl., VEB Deutscher Verlag Wiss., Berlin, 1964.
- [2] *A course in differential and integral calculus*. Vol. II, 4th ed., Fizmatgiz, Moscow, 1959; German transl., VEB Deutscher Verlag Wiss., Berlin, 1964.

FRAENKEL, ABRAHAM A. AND BAR-HILLEL, YEHOShUA

- [1] *Foundations of set theory*, North-Holland, 1958.

FRIEDBERG, RICHARD M.

- [1] *Un contre-exemple relatif aux fonctionnelles récurrentes*, C. R. Acad. Sci. Paris **247** (1958), 852–854.
- [2] *4-quantifier completeness: A Banach-Mazur functional not uniformly partial recursive*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **6** (1958), 1–5.

GELBAUM, BERNARD R. AND OLMSTEAD, JOHN M. H.

- [1] *Counterexamples in analysis*, Holden-Day, San Francisco, Calif., 1964.

GOODSTEIN, R. L.

- [1] *Recursive number theory: A development of recursive arithmetic in a logic-free equation calculus*, North-Holland, 1957.
- [2] *Recursive analysis*, North-Holland, 1961.
- [3] *A constructive form of the second Gauss proof of the fundamental theorem of algebra*, Constructive Aspects of the Fundamental Theorem of Algebra (Proc. Sympos., Zürich-Rüschlikon, 1967; B. Dejon and P. Henrici, editors), Wiley, 1969, pp. 69–76.
- [4] *Polynomials with computable coefficients*, Notre Dame J. Formal Logic **11** (1970), 447–448.
- [5] *Recursive mathematical analysis*, “Nauka”, Moscow, 1970. (Russian)*

GOODSTEIN, R. L. AND HOOLEY, J.

- [1] *On recursive transcendence*, Notre Dame J. Formal Logic **1** (1960), 127–137.

GRZEGORCZYK, ANDRZEJ

- [1] *Elementarily definable analysis*, Fund. Math. **41** (1955), 311–338.
- [2] *Computable functionals*, Fund. Math. **42** (1955), 168–202.
- [3] *On the definition of computable functionals*, Fund. Math. **42** (1955), 232–239.
- [4] *On the definitions of computable real continuous functions*, Fund. Math. **44** (1957), 61–71.
- [5] *Some approaches to constructive analysis*, in Heyting [4], pp. 43–61.

HAUCK, JÜRGEN

- [1] *Ein Kriterium für die Annahme des Maximums in der berechenbaren Analysis*, Z. Math. Logik Grundlagen Math. **17** (1971), 193–196.

HEYTING, A.

- [1] *Die formalen Regeln der intuitionistischen Logik*, S.-B. Preuss. Akad. Wiss. (Berlin) Phys.-Math. Kl. **1930**, 42–56.

*Russian translation of [1], [2] and other papers; also includes Shanin [8]. For the details, see MR **50** #4261.

- [2] *Die formalen Regeln der intuitionistischen Mathematik*, S.-B. Preuss. Akad. Wiss. (Berlin) Phys.-Math. Kl. **1930**, 57–71.
- [3] *Intuitionism. An introduction*, North-Holland, 1956.
- [4] (editor), *Constructivity in mathematics: Proceedings of the colloquium held at Amsterdam, 1957*, North-Holland, 1959.

HILBERT, DAVID

- [1] *Über das Unendliche*, Math. Ann. **95** (1925/26), 161–190.

IL'IN, V. A. AND POZNYAK, È. G.

- [1] *Foundations of mathematical analysis*, "Nauka", Moscow, 1965. (Russian)

ILSE, DIETER

- [1] *Zur Stetigkeit berechenbarer reeller Funktionen*, Z. Math. Logik Grundlagen Math. **11** (1965), 297–342.

KAMKE, E.

- [1] *Das Lebesgue-Stieltjes Integral*, Teubner, Leipzig, 1956.

KANOVICH, M. I. AND KUSHNER, B. A.

- [1] *Estimation of the complexity of certain mass problems of analysis*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 81–90; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **16** (1969).

KHACHATRYAN, M. A.

- [1] *On constructive number series*, Trudy Vychisl. Tsentr. Akad. Nauk Armyan. SSR i Erevan. Gos. Univ. **5** (1968), 7–25. (Russian)
- [2] *An example of a constructive nondifferentiable monotone function*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. **4** (1969), 296–299. (Russian)

KLAUA, DIETER

- [1] *Berechenbare Analysis*, Z. Math. Logik Grundlagen Math. **2** (1956), 265–303.
- [2] *Die Präzisierung des Berechenbarkeitsbegriffes in der Analysis mit Hilfe rationaler Funktionale*, Z. Math. Logik Grundlagen Math. **5** (1959), 33–96.
- [3] *Berechenbare Reihen*, Z. Math. Logik Grundlagen Math. **6** (1960), 143–161.
- [4] *Konstruktive Analysis*, VEB Deutscher Verlag Wiss., Berlin, 1961.

KLEENE, STEPHEN COLE

- [1] *On the integration of intuitionistic number theory*, J. Symbolic Logic **10** (1945), 109–124.

- [2] *Recursive functions and intuitionistic mathematics*, Proc. Internat. Congr. Math. (Cambridge, Mass., 1950), Vol. 1, Amer. Math. Soc., Providence, R.I., 1952, pp. 679–685.
- [3] *A note on computable functionals*, Nederl. Akad. Wetensch. Proc. Ser. A **59**=Indag. Math. **18** (1956), 275–280.
- [4] *Introduction to mathematics*, Van Nostrand, 1952.

KOLMOGOROV, A. N.

- [1] *On the principle of tertium non datur*, Mat. Sb. **32** (1925), 646–667; English transl. in *From Frege to Gödel: A source-book in mathematical logic, 1879–1931* (J. van Keijenoort, editor), Harvard Univ. Press, Cambridge, Mass., 1967.
- [2] *Zur Deutung der intuitionistischen Logik*, Math. Z. **35** (1932), 58–65.
- [3] *Three approaches to the definition of the concept of the “amount of information”*, Problemy Peredachi Informatsii **1** (1965), no. 1, 3–11; English transl. in *Selected Transl. Math. Statist. and Probab.*, vol. 7, Amer. Math. Soc., Providence, R.I., 1968.

KOLMOGOROV, A. N. AND FOMIN, S. V.

- [1] *Elements of the theory of functions and functional analysis*, 3rd. ed., “Nauka”, Moscow, 1972; English transl. of 1st ed., Vols. 1, 2, Graylock Press, Albany, N.Y., 1957, 1961.

KÖNIG, DÉNES

- [1] *Über eine Schlussweise aus dem Endlichen ins Unendliche*, Acta Litt. Sci. Regiae Univ. Hungar. Francisco-Joseph. Sect. Math. [=Acta. Sci. Math. (Szeged)] **3** (1927), 121–130.

KOSOVSKIĬ, N. K.

- [1] *Necessary and sufficient conditions for Specker properties of a probabilistic space*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 91–96; English transl. in *Sem. Math. V. A. Steklov Math. Inst. Leningrad* **16** (1969).
- [2] *Integrable FR-constructs over a probabilistic space*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 97–104; English transl. in *Sem. Math. V. A. Steklov Math. Inst. Leningrad* **16** (1969).
- [3] *Laws of large numbers in constructive probability theory*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 105–113; English transl. in *Sem. Math. V. A. Steklov Math. Inst. Leningrad* **16** (1969).

- [4] *Some questions in the constructive theory of normed Boolean algebras*, Trudy Mat. Inst. Steklov. **113** (1970), 3–38; English transl. in Proc. Steklov Inst. Math. **113** (1970).

KREISEL, GEORG AND LACOMBE, DANIEL

- [1] *Ensembles rékursivement mesurables et ensembles rékursivement ouverts ou fermés*, C. R. Acad. Sci. Paris **245** (1957), 1106–1109.

KREISEL, GEORG, LACOMBE, DANIEL AND SHOENFIELD, JOSEPH R.

- [1] *Fonctionnelles rékursivement définissables et fonctionnelles rékursives*, C. R. Acad. Sci. Paris **245** (1957), 399–402.
 [2] *Partial recursive functionals and effective operations*, in Heyting [4], pp. 290–297.

KUČERA, ANTONIN

- [1] *Weak convergence in constructive mathematics*, Comment. Math. Univ. Carolinae **11** (1970), 285–308. (Russian)
 [2] *Sufficient conditions for the normability of linear operators in constructive mathematics*, Comment. Math. Univ. Carolinae **12** (1971), 377–399. (Russian)

KURATOWSKI, K. AND MOSTOWSKI, A.

- [1] *Set theory*, PWN, Warsaw, and North-Holland, Amsterdam, 1968.

KUSHNER, B. A.

- [1] *Riemann integration in constructive analysis*, Dokl. Akad. Nauk SSSR **156** (1964), 255–257; English transl. in Soviet Math. Dokl. **5** (1964).
 [2] *On the existence of unbounded analytic constructive functions*, Dokl. Akad. Nauk SSSR **160** (1965), 29–31; English transl. in Soviet Math. Dokl. **6** (1965).
 [3] *Constructive theory of the Riemann integral*, Dokl. Akad. Nauk SSSR **65**, 1238–1240; English transl. in Soviet Math. Dokl. **6** (1965).
 [4] *Some properties of quasinumbers and of operators from quasinumbers into quasinumbers*, Dokl. Akad. Nauk SSSR **171** (1966), 275–277; English transl. in Soviet Math. Dokl. **7** (1966).
 [5] *Some relations between the properties of constructive functions and of operators from quasinumbers into quasinumbers*, Dokl. Akad. Nauk SSSR **177** (1967), 29–32; English transl. in Soviet Math. Dokl. **8** (1967).
 [6] *On constructive antiderivatives*, Mat. Zametki **2** (1967), 157–166; English transl. in Math. Notes **2** (1967).
 [7] *Some examples of quasidense but not dense sets of duplexes*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 95–102; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).

- [8] *A remark on the domains of definition of constructive functions*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 103–106; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).
- [9] *Some mass problems connected with the integration of constructive functions*, Trudy Mat. Inst. Steklov. **113** (1970), 39–72; English transl. in Proc. Steklov Inst. Math. **113** (1970).
- [10] *Coverings of separable sets*, Studies in the Theory of Algorithms and Mathematical Logic (P. S. Novikov Seventieth Birthday Coll.; A. A. Markov and N. V. Petri, editors), Vol. I, Vychisl. Tsentr. Akad. Nauk SSSR, Moscow, 1973, pp. 235–246. (Russian)
- [11] *Continuity theorems for certain types of computable operators*, Dokl. Akad. Nauk SSSR **208** (1973), 1031–1034; English transl. in Soviet Math. Dokl. **14** (1973).

KUSHNER, B. A. AND TSEĬTIN, G. S.

- [1] *Some properties of F -numbers*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 107–120; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).

KUZNETSOV, A. V. AND TRAKHTENBROT, B. A.

- [1] *Investigations of partially recursive operators by means of the theory of Baire space*, Dokl. Akad. Nauk SSSR **105** (1955), 897–900. (Russian)

LACHLAN, A. H.

- [1] *Recursive real numbers*, J. Symbolic Logic **28** (1963), 1–16.
- [2] *Effective operators in a general setting*, J. Symbolic Logic **29** (1964), 163–178.

LACHLAN, A. H. AND MADISON, E. W.

- [1] *Computable fields and arithmetically definable ordered fields*, Proc. Amer. Math. Soc. **24** (1970), 803–807.

LACOMBE, DANIEL

- [1] *Extension de la notion de fonction réursive aux fonctions d'une ou plusieurs variables réelles*. I, II, III, C. R. Acad. Sci. Paris **240** (1955), 2478–2480; **241** (1955), 13–14, 151–153.
- [2] *Remarques sur les opérateurs recursifs et sur les fonctions récursives d'une variables réelle*, C. R. Acad. Sci. Paris **241** (1955), 1250–1252.
- [3] *Quelques propriétés d'analyse réursive*. I, II, C. R. Acad. Sci. Paris **244** (1957), 838–840, 996–997.

- [4] *Les ensembles récursivement ouverts ou fermés, et leurs applications à l'analyse récursive*. I, II, C. R. Acad. Sci. Paris **245** (1957), 1040–1043; **246** (1958), 28–31.
- [5] *Sur les possibilités d'extension de la notion de fonction récursive aux fonctions d'une ou plusieurs variables réelles*, La Raisonement en Mathématiques et en Sciences Expérimentales, Colloques Internat. du Centre Nat. Recherche Sci., No. 70, Éditions du CNRS, Paris, 1958, pp. 67–75.
- [6] *Quelques procédés de définition en topologie récursive*, in Heyting [4], pp. 129–158.

LANDAU, EDMUND

- [1] *Grundlagen der Analysis*, Akademische Verlag, Leipzig, 1930; English transl., Chelsea, New York, 1951.
- [2] *Einführung in die Differentialrechnung und Integralrechnung*, Noordhoff, 1934; English transl., Chelsea, New York, 1950.

LEHMAN, R. S.

- [1] *On primitive recursive real numbers*, Fund. Math. **49** (1960/61), 105–118.

LIFSHITS, V. A.

- [1] *On constructive groups*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **4** (1967), 86–95; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **4** (1967).
- [2] *On constructive analytic functions of one real variable*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 121–131; English in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).
- [3] *The set of zeros of a constructive power series in the real domain*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 114–125; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **16** (1969).
- [4] *The investigation of constructive functions by the method of para-completion*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 61–79; English transl. in J. Soviet Math. **1** (1973), no. 1.
- [5] *A locally analytic constructive function that is not analytic*, Dokl. Akad. Nauk SSSR **202** (1972), 1265–1267; English transl. in Soviet Math. Dokl. **13** (1972).

LORENTS, A. A.

- [1] *Elements of constructive probability theory*, Z. Math. Logik Grundlagen Math. **15** (1969), 437–459. (Russian)

LORENZEN, P.

- [1] *Constructive mathematics as a philosophical problem*, *Compositio Math.* **20** (1968), 133–142.

LYUSTERNIK, L. A. AND SOBOLEV, V. I.

- [1] *Elements of functional analysis*, 2nd rev. ed., “Nauka”, Moscow, 1965; English transl. of 1st ed., Ungar, New York, 1961.

MAL'TSEV, A. I.

- [1] *Algorithms and recursive functions*, “Nauka”, Moscow, 1965; English transl., Noordhoff, 1970.

MANUKYAN, S. N.

- [1] *On constructive curves and contour integrals of a function of a complex variable*, *Izv. Akad. Nauk Armyan. SSR Ser. Mat.* **4** (1969), 137–143. (Russian)
- [2] *On interior points of nondegenerate constructive curves*, *Dokl. Akad. Nauk SSSR* **196** (1971), 768–769; English transl. in *Soviet Math. Dokl.* **12** (1971).

MARKOV, A. A.

- [1] *The theory of algorithms*, *Trudy Mat. Inst. Steklov.* **38** (1951), 176–189; English transl. in *Amer. Math. Soc. Transl. (2)* **15** (1960).
- [2] *The theory of algorithms*, *Trudy Mat. Inst. Steklov.* **42** (1954); English transl., Israel Program Sci. Transl., Jerusalem, 1961.
- [3] *On the continuity of constructive functions*, *Uspekhi Mat. Nauk* **9** (1954), no. 3 (61), 226–230. (Russian)
- [4] *On a principle of constructive mathematical logic*, *Proc. Third All-Union Math. Congr. (Moscow, 1956)*, Vol. II, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 146–147. (Russian)
- [5] *On constructive functions*, *Trudy Mat. Inst. Steklov.* **52** (1958), 315–348; English transl. in *Amer. Math. Soc. Transl. (2)* **29** (1963).
- [6] *On constructive mathematics*, *Trudy Mat. Inst. Steklov.* **67** (1962), 8–14; English transl. in *Amer. Math. Soc. Transl. (2)* **98** (1971).
- [7] *Commentary by the translation editor*, in the Russian transl. of Heyting [3], “Mir”, Moscow, 1965. (Russian)
- [8] *An approach to constructive mathematical logic*, *Logic, Methodology and Philos. Sci. III (Proc. Third Internat. Congr., Amsterdam, 1967)*, North-Holland, 1968, pp. 283–294.
- [9] *On the logic of constructive mathematics*, *Vestnik Moskov. Univ. Ser. I Mat. Mekh.* **1970**, no. 2, 7–29; English transl. in *Moscow Univ. Math. Bull.* **25** (1970).

MARTIN-LÖF, PER

- [1] *Notes on constructive mathematics*, Almqvist & Wiksell, Stockholm, 1970.

MATIYASEVICH, YU. V.

- [1] *A sufficient condition for the convergence of monotone sequences*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 97–103; English transl. in *J. Soviet Math.* **1** (1973), no. 1.

MAZUR, S.

- [1] *Computable analysis*, Dissertationes Math. Rozprawy Mat. **33** (1963).

MENDELSON, ELLIOTT

- [1] *Introduction to mathematical logic*, Van Nostrand, 1964.

MESCHKOWSKI, HERBERT

- [1] *Rekursive reelle Zahlen*, *Math. Z.* **66** (1956), 189–202.

MIHALJINEC, MIRKO

- [1] *On the continuity of constructive transformations*. I, II, *Glasnik Mat.-Fiz. Astr. Ser. II: Društvo Mat. Fiz. Hrvatske* **15** (1960), 21–29, 229–235.
- [2] *Some local properties of constructive real functions*, *Glasnik Mat.-Fiz. Astr. Ser. II: Društvo-Mat. Fiz. Hrvatske* **20** (1965), 33–37.
- [3] *Inverse upper bound theorems for constructive real functions*, *Glasnik Mat.-Fiz. Astr. Ser. II: Društvo Mat. Fiz. Hrvatsko* **20** (1965), 177–187.
- [4] *A generalization of constructively uniformly continuous functions*, Abstracts of Brief Scientific Communications, Sect. 1, Internat. Congr. Math., Moscow, 1966, p. 21. (Russian)
- [5] *A nonmonotonous constructive real C^∞ -differentiable function having no local maximum and no local minimum*, *Glasnik Mat. Ser. III* **3** (**23**) (1968), 155–164.

MINTS, G. E.

- [1] *The differentiability predicate and the differentiation operator in constructive mathematical analysis*, *Dokl. Akad. Nauk SSSR* **147** (1962), 1032–1034; English transl. in *Soviet Math. Dokl.* **3** (1962).
- [2] *On predicate and operator variants of the formation of the theories of constructive mathematics*, *Trudy Mat. Inst. Steklov.* **72** (1964), 383–436; English transl. in *Amer. Math. Soc. Transl. (2)* **100** (1972).
- [3] *Corrigenda and addenda to [2]*, *Trudy Mat. Inst. Steklov.* **93** (1967), 257–258; English transl. in *Proc. Steklov Inst. Math.* **93** (1967).

MOSCHOVAKIS, YIANNIS N.

- [1] *Recursive metric spaces*, Fund. Math. **55** (1964), 215–238.
- [2] *Notation systems and recursive ordered fields*, Compositio Math. **17** (1965), 40–71.

MOSTOWSKI, ANDRZEJ

- [1] *The present state of investigations in the foundations of mathematics*, Uspekhi Mat. Nauk **9** (1954), no. 3 (61), 3–38. (Russian)
- [2] *On computable sequences*, Fund. Math. **44** (1957), 37–51.
- [3] *On various degrees of constructivism*, in Heyting [4], pp. 178–194.

MYHILL, J.

- [1] *Criteria of constructibility for real numbers*, J. Symbolic Logic **18** (1953), 7–10.
- [2] *A recursive function, defined on a compact interval and having a continuous derivative that is not recursive*, Michigan Math. J. **18** (1971), 97–98.

MYHILL, J. AND SHEPHERDSON, J. C.

- [1] *Effective operations on partial recursive functions*, Z. Math. Logik Grundlagen Math. **1** (1955), 310–317.

NAGORNYĀ, N. M.

- [1] *On strengthening the reduction theorem of the theory of algorithms*, Dokl. Akad. Nauk SSSR **90** (1953), 341–342. (Russian)
- [2] *On a minimal alphabet of algorithms over a given alphabet*, Trudy Mat. Inst. Steklov. **52** (1958), 66–74; English trans. in Amer. Math. Soc. Transl. (2) **29** (1963).
- [3] *Some generalizations of the concept of a normal algorithm*, Trudy Mat. Inst. Steklov. **52** (1958), 7–65; English transl. in Amer. Math. Soc. Transl. (2) **29** (1963).

NATANSON, I. P.

- [1] *Theory of functions of a real variable*, 2nd rev. ed., GITTL, Moscow, 1957, English transl., Vols. 1, 2, Ungar, New York, 1955, 1961.

NOGINA, E. YU.

- [1] *On effectively topological spaces*, Dokl. Akad. Nauk SSSR **169** (1966), 28–31; English transl. in Soviet Math. Dokl. **7** (1966).
- [2] *Relations between certain classes of effectively topological spaces*, Mat. Zametki **5** (1969), 483–495; English transl. in Math. Notes **5** (1969).
- [3] *On a theorem of Moschovakis*, Proc. First Conf. Young Specialists. III, Vychisl. Tsentri Akad. Nauk Armyan. SSR and Erevan. Gos. Univ., Erevan, 1969, pp. 92–100. (Russian)

OREVKOV, V. P.

- [1] *A constructive mapping of a square onto itself that displaces every constructive point*, Dokl. Akad. Nauk SSSR **152** (1963), 55–58; English transl. in Soviet Math. Dokl. **4** (1963).
- [2] *On constructive mappings of a circle into itself*, Trudy Mat. Inst. Steklov. **72** (1964), 437–461; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [3] *Certain questions of the theory of polynomials with constructive real coefficients*, Trudy Mat. Inst. Steklov. **72** (1964), 462–487; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [4] *On constructive mappings of finite polyhedra*, Trudy Mat. Inst. Steklov. **93** (1967), 142–163; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [5] *On some types of continuity of constructive operators*, Trudy Mat. Inst. Steklov. **93** (1967), 154–187; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [6] *Some properties of homeomorphisms of constructive metric spaces*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **16** (1969), 157–164.
- [7] *The equivalence of two definitions of continuity*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 145–159; English transl. in J. Soviet Math. **1** (1973), no. 1.
- [8] *On the continuity of constructive functionals*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 160–169; English transl. in J. Soviet Math. **1** (1973), no. 1.

PÉTER, RÓZSA

- [1] *Zum Begriff der rekursiven reellen Zahl*, Acta Sci. Math. Szeged. **12** (1950), Part A, 239–245.
- [2] *Rekursive Funktionen*, Akad. Kiadó, Budapest, 1950; rev. English transl., Akad. Kiadó, Budapest, and Academic Press, New York, 1967.

PHAN DINH DIÊU

- [1] *Constructive locally convex linear topological spaces*, Dokl. Akad. Nauk SSSR **162** (1965), 766–769; English transl. in Soviet Math. Dokl. **6** (1965).
- [2] *The metrizable, normability and multinormability of constructive locally convex spaces*, Dokl. Akad. Nauk SSSR **162** (1965), 1011–1014; English transl. in Soviet Math. Dokl. **6** (1965).
- [3] *On spaces dual to constructive locally convex spaces*, Dokl. Akad. Nauk SSSR **166** (1966), 45–48; English transl. in Soviet Math. Dokl. **7** (1966).

- [4] *Constructive generalized functions*, Dokl. Akad. Nauk SSSR **174** (1967), 37–40; English transl. in Soviet Math. Dokl. **8** (1967).
- [5] *Certain properties of constructive generalized functions*, Dokl. Akad. Nauk SSSR **174** (1967), 298–301; English transl. in Soviet Math. Dokl. **8** (1967).
- [6] *On spaces of constructive infinitely differentiable functions, and on functionals in them*, Dokl. Akad. Nauk SSSR **180** (1968), 799–802; English transl. in Soviet Math. Dokl. **9** (1968).
- [7] *A language of constructive mathematics involving systems of sets*, Trudy Mat. Inst. Steklov. **93** (1967), 123–141; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [8] *On closed and open sets in constructive topological spaces*, Trudy Mat. Inst. Steklov. **93** (1967), 250–256; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [9] *Some questions in constructive functional analysis*, Trudy Mat. Inst. Steklov. **114** (1970); English transl., Proc. Steklov Inst. Math. **114** (1970).

RABIN, MICHAEL O.

- [1] *Computable algebra, general theory and theory of computable fields*, Trans. Amer. Math. Soc. **95** (1960), 341–360.

RICE, H. G.

- [1] *Recursive real numbers*, Proc. Amer. Math. Soc. **5** (1954), 784–791.

RICHARDSON, DANIEL

- [1] *Some undecidable problems involving elementary functions of a real variable*, J. Symbolic Logic **33** (1968), 514–520.

ROGERS, HARTLEY, JR.

- [1] *Theory of recursive functions and effective computability*, McGraw-Hill, 1967.

RUDIN, WALTER

- [1] *Principles of mathematical analysis*, 2nd ed., McGraw-Hill, 1964.

SCARPELLINI, BRUNO

- [1] *Zwei unentscheidbare Probleme der Analysis*, Z. Math. Logik Grundlagen Math. **9** (1963), 265–289.

SCHNORR, CLAUS-PETER

- [1] *Komplexität von Algorithmen mit Anwendung auf die Analysis*, Arch. Math. Logik Grundlagenforsch. **14** (1971), 54–68.

SHANIN, N. A.

- [1] *On constructive mathematical analysis*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. II, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 69–71. (Russian)
- [2] *Some problems of mathematical analysis in the light of constructive logic*, Z. Math. Logik Grundlagen Math. **2** (1956), 27–36. (Russian)
- [3] *On an algorithm for the constructive decoding of mathematical propositions*, Z. Math. Logik Grundlagen Math. **4** (1958), 293–303. (Russian)
- [4] *On the constructive interpretations of mathematical propositions*, Trudy Mat. Inst. Steklov. **52** (1958), 226–311; English transl. in Amer. Math. Soc. Transl. (2) **23** (1963).
- [5] *On constructive linear functionals in a constructive Hilbert space*, Z. Math. Logik Grundlagen Math. **5** (1959), 1–8. (Russian)
- [6] *Constructive real numbers and constructive function spaces*, Trudy Mat. Inst. Steklov. **67** (1962), 15–294; English transl., Amer. Math. Soc., Providence, R.I., 1968.
- [7] *Concerning the constructive interpretation of support formulas. I*, Trudy Mat. Inst. Steklov. **72** (1964), 348–379; English transl. in Amer. Math. Soc. Transl. (2) **99** (1972).
- [8] *On recursive mathematical analysis and R. L. Goodstein's calculus of arithmetical equations*, Introduction to Goodstein [5], pp. 7–76. (Russian)

SHAPIRO, N. Z.

- [1] *Recursively countable subsets of recursive metric spaces*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **17** (1969) 603–607.

SHURYGIN, V. A.

- [1] *Nontrivial constructive mappings of certain sets*, Dokl. Akad. Nauk SSSR **168** (1966), 40–42; English transl. in Soviet Math. Dokl. **7** (1966); errata, Soviet Math. Dokl. **7** (1966), no. 6, p.v.
- [2] *Constructive sets with equality and their mappings*, Dokl. Akad. Nauk SSSR **173** (1967), 54–57; English transl. in Soviet Math. Dokl. **8** (1967).
- [3] *Complete constructive sets with equality, and some of their properties*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **8** (1968), 272–280; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **8** (1968).
- [4] *Constructive sets with equality and their mappings*, Trudy Mat. Inst. Steklov. **113** (1970), 173–259; English transl. in Proc. Steklov Inst. Math. **113** (1970).

SLISENKO, A. O.

- [1] *Some properties of arithmetic operations on duplexes*, Dokl. Akad. Nauk SSSR **152** (1963), 292–295; English transl. in Soviet Math. Dokl. **4** (1963).
- [2] *On some algorithmic problems connected with arithmetic operations duplexes*, Trudy Mat. Inst. Steklov. (1964), 488–523; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [3] *Examples of a nondiscontinuous but not continuous constructive operator in a metric space*, Trudy Mat. Inst. Steklov. **72** (1964), 524–532; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [4] *On constructive nonseparable spaces*, Trudy Mat. Inst. Steklov. **72** (1964), 533–536; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [5] *The maximal continuity regulators of constructive functions*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **4** (1967), 201–208; English transl. in Sem. Math. V. A. Steklov Math. Inst. Leningrad **4** (1967).
- [6] *Arithmetical operations on certain sets of duplexes*, Trudy Mat. Inst. Steklov. **93** (1967), 187–207; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [7] *The construction of maximal continuity regulators for constructive functions*, Trudy Mat. Inst. Steklov. **93** (1967), 208–249; English transl. in Proc. Steklov Inst. Math. **93** (1967).
- [8] *Some questions on the approximation of maximal regulators of continuity*, Proc. Steklov Inst. Math. **113** (1970), 73–78; English transl. in Proc. Steklov Inst. Math. **113** (1970).

SOARE, ROBERT I.

- [1] *Cohesive sets and recursively enumerable Dedekind cuts*, Pacific J. Math. **31** (1969), 215–231.
- [2] *Recursion theory and Dedekind cuts*, Trans. Amer. Math. Soc. **140** (1969), 271–294.

SPECKER, ERNST

- [1] *Nicht konstruktiv beweisbare Sätze der Analysis*, J. Symbolic Logic **14** (1949), 145–158.
- [2] *Der Satz vom Maximum in der rekursiven Analysis*, in Heyting [4], pp. 254–265.

- [3] *The fundamental theorem of algebra in recursive analysis*, Constructive Aspects of the Fundamental Theorem of Algebra (Proc. Sympos., Zürich-Rüschlikon, 1967; B. Dejon and P. Henrici, editors), Wiley, 1969, pp. 321–329.

TRAKHTENBROT, B. A.

- [1] *Tabular representation of recursive operators*, Dokl. Akad. Nauk SSSR **101** (1955), 417–420. (Russian)
- [2] *Algorithms and automatic computing machines*, 2nd ed., Fizmatgiz, Moscow, 1960; English transl., Heath, Boston, Mass., 1963.
- [3] *Complexity of algorithms and computations*, Novosibirsk. Gos. Univ., Novosibirsk, 1967. (Russian)

TSEĬTIN, G. S.

- [1] *On Cauchy's theorem in constructive analysis*, Uspekhi Mat. Nauk **10** (1955), no. 4 (66), 207–209. (Russian)
- [2] *The nested segments theorem, Cauchy's theorem, and Rolle's theorem in constructive analysis*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. 1, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 186–187. (Russian)
- [3] *Uniform recursiveness of algorithmic operators on general recursive functions and a canonical representation for constructive functions of a real argument*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. 1, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 188–189. (Russian)
- [4] *Algorithmic operators in constructive complete separable metric spaces*, Dokl. Akad. Nauk SSSR **128** (1959), 49–52. (Russian)
- [5] *Algorithmic operators in constructive metric spaces*, Trudy Mat. Inst. Steklov. **67** (1962), 295–361; English transl. in Amer. Math. Soc. Transl. (2) **64** (1967).
- [6] *Mean-value theorems in constructive analysis*, Trudy Mat. Inst. Steklov. **67** (1962), 362–384; English in Amer. Math. Soc. Transl. (2) **98** (1971).
- [7] *A method of presenting the theory of algorithms and enumerable sets*, Trudy Mat. Inst. Steklov. **72** (1964), 69–98; English transl. in Amer. Math. Soc. Transl. (2) **99** (1972).
- [8] *Three theorems on constructive functions*, Trudy Mat. Inst. Steklov. **72** (1964), 537–543; English transl. in Amer. Math. Soc. Transl. (2) **100** (1972).
- [9] *Studies in constructive analysis (constructive real numbers and pointwise-defined functions)*, Author's summary of doctoral dissertation, Leningrad, 1968. (Russian)

- [10] *On upper bounds of recursively enumerable sets of constructive real numbers*, Trudy Mat. Inst. Steklov. **113** (1970), 102–172; English transl. in Proc. Steklov Inst. Math. **113** (1970).
- [11] *A pseudofundamental sequence that is not equilent to a monotone one*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 263–271; English transl. in J. Soviet Math. **1** (1973), no. 1.

TSEĬTIN, G. S., ZASLAVSKIĬ, I. D. AND SHANIN, N. A.

- [1] *Peculiarities of constructive mathematical analysis*, Abstracts of Reports on Invitation, Internat. Congr. Math., Moscow, 1966, pp. 171–177. (Russian)
- [2] *Peculiarities of constructive mathematical analysis*, Proc. Internat. Congr. Math. (Moscow, 1966), “Mir”, Moscow, 1968, pp. 253–261; English transl. in Amer. Math. Soc. Transl. (2) **70** (1968).

TURING, A. M.

- [1] *On computable numbers, with an application to the Entscheidungsproblem*, Proc. London Math. Soc. (2) **42** (1936/37), 230–265.
- [2] *Correction to [1]*, Proc. London Math. Soc. (2) **43** (1937), 544–546.

USPENSKIĬ, V. A.

- [1] *On the theorem of uniform continuity*, Uspekhi Mat. Nauk **12** (1957), no. 1 (73), 99–142; English transl. in Amer. Math. Soc. Transl. (2) **19** (1962).
- [2] *On the question of the relation between various systems of constructive real numbers*, Izv. Vyssh. Uchebn. Zaved. Matematika **1960**, no. 2 (15), 199–208. (Russian)
- [3] *Lectures on computable functions*, Fizmatgiz, Moscow, 1960; French transl., Actualités Sci. Indust., No. 1317, Hermann, Paris, 1966.

VANDIVER, H. S.

- [1] *Constructive derivation of the decomposition-field of a polynomial*, Ann. of Math. (2) **37** (1936), 1–6.
- [2] *On the ordering of real algebraic numbers by constructive methods*, Ann. of Math. (2) **37** (1936), 7–16.

WEYL, HERMANN

- [1] *Das Kontinuum*, Veit & Co., Leipzig, 1918.
- [2] *On the philosophy of mathematics*, ONTI, Moscow, 1934. (Russian)*

WHITEHEAD, ALFRED NORTH AND RUSSELL, BERTRAND

- [1] *Principia mathematica*. Vol. I, Cambridge Univ. Press, 1910.

*A collection of papers, translated from the German.

- [2] *Principia mathematica*. Vol. II, Cambridge Univ. Press, 1912.
 [3] *Principia mathematica*. Vol. III, Cambridge Univ. Press, 1913.

ZASLAVSKIĬ, I. D.

- [1] *Refutation of some theorems of classical analysis in constructive analysis*, Uspekhi Mat. Nauk **10** (1955), no. 4 (66), 209–210. (Russian)
 [2] *Some peculiarities of constructive functions of a real variable in comparison with classical functions*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. 1, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 181–182. (Russian)
 [3] *On constructive Dedekind cuts*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. 1, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 182–183. (Russian)
 [4] *Some properties of constructive real numbers and constructive functions*, Trudy Mat. Inst. Steklov. **67** (1962), 385–457; English transl. in Amer. Math. Soc. Transl. (2) **57** (1966).
 [5] *On some distinctions between basis and restricted variables in logico-mathematical languages*, Trudy Vychisl. Tsentra Akad. Nauk Armyan. SSR i Erevan. Gos. Univ. **1** (1963), 13–29. (Russian)
 [6] *On the differentiation and integration of constructive functions*, Dokl. Akad. Nauk SSSR **156** (1964), 25–27; English transl. in Soviet Math. Dokl. **5** (1964).
 [7] *On the rectifiability of constructive plane curves*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. **2** (1967), 69–82. (Russian)
 [8] *On the axiomatic definition of constructive objects and operations*, Izv., Akad. Nauk Armyan. SSSR Ser. Mat. **4** (1969), 153–181. (Russian)

ZASLAVSKIĬ, I. D. AND MANUKYAN, S. N.

- [1] *Partitionings of the plane by constructive curves*, Trudy Vychisl. Tsentra Akad. Nauk Armyan. SSR i Erevan. Gos. Univ. **5** (1968), 26–138. (Russian)

ZASLAVSKIĬ, I. D. AND TSEĬTIN, G. S.

- [1] *On relations between the basic properties of constructive functions*, Proc. Third All-Union Math. Congr. (Moscow, 1956), Vol. 1, Izdat. Akad. Nauk SSSR, Moscow, 1956, pp. 180–181. (Russian)
 [2] *On singular coverings and related properties of constructive functions*, Trudy Mat. Inst. Steklov. **67** (1962), 458–502; English transl. in Amer. Math. Soc. Transl. (2) **98** (1971).
 [3] *On the question of generalizations of the principle of constructive selection*, Trudy Mat. Inst. Steklov. **72** (1964), 344–347; English transl. in Amer. Math. Soc. Transl. (2) **99** (1972).

- [4] *A criterion for the rectifiability of constructive plane curves*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. **5** (1970), 434–440. (Russian)
- [5] *Yet another constructive variant of the Cauchy theorem*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **20** (1971), 36–39; English transl. in J. Soviet Math. **1** (1973), no. 1.

ZVONKIN, A. K. AND LEVIN, L. A.

- [1] *Complexity of finite objects and the algorithm-theoretic foundation of the notions of information and randomness*, Uspekhi Mat. Nauk **25** (1970), no. 6 (156), 85–127; English transl. in Russian Math. Surveys **25** (1970).

Index of Names

- Abel, N. H., 128
Aberth, O., 317
Adler, A., 317
Aleksandrov, P. S., 70, 186, 317
- Baire, R., 186, 273, 274, 295, 312
Banach, S., 7, 8, 279, 317
Bar-Hillel, Y., 2, 3, 319
Bernstein, S. N., 186
Bertrand, J., 128
Bishop, E., 3, 5, 12–13, 317–318
Bolzano, B., 1, 4, 189, 193–195
Borel, E., 2, 10, 24, 27, 134, 173, 229,
236, 303, 318
Brouwer, L. E. J., 2, 7, 18
Bunyakovskii, V. Ya., 265
- Cantor, G., 1, 117, 134
Cauchy, A. L., 1, 5, 70, 126, 128, 193–195,
200, 204, 265
Church, A., 6, 12, 17, 58
Cleave, J., 318
van der Corput, J. G., 171, 318
Curry, H. B., 7, 318
- D'Alembert, J., 128
Darboux, G., 217, 220
Dedekind, R., 1, 2
Demuth, O., 13, 235, 318–319
Detlovs, V. K., 74, 319
Dirichlet, P. G. L., 128, 159
- Feferman, S., 6, 319
Fermat, P., 4, 19
Fikhtengol'ts, G. M., 4, 128, 191, 194,
216, 319
- Fomin, S. V., 265, 277, 279, 322
Fraenkel, A. A., 2, 3, 319
Friedberg, R. M., 9, 10, 284, 306, 313, 320
- Gauss, C. F., 2
Gelbaum, B. R., 152, 276, 320
Gödel, K., 6
Goodstein, R. L., 11–12, 125, 159, 164,
166, 171, 195, 320
Grzegorzczuk, A., 8–11, 159, 166, 179, 193,
305, 320
- Hauck, J., 320
Herbrand, J., 6
Heyting, A., 2, 11, 18, 20, 21, 320–321
Hilbert, D., 14, 321
Hooley, J., 320
- Il'in, V. A., 203, 321
Ilse, D., 9, 321
- Kamke, E., 298, 321
Kanovich, M. I., 321
Khachatryan, M. A., 125, 321
Klaua, D., 9, 10, 321
Kleene, S. C., 2, 6, 9, 10, 17, 20, 58, 62,
159, 172, 229, 289, 304–305, 321–322
Kolmogorov, A. N., 2, 14, 18, 20, 265, 322
König, D., 229, 322
Kosovskii, N. K., 13, 322–323
Kreisel, G., 10, 229, 232, 234, 284, 289,
304, 323
Kronecker, L., 2
Kučera, A., 323
Kummer, E., 127
Kuratowski, K., 229, 323
Kushner, B. A., 89, 153, 172, 223, 245,
247, 278, 295, 298, 303, 321, 323–324
Kuznetsov, A. V., 266, 324

- Lachlan, A. H., 9, 304, 334
 Lacombe, D., 10, 159, 166, 171, 193, 229,
 232, 234, 241, 244, 261, 284, 289,
 301-302, 304, 323, 324-325
 Lagrange, J. L., 200, 204, 205
 Landau, E., 79, 325
 Lebesgue, H., 235
 Lehman, R. S., 9, 325
 Leibniz, G. W., 127, 224, 228
 Levin, L. A., 14, 336
 Lifshits, V. A., 13, 134, 171, 244, 245,
 303, 325
 Lindelöf, E., 298
 Lipschitz, R., 203
 Lorents, A. A., 13, 325
 Lorenzen, P., 326
 Lyusternik, L. A., 279, 326

 Madison, E. W., 324
 Mal'tsev, A. I., 29, 58, 67, 137, 326
 Manukyan, S. N., 13, 164, 326, 335
 Markov, A. A., 1, 9, 13-15, 17-20, 27, 29,
 30, 32, 34, 41-44, 47, 50, 52, 55-58, 72,
 73, 101, 103, 159, 160, 164, 165, 216,
 229, 245, 261, 284, 289, 304-305, 314,
 326
 Martin-Löf, P., 13, 327
 Matiyasevich, Yu. V., 327
 Mazur, S., 7-9, 132, 164, 317, 327
 Mendelson, E., 327
 Méray, Ch., 1
 Meschkowski, H., 8, 327
 Mihajlinec, M., 242, 327
 Mints, G. E., 150, 151, 208, 261, 327
 Moschovakis, Y. N., 261, 268, 280, 289,
 292, 294-298, 301, 306, 323, 328
 Mostowski, A., 7, 9, 155, 157, 229, 328
 Muchnik, A. A., 306, 313
 Myhill, J. R., 8, 284, 289, 328

 Nagornyi, N. M., 41, 328
 Natanson, I. P., 186, 328
 Newton, I., 224, 228
 Nogina, E. Yu., 14, 261, 303, 328

 Olmsted, J. M. H., 152, 276, 320
 Orevkov, V. P., 13, 171, 261, 268, 316, 329

 Péter, R., 8, 329
 Phan, Đình Diệu, 13, 319, 329-330
 Post, E. L., 6

 Poznyak, È. G., 203, 327

 Raabe, J. L., 128
 Rabin, M. O., 330
 Rasiowa, H., 8
 Rice, H. G., 330
 Richard, J., 2
 Richardson, D., 330
 Riemann, B., 169, 212-214
 Roche, E., 203
 Rogers, H. Jr., 330
 Rolle, M., 200, 202, 205
 Rudin, W., 129, 330
 Russell, B., 6, 62, 334-335

 Scarpellini, B., 330
 Schlömilch, O., 203
 Schnorr, C.-P., 330
 Shanin, N. A., 1, 11, 13, 20-22, 88, 89,
 164, 235, 261, 263, 331, 334
 Shapiro, N. Z., 331
 Shepherdson, J. C., 284, 289, 328
 Shoenfield, J. R., 10, 284, 289, 304, 323
 Shurygin, V. A., 331
 Slisenko, A. O., 160, 164, 274, 314, 332
 Soare, R. I., 332
 Sobolev, V. I., 279, 326
 Specker, E., 5, 7, 25, 129, 131, 159, 171,
 241, 332-333

 Taylor, B., 200, 203-205
 Trakhtenbrot, B. A., 26, 266, 324, 333
 Tseitin, G. S., 1, 5, 10, 13-15, 25, 53, 59,
 61, 66, 89, 131, 142-143, 145, 147, 149,
 151-153, 164, 165, 173-175, 180, 186,
 189, 194, 197, 205, 207, 229, 231, 232,
 234, 239, 244, 248, 252, 261, 284, 286,
 289, 290, 26-298, 303-306, 324,
 333-334, 335-336
 Turing, A. M., 6-8, 12, 17, 58, 153, 157,
 334

 Uspenkiĭ, V. A., 7, 9, 28, 58, 67, 79, 125,
 153, 155, 157, 241, 266, 305, 334

 Vaĭnberg, Yu. R., 294
 Vandiver, H. S., 334

 Weierstrass, C., 1, 4, 132
 Weyl, H., 2, 6, 21, 334

Whitehead, A. N., 6, 334–335

Zaslavskii, I. D., 1, 10, 13, 14, 24, 79, 125,
131, 153, 164, 166, 174, 186, 193, 229,
231, 232, 234–239, 241, 244, 248, 250,
258, 259, 334, 335–336

Zvonkin, A. K., 14, 336

This page intentionally left blank

Subject Index

- abstraction of potential realizability, 15
agglutination, 239
algorithm, 16–17
 _____ , in a given alphabet, 30
 _____ , normal, 30, 33–35
 _____ , _____ , arithmetically complete, 69
 _____ , _____ , completable, 65
 _____ , _____ , complete, 65
 _____ , _____ , consistent (in a CMS), 280
 _____ , _____ , in an alphabet, 33–35
 _____ , _____ , incompletable, 65
 _____ , _____ , metric (CMS), 263
 _____ , _____ , non-self-applicable, 62
 _____ , _____ , of passage to the limit (in a CMS), 268
 _____ , _____ , of weak passage to the limit (in a CMS), 268
 _____ , _____ , orderly, 70
 _____ , _____ , self-applicable, 62
 _____ , _____ , universal, 55
 _____ of type $\mathcal{M}_1 \rightarrow \mathcal{M}_2$, 31
 _____ of type $\mathcal{M}_1 \rightarrow \mathcal{M}_2$, 31
 _____ over an alphabet, 31
algorithmic limit point, 291
algorithmic operator, 282
algorithmically closed, 291
almost continuous, 172
alphabet, 29

Baire space, 266
ball (in a CMS), 268
 _____ , closed, 268
base of a CRN, 94–95
bound, 167
 _____ , lower, 167
 _____ , _____ , greatest, 167
 _____ , upper, 167

 _____ , _____ , least, 167

capture principle, 286–287
characteristic algorithm of a covering, 230
closed set, 309
closure of a normal algorithm, 35
code, uniform, 166
 _____ , polygonal, 163
completion of a CMS, 270
computable real number, 7–8
conditional equality, 31
consistent set, 280
constructive
 _____ axis, 112
 _____ continuity, 165
 _____ continuum, 112
 _____ differentiability, 197
 _____ discontinuity, 165, 284
 _____ function (CF), 159
 _____ differentiable, 197–199
 _____ Riemann integrable (R-integrable), 212
 _____ linear, 162
 _____ continuous, 165
 _____ polygonal, 163
 _____ pseudopolygonal, 180
 _____ uniformly continuous, 166
 _____ corner, 179
 _____ effectively not integrable, 245
 _____ effectively nonuniformly continuous, 241
 _____ line, 112
 _____ metric space (CMS), 263
 _____ _____ , complete, 268
 _____ _____ , perfect, 278
 _____ _____ , separable, 269
 _____ _____ , weakly complete, 268
 _____ object, 14
 _____ real number, 87
 _____ Riemann integrability, 212
 _____ uniform continuity, 166

- covering
 ———, ε -bounded, 231
 ———, exact, 231
 ———, interval, 230
 ———, nondegenerate, 231
 ———, rational, 231
 ———, segment, 230
 ———, disjoint, 230
 ———, singular, 231
- decidable set, 68
 dense set, 269
 derivative, 198
 derivate, 197
 disjoint covering, 230
- effective covering, 298–299
 effective functional, 283
 effectively
 ——— nonuniformly continuous, 241
 ——— nowhere dense, 276
 ——— open, 294
 ——— uncountable, 134
- equal, 29
 equality, conditional, 31
 equivalence of algorithms, 31
 ———, full, 31
 equivalence of points of a CMS, 263
 exact covering, 231
 expansion, systematic, 153
 ———, n -ary, 153
- finite, 73
 first category, 277
 F -number, 87
 formula, substitution, 33
 ———, simple, 33
 ———, terminal, 33
 FR -number, 87
 function
 ———, constructive, 159
 ———, corner, 179
 ———, linear, 162
 ———, monotonic, 169
 ———, ———, piecewise, 170
 ———, polygonal, 163
 ———, ———, completely, 185
- generalized integral, 248
 graphical equality, 30
 graphical inequality, 30
- greatest lower bound, 167
- indicator of fundamentality, 149
 indicator of integrability, 222
 infinite set, 73
 integer, 81
 integral code, 216
 integral, generalized, 248
 integral sum, 212
 interval, 113
- Lacombe set, 301
 least upper bound, 167
 length of a word, 30
 letter, 29
 limit point, 309
 limit of a sequence, 118
 limiting word, 290
 lower bound, 167
- Markov's principle, 18–19
- natural number, 17, 79–81
 nondegenerate covering, 231
 nonlimiting word, 290
 norm of a partition (or of an integral sum), 212
 normalization principle, 57
 nowhere dense set, 312
 number series, 120
- occurrence, 32
 ———, first, 32
- partial recursive operator, 305
 partition, 162
 ———, integral, 211
 ——— of a segment, 163
 ———, positive, 162
 ———, proper, 211
 ———, rational, 162
 ——— theorem, 245
 P -operator, 170
 ———, almost continuous, 172
 point image of a set, 277
 point of a CMS, 263
 ———, isolated, 278
 ———, limit, 309
 polygonal code, 163
 polygonal function, 163

- principle
 - , capture, 286–287
 - , Markov's, 17, 19–20
 - , normalization, 57
 - of constructive choice, 19
- problem of recognizing applicability, 63
- pseudonumber, 88
- pseudo-operator, 314

- Q*-operator, 171
 - , almost continuous, 172
- quasinumber, 87
- quasi-operator, 314

- rational covering, 231
- rational number, 83
- real image of a rational number, 89
- regulator of
 - continuity, 165
 - convergence in itself, 87, 117
 - convergence to a CRN, 118
 - fundamentality, 87, 117
 - integrability, 212
 - uniform continuity, 166
- representation of a normal algorithm, 55

- saturated, 264
- scheme of a normal algorithm, 34
- segment, 113
 - covering, 230
- separable set, 269
- sequence of
 - balls, nested regular, 275
 - consistent sets, 281
 - constructive real numbers (SCN), 117
 - ———, convergent, 118
 - ———, fundamental, 117
 - ———, pseudofundamental, 118
 - ———, quasifundamental, 117
 - ———, Specker, 131
 - enumerable sets, 76
 - functions, 178
 - ———, compatible, 178
 - ——— compatible with a covering, 239
 - ———, expanding, 186
 - intervals, 123
 - ———, ε -bounded, 231
 - ———, nested, 123
 - ———, universal, 231
 - ——— natural numbers (SNN), 87
 - ——— points of a CMS, 267
 - ———, convergent, 267
 - ———, fundamental, 267
 - ———, regular, 267
 - ———, regularly convergent, 267
 - rational numbers (SRN), 87
 - ———, fundamental, 87
 - ———, effectively nonconvergent, 236
 - ———, pseudofundamental, 87
 - ———, quasifundamental, 87
 - ———, segments, 123
 - ———, ε -bounded, 231
 - ———, nest, 124
 - ———, regular, 124
 - ———, sets, 277
- series, number, 120
 - , convergent, 121
 - , divergent, 126
 - , Specker, 132
- set, 21
 - , decidable, 68
 - , effectively uncountable, 134
 - , enumerable, 68–69
 - , finite, 73
 - , infinite, 73
 - , nonfinite, 73
- set of points of a CMS
 - , algorithmically closed, 291
 - , closed, 309
 - , consistent, 280
 - , dense, 269
 - , effectively nowhere dense, 276
 - , effectively open, 294
 - , Lacombe, 301
 - , nowhere dense, 312
 - of the first category, 277
 - , saturated, 264
 - , separable, 269
 - , trackable, 292
- singular covering, 231
- span, 114
- subspace, 264
 - , saturated, 264
- substitution formula, 33
- support of a CMS, 263
- system of
 - intervals, 174
 - segments, 174
 - words, 16
- systematic expansion, 153

- theorem
 _____ , branching, 50–51
 _____ , choice of an enumerable covering, 298
 _____ , completeness of the constructive continuum, 121
 _____ , completion of a CMS, 270
 _____ , composition, 45
 _____ , continuity, 165, 298, 303
 _____ , incompletable algorithm, 65
 _____ , Mostowski-Uspenskii, 155
 _____ , nested balls, 275
 _____ , non-discontinuity, 165
 _____ , reduction, 43
 _____ , repetition, 51–53
 _____ , separation, 292
 _____ , Specker, 129
 _____ , translation, 42
 _____ , Tseitin's on approximation of constructive functions, 180
 _____ , union, 48
 _____ , universal algorithm, 55
- theorems, combination, 44
 trackable set, 292
 transcription of a normal algorithm, 22
 56–57, 77
- uniform code, 166
 uniformly continuous
 _____ , constructively, 165
 _____ , effectively not, 244
 union of a sequence of sets, 277
 upper bound, 167
 _____ , least, 167
- value of an integral sum, 212
 vector, 16
- word, 15–16, 29
 _____ , empty, 30
 _____ in a given alphabet, 30
 _____ , limiting, 290
 _____ , nonlimiting, 290

Index of Notation

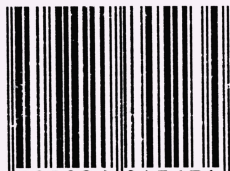
- $\forall, \&, \supset, \forall, \exists, \equiv, \neg, 18$
 $\in, 22$
 $\mathcal{M}_1 \subseteq \mathcal{M}_2, \mathcal{M}_1 \subset \mathcal{M}_2, \mathcal{M}_1 \cap \mathcal{M}_2, \mathcal{M}_1 \cup \mathcal{M}_2$
 (inclusion, strict inclusion,
 intersection, and union of sets \mathcal{M}_1 and
 \mathcal{M}_2), 22
 $\overline{\mathcal{M}}$ (complement of the set \mathcal{M}), 22
 $\mathcal{M}_1 \times \mathcal{M}_2 \times \dots \times \mathcal{M}_k$ (Cartesian product
 of sets $\mathcal{M}_1, \dots, \mathcal{M}_k$), 22
 \mathcal{M}^k (Cartesian power of the set \mathcal{M}), 22
 $\{\mathfrak{A}\}$ (transcription of the algorithm \mathfrak{A}),
 22, 56, 77
 \equiv, \neq (graphical equality, graphical
 inequality), 30
 \wedge (empty word), 30
 \simeq (conditional equality), 31
 $!\mathfrak{A}(P)$ (applicability of the algorithm \mathfrak{A}
 to input P), 30
 $P \rightarrow Q, P \rightarrow \cdot Q$ (simple and terminal
 substitution formulas), 33
 \mathfrak{A} (closure of a normal algorithm), 35
 $(\mathfrak{B} \circ \mathfrak{A})$ (composition of normal
 algorithms \mathfrak{A} and \mathfrak{B}), 47
 $[\mathfrak{A}]_\alpha, 59$
 $[\mathfrak{A}], 60$
 $\mathfrak{A}_P, 61, 77$
 $\mathfrak{A}_P, 62, 77$
 $\Psi_1, \Psi, \Psi^a, 77$
 \mathbf{N} (the set of natural numbers), 80
 $=_{\mathbf{N}}, <_{\mathbf{N}}, >_{\mathbf{N}}, \leq_{\mathbf{N}}, \geq_{\mathbf{N}}, 80$
 $+_{\mathbf{N}}, \cdot_{\mathbf{N}}, 81$
 \mathbf{Z} (the set of integers), 81
 $\text{mod}_{\mathbf{Z}}, 81$
 $=_{\mathbf{Z}}, <_{\mathbf{Z}}, >_{\mathbf{Z}}, \leq_{\mathbf{Z}}, \geq_{\mathbf{Z}}, 82$
 $+_{\mathbf{Z}}, 82$
 $\cdot_{\mathbf{Z}}, 83$
 \mathbf{Q} (the set of rational numbers), 83
 $=_{\mathbf{Q}}, <_{\mathbf{Q}}, >_{\mathbf{Q}}, \leq_{\mathbf{Q}}, \geq_{\mathbf{Q}}, 83-84$
 $+_{\mathbf{Q}}, \cdot_{\mathbf{Q}}, 84$
 in, 85
 $-_{\mathbf{Q}}, :_{\mathbf{Q}}, 85$
 $\text{mod}_{\mathbf{Q}}, 85$
 $\max_{\mathbf{Q}}, \min_{\mathbf{Q}}, 86, 87$
 \mathbf{R} (the set of constructive real numbers),
 89
 $\text{Id}, 89$
 $\overline{x}, x, 90$
 x_n (used only in Chapter 2), 90
 $=_{\mathbf{R}}, >_{\mathbf{R}}, <_{\mathbf{R}}, \geq_{\mathbf{R}}, \leq_{\mathbf{R}}, \neq_{\mathbf{R}}, 90, 91$
 $\text{bas}, 94$
 $\text{Nr}, 100$
 $\text{sgn}^{(2)}, 100$
 $\text{sgn}, 101$
 $Ds, 103$
 $G, 104$
 $+_{\mathbf{R}}, -_{\mathbf{R}}, 107$
 $\text{mod}_{\mathbf{R}}, 107$
 $G^+, 108$
 $\cdot_{\mathbf{R}}, 109$
 $G^-, \overline{G}^-, 110$
 $\text{inv}, 111$
 $\cdot_{\mathbf{R}}, 111$
 $\max_{\mathbf{R}}, \min_{\mathbf{R}}, 112$
 $x \triangle y, x \nabla y, x \overline{\times} y$ (segment, interval,
 span), 113-114
 $E^L, E^R, 113-114$
 $\text{Le}, 114$
 $\text{Rat}, 115$
 $\text{mod}^{(S)}, 121$
 $D^+, D^-, 121$
 $\text{lim}, \text{lim}^{(1)}, 122$
 $\text{lim}^{(2)}, 124$
 $\cong, 160$
 $\{f + g\}, \{f - g\}, \{f \cdot g\}, 161$
 $\{\frac{f}{g}\}, \{\max(f, g)\}, \{\min(f, g)\}, \{|f|\}, 161$
 $\text{Der}(t, f, z, \delta), \text{Der}(t, f, z), 197, 198$
 $\text{Der}(x \overline{\times} y, t, f, z, \delta), 197$
 $\text{Der}(x \overline{\times} y, t, f, z), 198$
 $\text{Der}(-\infty \nabla +\infty, f, f'), 198$

$\text{Der}(-\infty \nabla +\infty, f, f', W)$, 198
 $\text{Der}(x \bar{\Delta} y, f, f')$, 198
 $\text{Der}(x \bar{\Delta} y, f, f', W)$, 199
 δ, π , 212
 $D, \text{Ю}, \text{И}, \text{И}_1$, 212
 $z =_{(R)} \int_x^y f, \int_x^y f =_{(R)} z$, 214
 $\text{Md}(f, F, \mathcal{D}, \gamma)$, 253
 $=_M, \neq_M$, 263
 H, E_1, E_n , 264
 E_n^1, E_n^2, C, B , 265, 266
 $\text{Nd}(\mathcal{X}, \omega)$, 276
 $\text{Cat}(\mathcal{L}, \{\mathcal{X}_n\}, \gamma)$, 277
 \tilde{M}_1 (point image of a set M_1), 277
 $\text{Cons}(\mathcal{X}, \mathfrak{A})$, 280
 sep , 297
 tr , 297

ABBREVIATIONS

SNN—sequence of natural numbers, 87
 SRN—sequence of rational numbers, 87
 CRN—constructive real number, 87
 SCN—sequence of constructive real numbers, 117
 CF—constructive function, 160
 R-integrable F Riemann-integrable, 212
 CMS—constructive metric space, 263

ISBN 978-0-8218-4513-4



9 780821 845134

MMONO/60