Translations of

MATHEMATICAL MONOGRAPHS

Volume 62

Kleinian Groups and Uniformization in Examples and Problems

S. L. Krushkal′
B. N. Apanasov
N. A. Gusevskiĭ

American Mathematical Society

Kleinian Groups and Uniformization in Examples and Problems

This page intentionally left blank

10.1090/mmono/062

TRANSLATIONS OF
MATHEMATICAL
MONOGRAPHS
VOLUME 62

S. L. Krushkal′
B. N. Apanasov
N. A. Gusevskiĭ

Kleinian Groups and Uniformization in Examples and Problems

American Mathematical Society

КЛЕЙНОВЫ ГРУППЫ И УНИФОРМИЗАЦИЯ
В ПРИМЕРАХ И ЗАДАЧАХ
С. Л. КРУШКАЛЬ, Б. Н. АПАНАСОВ И Н. А. ГУСЕВСКИЙ

«НАУКА», НОВОСИБИРСК, 1981

Translated from the Russian by H. H. McFaden
Translation edited by Bernard Maskit

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 30-01, 30-02, 20H10, 30F10, 30F35, 30F40; Secondary 30C25, 30C60, 30F20, 32G15, 57M12, 58H15.

ABSTRACT. This monograph gives a unified exposition of all the main areas and methods of the theory of Kleinian groups and the theory of uniformization of manifolds. It features a large number of examples, problems, and unsolved problems, many presented for the first time. This is the first publication of this kind in the literature. The book is intended for researchers, graduate students, and undergraduates. It permits nonspecialists to quickly acquaint themselves with contemporary problems in this area of mathematics.
Bibliography: 310 titles. Figures: 67.

Library of Congress Cataloging-in-Publication Data

Krushkal', S. L. (Samuil Leĭbovich)
Kleinian groups and uniformization in examples and problems.
(Translations of mathematical monographs; v. 62)
Translation of: Kleĭnovy gruppy i uniformizat͡siia͡ v primerakh i zadachakh.
Bibliography: p.
Includes index.
1. Kleinian groups. I. Apanasov, B. N. (Boris Nikolaevich) II. Gusevskiĭ, N. A. (Nikolaĭ Aleksandrovich) III. Title. IV. Series.
QA331.K78513 1986 516.3'73 85-15058
ISBN 0-8218-4516-0

10 9 8 7 6 5 4 3 02 01 00 99 98

Contents

Foreword

The theory of Kleinian groups was established as far back as the end of the last century, mainly in work of Klein and Poincaré. This theory was created and developed due to the needs of various areas of mathematics: the theory of differential equations, function theory, geometry, topology, number theory, and so on. The theory of Kleinian groups and functions automorphic with respect to them is now one of the most beautiful areas of mathematics and has progressed very far. One of the stimuli promoting the evolution of this theory was the classical problem of uniformization of Riemann surfaces and multivalent analytic functions, solved with different methods by Klein, Poincaré, and Koebe. This has been a source for the development of many fruitful ideas and methods in mathematics.

In the past 20 years the theory of general Kleinian groups and the corresponding theory of uniformization have experienced a rejuvenation. The reawakened interest in these questions was due, in particular, to the development of new powerful methods in topology, the theory of functions of several complex variables, and the theory of quasiconformal mappings.

The number of papers in this area is constantly growing, and it is difficult to imagine a book that could completely reflect the modern state of the theory. Although there are good expositions of its foundations in the well-known monographs by Fricke and Klein, Ford, Appell and Goursat, Lehner, Nevanlinna, and others, none of them is entirely satifactory for the modern reader.

Our set purpose is to give the reader the main contours of this extensive area of research, with special emphasis on examples and problems (which, as a rule, are passed over in the established forms of exposition). They are of varying degrees of difficulty and, in our view, clear up nicely some subtle points of the theory. Most of the examples are also of independent interest.

We count on an active and critical reader, and as food for reflection we provide a large number of exercises, problems, and even unsolved problems. The exercises bear an academic character on the whole, but the problems present more essential results not mentioned in the general theory, with the original

sources indicated. The large number of examples and problems enable us to quickly introduce the reader, not only to the actual state of the contemporary theory, but also to the problems and methods which the theory uses.

The main part of the material presented is also contained in numerous journal articles. Some of the material, especially the examples, is presented for the first time. The same applies to many of the exercises. The book was conceived from various special courses and seminar discussions at the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of the USSR and at Novosibirsk State University.

We hope that this book will enable beginning mathematicians to rapidly enter the contemporary circle of problems, and at the same time will be useful both to specialists in the area and to mathematicians working in many other areas.

The authors
Akademgorodok,
Novosibirsk,
November 1979

Editor's Preface to the English Translation

This most unusual book is both a list of basic facts concerning Kleinian groups and uniformization, and a comprehensive compendium of the primary literature in these and related topics. We expect it to be a very valuable resource—as a list of important results, many of which are not elsewhere in print, as a general guide to the literature, and as a specific guide to the Russian literature, which is not as well known here as it deserves to be. The mathematical community owes a debt of gratitude to the three authors.

The book covers an enormous amount of material; while the statements are in general clearly written, almost all of them come without proof. The authors provide references to primary sources for most of the material; however some of their statements are unsupported.

The editor has tried to find the appropriate balance between respecting the flavor of the original, and seeing to it (as best he can) that the book not only is correct, but also does not mislead the unwary. While the editor has tried to make appropriate comments where he found incorrect or misleading statements, he cannot guarantee that he has found them all, especially since some of the errors or gaps may occur in the original literature. Clearly, the specialist should consult the primary sources, with the usual caution.

This page intentionally left blank

Bibliography

[1] Herbert Abels, *Geometrische Erzeugung von diskontinuierlichen Gruppen*, Schriftenreihe Math. Inst. Univ. Münster No. 33 (1966).

[2] William Abikoff, *Some remarks on Kleinian groups*, Advances in the Theory of Riemann Surfaces (Proc. Conf., Stony Brook, N. Y., 1969; L. V. Ahlfors et al., editors), Ann. of Math. Studies, vol. 66, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1971, pp. 1–5.

[3] ——, *Remarks on a paper of Maskit*, Preprint, Columbia Univ., New York, 1973.

[4] ——, *The residual limit sets of Kleinian groups*, Acta Math. **130** (1973), 127–144.

[5] ——, *Constructability and Bers stability of Kleinian groups*, Discontinuous Groups and Riemann Surfaces (Proc Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 3–12.

[6] ——, *On boundaries of Teichmüller spaces and on Kleinian groups.* III, Acta Math. **134** (1975), 211–237.

[7] ——, *Degenerating families of Riemann surfaces*, Ann. of Math. (2) **105** (1977), 29–44.

[8] William Abikoff and Bernard Maskit, *Geometric decompositions of Kleinian groups*, Amer. J. Math. **99** (1977), 687–697.

[9] Robert D. M. Accola, *Invariant domains for Kleinian groups*, Amer. J. Math. **88** (1966), 329–334.

[10] ——, *Automorphisms of Riemann surfaces*, J. Analyse Math. **18** (1967), 1–5.

[11] ——, *On the number of automorphisms of a closed Riemann surface*, Trans. Amer. Math. Soc. **131** (1968), 398–408.

[12] ——, *Strongly branched coverings of closed Riemann surfaces*, Proc. Amer. Math. Soc. **26** (1970), 315–322.

[13] Lars V. Ahlfors, *On quasiconformal mappings*, J. Analyse Math. **3** (1953/54), 1–58, 207–208.

[14] ——, *Curvature properties of Teichmüller's space*, J. Analyse Math. 9 (1961), 161–176.

[15] ——, *Finitely generated Kleinian groups*, Amer. J. Math. **86** (1964), 413–429; **87** (1965), 759.

[16] ____, *Fundamental polyhedrons and limit point sets of Kleinian groups*, Proc. Nat. Acad. Sci. U.S.A. **55** (1966), 251–254.

[17] ____, *Kleinsche Gruppen in der Ebene und im Raum*, Festband zum 70. Geburtstag von Rolf Nevanlinna (H. P. Kunzi and A. Pfluger, editors), Springer-Verlag, 1966, pp. 7–15.

[18] ____, *Eichler integrals and the area theorem of Bers*, Michigan Math. J. **15** (1968), 257–263.

[19] ____, *Lectures on quasiconformal mappings*, Van Nostrand, 1966.

[20] ____, *The structure of a finitely generated Kleinian group*, Acta Math. **122** (1969), 1–17.

[21] ____, *Two lectures on Kleinian groups*, Proc. Romanian-Finnish Sem. Teichmüller Spaces and Quasiconformal Mappings (Brasov, 1969; C. A. Cazacu, editor), Publ. House Acad. Socialist Republic of Romania, Bucharest, 1971, pp. 49–64.

[22] Lars V. Ahlfors and Leo Sario, *Riemann surfaces*, Princeton Univ. Press, Princeton, N. J., 1960.

[23] L. Ahlfors and G. Weill, *A uniqueness theorem for Beltrami equations*, Proc. Amer. Math. Soc. **13** (1962), 975–978.

[24] Tohru Akaza, *Poincaré theta series and singular sets of Schottky groups*, Nagoya Math. J. **24** (1964), 43–65.

[25] A. D. Aleksandrov, *On the filling of space by polyhedra*, Vestnik Leningrad. Univ. **1954**, no. 2 (Ser. Mat. Fiz. Khim. vyp. 1), 33–43. (Russian)

[26] B. N. Apanasov, *Some characteristics of conformal mappings in R^n*, Metric Questions of the Theory of Functions and Mappings, No. 5, "Naukova Dumka", Kiev, 1974, pp. 3–13. (Russian)

[27] ____, *On a class of Kleinian groups in R^n*, Dokl. Akad. Nauk SSSR **215** (1974), 509–510=Soviet Math. Dokl. **15** (174), 515–517.

[28] ____, *On an analytic method in the theory of Kleinian groups on a multidimensional Euclidean space*, Dokl. Akad. Nauk SSSR **222** (1975), 11–14=Soviet Math. Dokl. **16** (1975), 553–556.

[29] ____, *Kleinian groups in space*, Sibirsk. Mat. Zh. **16** (1975), 891–898=Siberian Math. J. **16** (1975), 679–684.

[30] ____, *A universal property of Kleinian groups in the hyperbolic metric*, Dokl. Akad. Nauk SSSR **225** (1975), 15–17=Soviet Math. Dokl. **16** (1975), 1418–1421.

[31] ____, *On the relation of the group $SL(n, \mathbf{Z})$ with the n-dimensional modular group*, Dokl. Akad. Nauk SSSR **231** (1976), 1033–1036=Soviet Math. Dokl. **17** (1976), 1670–1674.

[32] ____, *On an automorphic mapping in space*, Some Questions of Contemporary Function Theory (Conf. Materials), Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1976. (Russian)

[33] ____, *On Mostow's rigidity theorem*, Dokl. Akad. Nauk SSSR **243** (1978), 829–832=Soviet Math. Dokl. **19** (1978), 1408–1412.

[34] ____, *Entire automorphic forms in R^n*, Sibirsk. Mat. Zh. **19** (1978), 735–748=Siberian Math. J. **19** (1978), 518–528.

[35] ____, *On the problem of the existence of automorphic mappings in n dimensions*, Questions in the Metric Theory of Mappings and Its Application (Proc. Fifth Colloq., Donetsk, 1976; G. D. Suvorov and V. I. Biliĭ, editors), "Naukova Dumka", Kiev, 1978, pp. 3–9. (Russian)

[36] ____, *Nontriviality of Teichmüller space for Kleinian group in space*, Riemann Surfaces and Related Topics: Proc. 1978 Stony Brook Conf. (I. Kra and B. Maskit, editors), Ann. of Math. Studies, vol. 97, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1981, pp. 21–31.

[37] ____, *Kleinian groups, Teichmüller space, and Mostow's rigidity theorem*, Sibirsk. Mat. Zh. **21** (1980), no. 4, 3–15=Siberian Math. J. **21** (1980), 483–491.

[38] B. N. Apanasov and A. V. Tetenov, *On the existence of nontrivial quasiconformal deformations of Kleinian groups in space*, Dokl. Akad. Nauk SSSR **239** (1978), 14–17=Soviet Math. Dokl. **19** (1978), 242–245.

[39] Paul Appell and Édouard Goursat, *Théorie des fonctions d'une variable et des transcendantes qui s'y rattachent.* Vol. II: *Fonctions automorphes*, 2nd ed. (rev. by P. Fatou), Gauthier-Villars, Paris, 1930.

[40] *Arithmetic groups and automorphic functions*, "Mir", Moscow, 1979. (Russian) This is an abridged Russian translation of the following three items:

a. *Algebraic groups and discontinuous subgroups*, Proc. Sympos. Pure Math., vol. 9, Amer. Math. Soc., Providence, R. I., 1966.

b. Roger Godement, *Introduction à la théorie de Langlands*, Sém. Bourbaki 1966/67, Exposé 321, Secrétariat Math., Paris, 1967; reprint, Benjamin, 1968.

c. Atle Selberg, *Discrete groups and harmonic analysis*, Proc. Internat. Congr. Math. (Stockholm, 1962), Inst. Mittag-Leffler, Djursholm, 1963, pp. 177–189.

[41] Walter L. Baily, Jr., *Introductory lectures on automorphic forms*, Publ. Math. Soc. Japan, vol. 12, Iwanami Shoten, Tokyo, and Princeton Univ. Press, Princeton, N. J., 1973.

[42] A. F. Beardon, *The Hausdorff dimension of singular sets of properly discontinuous groups*, Amer. J. Math. **88** (1966), 722–736.

[43] Alan F. Beardon and Troels Jørgensen, *Fundamental domains for finitely generated Kleinian groups*, Math. Scand. **36** (1975), 21–26.

[44] Alan F. Beardon and Bernard Maskit, *Limit points of Kleinian groups and finite sided fundamental polyhedra*, Acta Math. **132** (1974), 1–12.

[45] P. P. Belinskiĭ, *General properties of quasiconformal mappings*, "Nauka", Novosibirsk, 1974. (Russian)

[46] ____, *An example of a universal Riemann surface containing any compact surface with boundary as a subregion*, Sibirsk. Mat. Zh. **21** (1980), no. 3, 224–225. (Russian)

[47] Lipman Bers, *Quasiconformal mappings and Teichmüller's theorem*, Analytic Functions (Conf., Princeton, N. J., 1957; by R. Nevanlinna et al.), Princeton Univ. Press, Princeton, N. J., 1960, pp. 89–119.

[48] ——, *On moduli of Riemann surfaces*, Lectures, Forschungsinst. Math., E.T.H., Zürich, 1964.

[49] ——, *Inequalities for finitely generated Kleinian groups*, J. Analyse Math. **18** (1967), 23–41.

[50] ——, *Spaces of Kleinian groups*, Several Complex Variables (Proc. Conf., College Park, Md., 1970), Lecture Notes in Math., vol. 155, Springer-Verlag, 1970, pp. 9–34.

[51] ——, *On boundaries of Teichmüller spaces and on Kleinian groups. I*, Ann. of Math. (2) **91** (1970), 570–600.

[52] ——, *A non-standard integral equation with applications to quasiconformal mappings*, Acta Math. **116** (1966), 113–134.

[53] ——, *A remark on Mumford's compactness theorem*, Israel J. Math. **12** (1972), 400–407.

[54] ——, *Uniformization, moduli, and Kleinian groups*, Bull. London Math. Soc. **4** (1972), 257–300.

[55] ——, *On moduli of Kleinian groups*, Uspekhi Mat. Nauk **29** (1974), no. 2 (1976), 86–102=Russian Math. Surveys **29** (1974), no. 2, 88–102.

[56] ——, *Deformations and moduli of Riemann surfaces with nodes and signatures*, Math. Scand. **36** (1975), 12–16.

[57] ——, *On Hilbert's 22nd problem*, Mathematical Developments Arising from Hilbert Problems, Proc. Sympos. Pure Math., vol. 28, Amer. Math. Soc., Providence, R. I., 1976, pp. 559–609.

[58] Lipman Bers and Leon Ehrenpreis, *Holomorphic convexity of Teichmüller spaces*, Bull. Amer. Math. Soc. **70** (1964), 761–764.

[59] L. Bers and I. Kra (editors), *A crash course on Kleinian groups* (Lectures, San Francisco, 1974), Lecture Notes in Math., vol. 400, Springer-Verlag, 1974.

[60] A. Beurling and L. Ahlfors, *The boundary correspondence under quasiconformal mappings*, Acta Math. **96** (1956), 125–142.

[61] Joan S. Birman, *Abelian quotients of the mapping class group of a 2-manifold*, Bull. Amer. Math. Soc. **76** (1970), 147–150.

[62] E. M. Brown, *Unknotting in $M^2 \times I$*, Trans. Amer. Math. Soc. **123** (1966), 480–505.

[63] Henri Cartan, *Quotient d'un espace analytique par un groupe d'automorphismes*, Algebraic Geometry and Topology (Sympos. in Honor of S. Lefschetz, Princeton, N. J., 1954; R. Fox, D. Spencer and A. Tucker, editors), Princeton Univ. Press, Princeton, N. J., 1957, pp. 90–102.

[64] N. G. Chebotarev, *Theory of algebraic functions*, GITTL, Moscow, 1948. (Russian)

[65] Su-Shing Chen, *A theorem of Ahlfors for hyperbolic spaces*, Trans. Amer. Math. Soc. **242** (1978), 401–406.

[66] Vicki Chuckrow, *On Schottky groups with applications to Kleinian groups*, Ann. of Math. (2) **88** (1968), 47–61.

[67] V. V. Chueshev, *Schottky spaces of the type* (g, s, m), Dokl. Akad. Nauk SSSR **223** (1975), 1326–1328=Soviet Math. Dokl. **16** (1975), 1107–1110.

[68] ——, *Spaces of Kleinian groups and Riemann surfaces*, Candidate's Dissertation, Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1979. (Russian)

[69] Richard H. Crowell and Ralph H. Fox, *Introduction to knot theory*, Ginn, Boston, Mass., 1963.

[70] Clifford J. Earle, *On holomorphic cross-sections in Teichmüller spaces*, Duke Math. J. **36** (1969), 409–415.

[71] Clifford J. Earle and James Eells, *A fibre bundle description of Teichmüller theory*, J. Differential Geometry **3** (1969), 19–43.

[72] Clifford J. Earle and Irwin Kra, *On holomorphic mappings between Teichmüller spaces*, Contributions to Analysis (Collection Dedicated to Lipman Bers; L. Ahlfors et al., editors), Academic Press, 1974, pp. 107–124.

[73] ——, *On sections of some holomorphic families of closed Riemann surfaces*, Acta Math. **137** (1976), 49–79.

[74] D. B. A. Epstein, *Finite presentations of groups and 3-manifolds*, Quart J. Math. Oxford Ser. (2) **12** (1961), 205–212.

[75] ——, *Projective planes in 3-manifolds*, Proc. London Math. Soc. (3) **11** (1961), 469–484.

[76] Benny Evans and Louise Moser, *Solvable fundamental groups of compact 3-manifolds*, Trans. Amer. Math. Soc. **168** (1972), 189–210.

[77] W. Fenchel and J. Nielsen, *On discontinuous groups of isometric transformations of the non-Euclidean plane*, Studies and Essays Presented to R. Courant on his 60th Birthday (K. Friedrichs, O. Neugebauer and J. Stoker, editors), Interscience, 1948, pp. 117–128.

[78] L. R. Ford, *Automorphic functions*, McGraw-Hill, 1929.

[79] Robert Fricke and Felix Klein, *Vorlesungen über die Theorie der automorphen Functionen*. Vols. I, II, 2nd ed., Teubner, Leipzig, 1926; reprint, 1965.

[80] F. Gardiner and I. Kra, *Stability of Kleinian groups*, Indiana Univ. Math. J. **21** (1971/72), 1037–1059.

[81] H. Garland and M. S. Raghunathan, *Fundamental domains for lattices in (R-) rank 1 semisimple Lie groups*, Ann. of Math. (2) **92** (1970), 279–326.

[82] F. W. Gehring, *Spirals and the universal Teichmüller space*, Acta Math. **141** (1978), 99–113.

[83] ——, *Univalent functions and the Schwarzian derivative*, Comment. Math. Helv. **52** (1977), 561–572.

[84] V. V. Golubev, *Lectures on the analytic theory of differential equations*, 2nd ed., GITTL, Moscow, 1950. (Russian)

[85] ——, *Single-valued analytic functions. Automorphic functions*, Fizmatgiz, Moscow, 1961. (Russian)

[86] Leon Greenberg, *Discrete subgroups of the Lorentz group*, Math. Scand. **10** (1962), 85–107.

[87] ——, *Fundamental polyhedra for Kleinian groups*, Ann. of Math. (2) **84** (1966), 433–441.

[88] ——, *Commensurable groups of Moebius transformations*, Discontinuous Groups and Riemann Surfaces (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 227–237.

[89] ——, *Maximal groups and signatures*, Discontinuous Groups and Riemann Surfaces (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 207–226.

[90] Phillip A. Griffiths, *Complex-analytic properties of certain Zariski-open sets on algebraic varieties*, Ann. of Math. (2) **94** (1971), 21–51.

[91] N. A. Gusevskiĭ, *On a problem in the theory of Kleinian groups*, Dokl. Akad. Nauk SSSR **234** (1977), 277–279=Soviet Math. Dokl. **18** (1977), 629–631.

[92] ——, *On two classes of Kleinian groups*, Dokl. Akad. Nauk SSSR **241** (1978), 753–756=Soviet Math. Dokl. **19** (1978), 919–922.

[93] ——, *Topological characterization of Schottky groups*, Sibirsk. Mat. Zh. **20** (1979), 651–655=Siberian Math. J. **20** (1979), 454–457.

[94] ——, *Kleinian groups and 3-manifolds*, Candidate's Dissertation, Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1979. (Russian)

[95] V. Ya. Gutlyanskiĭ, *On the area principle for a class of quasiconformal mappings*, Dokl. Akad. Nauk SSSR **212** (1973), 540–543=Soviet Math. Dokl. **14** (1973), 1401–1406.

[96] Zh. Adamar [Jacques Hadamard], *Non-Euclidean geometry in the theory of automorphic functions*, GITTL, Moscow, 1951. (Russian)

[97] W. J. Harvey (editor), *Discrete groups and automorphic functions* (Proc. Instructional Conf., Cambridge, 1975), Academic Press, 1977.

[98] Dennis A. Hejhal, *On Schottky and Teichmüller spaces*, Advances in Math. **15** (1975), 133–156.

[99] ——, *The variational theory of linearly polymorphic functions*, J. Analyse Math. **30** (1976), 215–264.

[100] John Hempel, *3-manifolds*, Ann. of Math. Studies, vol. 86, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1976.

[101] John Hempel and William Jaco, *3-manifolds which fiber over a surface*, Amer. J. Math. **94** (1972), 189–205.

[**102**] ____, *Fundamental groups of 3-manifolds which are extensions*, Ann. of Math. (2) **95** (1972), 86–98.

[**103**] John Hamal Hubbard, *Sur les sections analytiques de la courbe universelle de Teichmüller*, Mem. Amer. Math. Soc. **4** (1976), no. 166.

[**104**] A. Hurwitz, *Über algebraische Gebilde mit eindeutigen Transformationen in sich*, Math. Ann. **41** (1893), 403–442.

[**105**] Yu. G. Il'yashenko, *Nondegenerate B-groups*, Dokl. Akad. Nauk SSSR **208** (1973), 1020–1022=Soviet Math. Dokl. **14** (1973), 207–210.

[**106**] ____, *Nondegenerate B-groups and simultaneous uniformization*, Some Questions of Contemporary Function Theory (Conf. Materials, Novosibirsk, 1976; P. P. Belinskiĭ, editor), Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1976, pp 67–74. (Russian)

[**107**] William Jaco, *Finitely presented subgroups of three-manifold groups*, Invent. Math. **13** (1971), 335–346.

[**108**] Troels Jørgensen, *Some remarks on Kleinian groups*, Math. Scand. **34** (1974), 101–108.

[**109**] ____, *On discrete groups of Möbius transformations*, Amer. J. Math. **98** (1976), 739–749.

[**110**] ____, *Compact 3-manifolds of constant negative curvature fibering over the circle*, Ann. of Math. (2) **106** (1977), 61–72.

[**111**] Troels Jørgensen and Maire Kiikka, *Some extreme discrete groups*, Ann. Acad. Sci. Fenn. Ser. A I Math. **1** (1975), 245–248.

[**112**] Troels Jørgensen and Albert Marden, *Two doubly degenerate groups*, Quart. J. Math. Oxford Ser. (2) **30** (1979), 143–156.

[**113**] T. Jørgensen, A. Marden and B. Maskit, *The boundary of classical Schottky space*, Duke Math. J. **46** (1979), 441–446.

[**114**] Linda Keen, *Canonical polygons for finitely generated Fuchsian groups*, Acta Math. **115** (1965), 1–16.

[**115**] ____, *Intrinsic moduli on Riemann surfaces*, Ann. of Math. (2) **84** (1966), 404–420.

[**116**] ____, *A rough fundamental domain for Teichmüller spaces*, Bull. Amer. Math. Soc. **83** (1977), 1199–1226.

[**117**] Felix Klein, *Neue Beiträge zur Riemann'schen Functionentheorie*, Math. Ann. **21** (1883), 141–218.

[**118**] ____, *Gesammelte mathematische Abhandlungen*. Vol. 3, Springer-Verlag, 1923.

[**119**] Paul Koebe, *Über die Uniformisierung der algebraischen Kurven*. I, II, III, IV, Math. Ann. **67** (1909), 145–224; **69** (1910), 1–81; **72** (1912), 437–516; **75** (1914), 42–129.

[**120**] Irwin Kra, *On spaces of Kleinian groups*, Comment. Math. Helv. **47** (1972), 53–69.

[**121**] ____, *Automorphic forms and Kleinian groups*, Benjamin, 1972.

[**122**] ____, *On new kinds of Teichmüller spaces*, Israel J. Math. **16** (1973), 237–257.

[**123**] ____, *Varieties of Kleinian groups*, Complex Analysis 1972, Vol. II (Proc. Conf., Houston, Texas; H. Resnikoff and R. Wells, editors), Rice Univ. Studies **59** (1973), no. 2, 41–56.

[**124**] I. Kra and B. Maskit, *Involutions on Kleinian groups*, Bull. Amer. Math. Soc. **78** (1972), 801–805.

[**125**] Wilhelm Kraus, *Über die Zusammenhang einiger Charakteristiken eines einfach zusammenhängenden Bereiches mit der Kreisabbildung*, Mitt. Math. Sem. Univ. Giessen **21** (1932).

[**126**] S. N. Krushkal', *On the theory of extremal problems for quasiconformal mappings of closed Riemann surfaces*, Dokl. Akad. Nauk SSSR **171** (1966), 784–787=Soviet Math. Dokl **7** (1966), 1541–1544.

[**127**] ____, *Teichmüller's theorem on extremal quasiconformal mappings*. Sibirsk. Mat. Zh. **8** (1967), 313–332=Siberian Math. J. **8** (1967), 231–244.

[**128**] ____, *Moduli of Riemann surfaces*, Sibirsk. Mat. Zh. **13** (1972) 349–367=Siberian Math. J. **13** (1972), 241–253.

[**129**] ____, *Variational methods in the theory of quasiconformal mappings* Novosibirsk. Gos. Univ., Novosibirsk, 1974. (Russian)

[**130**] ____, *Quasiconformal mappings and Riemann surfaces*, "Nauka" Novosibirsk, 1975. (Russian; English transl. listed separately as [137])

[**131**] ____, *On a property of limit sets of Kleinian groups*, Dokl Akad Nauk SSSR **225** (1975), 500–502; **237** (1977), 256=Soviet Math. Dokl. **16** (1975), 1497–1499; **18** (1977), no. 4, iv.

[**132**] ____, *On the supports of Beltrami differentials for Kleinian groups*. Dokl. Akad. Nauk SSSR **231** (1976), 799–801=Soviet Math. Dokl. **17** (1976), 1642–1644.

[**133**] ____, *Two theorems on Teichmüller spaces*, Dokl. Akad. Nauk SSSR **228** (1976), 290–292=Soviet Math. Dokl. **17** (1976), 704–707.

[**134**] ____, *Some remarks on Kleinian groups*, Riemann Surfaces and Related Topics: Proc. 1978 Stony Brook Conf. (I. Kra and B. Maskit, editors), Ann. of Math. Studies, vol. 97, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1981, pp. 361–366.

[**135**] ____, *Some rigidity theorems for discontinuous groups*, Math. Anal. and Related Math. Questions (A. A. Borovkov, editor), "Nauka", Novosibirsk, 1978, pp. 69–82; English transl., Amer. Math. Soc. Transl. (2) **122** (1984), 75–83.

[**136**] ____, *Quasiconformal stability of Kleinian groups*, Sibirsk. Mat. Zh. **20** (1979), 322–329=Siberian Math. J. **20** (1979), 229–234.

[**137**] ____, *Quasiconformal mappings and Riemann surfaces*, Wiley, 1979.

[**138**] ____, *To the problem of the supports of Beltrami differentials for Kleinian groups*, Romanian-Finnish Sem. Complex Anal. (Proc., Bucharest, 1976), Lecture Notes in Math., vol. 743, Springer-Verlag, 1979, pp. 132–139.

[**139**] ——, *A remark on rigidity of quasiconformal deformations of discrete isometry groups of hyperbolic spaces*, Sibirsk. Mat. Zh. **21** (1980), no. 5, 52–57=Siberian Math. J. **21** (1980), 683–687.

[**140**] S. L. Krushkal′, B. N. Apanasov and N. A. Gusevskiĭ, *Kleinian groups in examples and problems*, Novosibirsk. Gos. Univ., Novosibirsk, 1978. (Russian)

[**141**] ——, *Uniformization and Kleinian groups*, Novosibirsk. Gos. Univ., Novosibirsk, 1979. (Russian)

[**142**] R. Kühnau, *Geometrie der konformen Abbildung auf der hyperbolischen und der elliptischen Ebene*, VEB Deutscher Verlag Wiss., Berlin, 1974.

[**143**] N. H. Kuiper, *On compact conformally Euclidean spaces of dimension* > 2, Ann. of Math. (2) **52** (1950), 478–490.

[**144**] ——, *On conformally-flat spaces in the large*, Ann. of Math. (2) **50** (1949), 916–924.

[**145**] R. S. Kulkarni, *Infinite regular coverings*, Duke Math. J. **45** (1978), 781–796.

[**146**] ——, *On the principle of uniformization*, J. Differential Geometry **13** (1978), 109–138.

[**147**] Herbert Lange, *Moduli spaces of algebraic curves with rational maps*, Math. Proc. Cambridge Philos. Soc. **78** (1975), 283–292.

[**148**] M. A. Lavrent′ev, *Variational methods for boundary value problems for systems of elliptic equations*, Izdat. Akad. Nauk SSSR, Moscow, 1962; English transl., Noordhoff, 1963.

[**149**] Joseph Lehner, *Discontinuous groups and automorphic functions*, Math. Surveys, Vol. 8, Amer. Math. Soc., Providence, R. I., 1964; reprinted with corrections, 1982.

[**150**] Olli Lehto, *Univalent functions and Teichmüller theory*, Proc. First Finnish-Polish Summer School Complex Anal. (Podlesice, 1977; J. Ławrynowicz and O. Lehto, editors), Part 1, Univ. Łódź, Łódź, 1978, pp. 11–33.

[**151**] O. Lehto and K. I. Virtanen, *Quasikonforme Abbildungen*, Springer-Verlag, 1965.

[**152**] Armin Leutbecher, *Über die Heckeschen Gruppen* $\mathfrak{G}(\lambda)$, Abh. Math. Sem. Univ. Hamburg **31** (1967), 199–205.

[**153**] W. B. R. Lickorish, *A finite set of generators of the homeotopy group of a 2-manifold*, Proc. Cambridge Philos. Soc. **60** (1964), 769–778; **62** (1966), 679–681.

[**154**] Michele Linch, *A comparison of metrics on Teichmüller space*, Proc. Amer. Math. Soc. **43** (1974), 349–352.

[**155**] G. M. Lyan, *On the kernel of Poincaré's* θ*-operator*, Dokl. Akad. Nauk SSSR **230** (1976), 269–270=Soviet Math. Dokl. **17** (1976), 1283–1285.

[**156**] ——, *A constructive description of the kernel of the Poincaré* θ*-operator*, Dokl. Akad. Nauk UzSSR **1977**, no. 12, 7–8. (Russian)

[**157**] ____, *Teichmüller space and the Poincaré Θ-operator*, Dokl. Akad. Nauk SSSR **250** (1980), 1053–1056=Soviet Math. Dokl. **21** (1980), 260–262.

[**158**] A. M. Macbeath, *On a theorem of Hurwitz*, Proc. Glasgow Math. Assoc. **5** (1961/62), 90–96.

[**159**] ____, *On a curve of genus* 7, Proc. London Math. Soc. (3) **15** (1965), 527–542.

[**160**] Colin Maclachlan, *A bound for the number of automorphisms of a compact Riemann surface*, J. London Math. Soc. **44** (1969), 265–272.

[**161**] ____, *Modulus space is simply-connected*, Proc. Amer. Math. Soc. **29** (1971), 85–86.

[**162**] ____, *Maximal normal Fuchsian groups*, Illinois J. Math. **15** (1971), 104–113.

[**163**] C. Maclachlan and W. J. Harvey, *On mapping-class groups and Teichmüller spaces*, Proc. London Math. Soc. (3) **30** (1975), 496–512.

[**164**] Wilhelm Magnus, Abraham Karrass and Donald Solitar, *Combinatorial group theory*, Interscience, 1966.

[**165**] V. S. Makarov, *A certain class of discrete Lobachevsky space groups with an infinite fundamental region of finite measure*, Dokl. Akad. Nauk SSSR **167** (1966), 30–33=Soviet Math. Dokl. **7** (1966), 328–331.

[**166**] Albert Marden, *On homotopic mappings of Riemann surfaces*, Ann. of Math. (2) **90** (1969), 1–8.

[**167**] ____, *An inequality for Kleinian groups*, Advances in the Theory of Riemann Surfaces (Proc. Conf., Stony Brook, N. Y., 1969; L. V. Ahlfors et al., editors), Ann. of Math. Studies, vol. 66, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1971, pp. 295–296.

[**168**] ____, *The geometry of finitely generated Kleinian groups*, Ann. of Math. (2) **99** (1974), 383–462.

[**169**] ____, *Universal properties of Fuchsian groups in the Poincaré metric*, Discontinuous Groups and Riemann Surfaces (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 315–339.

[**170**] ____, *Schottky groups and circles*, Contributions to Analysis (Collection Dedicated to Lipman Bers; L. Ahlfors et al., editors), Academic Press, 1974, pp. 273–278.

[**171**] A. Marden and B. Maskit, *On the isomorphism theorem for Kleinian groups*, Invent. Math. **51** (1979), 9–14.

[**172**] G. A. Margulis, *Isometry of closed manifolds of constant negative curvature with the same fundamental group*, Dokl. Akad. Nauk SSSR **192** (1970), 736–737=Soviet Math. Dokl. **11** (1970), 722–723.

[**173**] ____, *Arithmetic properties of discrete subgroups*, Uspekhi Mat. Nauk **29** (1974), no. 1 (174), 49–98=Russian Math. Surveys **29** (1974), no. 1, 107–156.

[**174**] Olli Martio and Uri Srebro, *On the existence of automorphic quasimeromorphic mappings in* R^n, Ann. Acad Sci. Fenn. Ser. A I Math. **3** (1977), 123–130.

[**175**] ——, *Automorphic quasimeromorphic mappings in* R^n, Acta Math. **135** (1975), 221–247.

[**176**] Bernard Maskit, *A theorem on planar covering surfaces with applications to* 3*-manifolds*, Ann. of Math. (2) **81** (1965), 341–355.

[**177**] ——, *A characterization of Schottky groups*, J. Analyse Math. **19** (1967), 227–230.

[**178**] ——, *The conformal group of a plane domain*, Amer. J. Math. **90** (1968), 718–722.

[**179**] ——, *On a class of Kleinian groups*, Ann. Acad. Sci. Fenn. Ser. A I Math. No. 442 (1969).

[**180**] ——, *On boundaries of Teichmüller spaces and on Kleinian groups.* II, Ann. of Math. (2) **91** (1970), 607–639.

[**181**] ——, *On Klein's combination theorem.* III, Advances in the Theory of Riemann Surfaces (Proc. Conf., Stony Brook, N. Y., 1969; L. V. Ahlfors et al., editors), Ann. of Math. Studies, vol. 66, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1971, pp. 297–316.

[**182**] ——, *Self-maps on Kleinian groups*, Amer. J. Math. **93** (1971), 840–856.

[**183**] ——, *On Poincaré's theorem for fundamental polygons*, Advances in Math. **7** (1971), 219–230.

[**184**] ——, *Decompositions of certain Kleinian groups*, Acta Math. **130** (1973), 243–263.

[**185**] ——, *Intersections of component subgroups of Kleinian groups*, Discontinuous Groups and Riemann Surfaces (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 349–367.

[**186**] ——, *Uniformizations of Riemann surfaces*, Contributions to Analysis (Collection Dedicated to Lipman Bers; L. Ahlfors et al., editors), Academic Press, 1974, pp. 293–312.

[**187**] ——, *Moduli of marked Riemann surfaces*, Bull. Amer. Math. Soc. **80** (1974), 773–777.

[**188**] ——, *On the classification of Kleinian groups.* I, II, Acta Math. **135** (1975), 249–270; **138** (1977), 17–42.

[**189**] A. D. Mednykh, *On semidirect products of discontinuous transformation groups*, Dokl. Akad. Nauk SSSR **225** (1975), 1016–1017=Soviet Math. Dokl. **16** (1975), 1578–1580.

[**190**] ——, *On branched coverings of Riemann surfaces with the trivial group of covering transformations*, Dokl. Akad. Nauk SSSR **235** (1977), 1267–1269=Soviet Math. Dokl. **18** (1977), 1156–1158.

[**191**] ____, *On an example of a compact Riemann surface with trivial automorphism group*, Dokl. Akad. Nauk SSSR **237** (1977), 32–34=Soviet Math. Dokl. **18** (1977), 1396–1398.

[**192**] ____, *Determination of the number of nonequivalent coverings over a compact Riemann surface*, Dokl. Akad. Nauk SSSR **239** (1978), 269–271=Soviet Math. Dokl. **19** (1978), 318–320.

[**193**] ____, *On unramified coverings of compact Riemann surfaces*, Dokl. Akad. Nauk SSSR **244** (1979), 529–532=Soviet Math. Dokl. **20** (1979), 85–88.

[**194**] G. D. Mostow, *Strong rigidity of locally symmetric spaces*, Ann. of Math. Studies, vol. 78, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1973.

[**195**] David Mumford, *A remark on Mahler's compactness theorem*, Proc. Amer. Math. Soc. **28** (1971), 289–294.

[**196**] ____, *Stability of projective varieties*, L'Enseignement Math. (2) **23** (1977), 39–110; reprint, Monogr. Enseignement Math., no. 24, L'Enseignement Math., Geneva, 1977.([19])

[**197**] P. J. Myrberg, *Die Kapazität der singulären Menge der linearen Gruppen*, Ann. Acad. Sci. Fenn. Ser. A I Math.-Phys. No. 10 (1941).

[**198**] Zeev Nehari, *The Schwarzian derivative and schlicht functions*, Bull. Amer. Math. Soc. **55** (1949), 545–551.

[**199**] Rolf Nevanlinna, *Uniformisierung*, Springer-Verlag, 1953.

[**200**] S. P. Novikov, *Manifolds with free Abelian fundamental groups and their applications (Pontryagin classes, smoothness, multidimensional knots)*, Izv. Akad. Nauk SSSR Ser. Mat. **30** (1966), 207–246; English transl., Amer. Math. Soc. Transl. (2) **71** (1968), 1–42.

[**201**] William F. Osgood, *A Jordan curve of positive area*, Trans. Amer. Math. Soc. **4** (1903), 107–112.

[**202**] C. D. Papakyriakopoulos, *On Dehn's lemma and the asphericity of knots*, Ann. of Math. (2) **66** (1957), 1–26.

[**203**] ____, *On solid tori*, Proc. London Math. Soc. (3) **7** (1957), 281–299.

[**204**] H. Poincaré, *Mémoire sur les groupes kleinéens*, Acta Math. **3** (1883/84), 49–92.

[**205**] ____, *Cinquième complément à l'Analysis situs*, Rend. Circ. Mat. Palermo **18** (1904), 45–110.

[**206**] ____, *Sur l'uniformisation des fonctions analytiques*, Acta Math. **31** (1908), 1–63.

[**207**] ____, *Oeuvres*. Vol. II, Gauthier-Villars, Paris, 1914.

[**208**] Christian Pommerenke, *On the Green's fundamental domain*, Math. Z. **156** (1977), 157–164.

([19])The authors cite a preprint.

[**209**] H. E. Rauch, *A transcendental view of the space of algebraic Riemann surfaces*, Bull. Amer. Math. Soc. **71** (1965), 1–39.

[**210**] Yu. G. Reshetnyak, *Representations in space with bounded distortion*, Internat. Congr. Math. (Nice, 1970): Papers of Soviet Mathematicians, "Nauka", Moscow, 1972, pp. 258–263. (Russian)

[**211**] Robert Riley, *Discrete parabolic representations of link groups*, Mathematika **22** (1975), 141–150.

[**212**] C. P. Rourke and B. J. Sanderson, *Introduction to piecewise-linear topology*, Springer-Verlag, 1972.

[**213**] H. L. Royden, *Automorphisms and isometries of Teichmüller spaces*, Advances in the Theory of Riemann Surfaces (Proc. Conf., Stony Brook, N. Y., 1969; L. A. Ahlfors et al., editors), Ann. of Math. Studies, vol. 66, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1971, pp. 369–383.

[**214**] V. V. Savin, *On moduli of Riemann surfaces*, Dokl. Akad. Nauk SSSR **196** (1971), 783–785=Soviet Math. Dokl. **12** (1971), 267–270.

[**215**] G. P. Scott, *Finitely generated 3-manifold groups are finitely presented*, J. London Math. Soc. (2) **6** (1972/73), 437–440.

[**216**] ——, *Compact submanifolds of 3-manifolds*, J. London Math. Soc. (2) **7** (1973/74), 246–250.

[**217**] H. Seifert and W. Threlfall, *Lehrbuch der Topologie*, Teubner, Leipzig, 1934.

[**218**] C. Weber and H. Seifert, *Die beiden Dodekaederräume*, Math. Z. **37** (1933), 237–253.

[**219**] Atle Selberg, *On discontinuous groups in higher-dimensional symmetric spaces*, Contributions to Function Theory (Internat. Colloq., Bombay, 1960), Tata Inst. Fund. Res., Bombay, 1960, pp. 147–164.

[**220**] V. G. Sheretov, *Invariant subspaces in a Teichmüller space*, Perm. Gos. Univ. Uchen. Zap. Mat. No. 103 (1963), 156–159. (Russian)

[**221**] Edwin H. Spanier, *Algebraic topology*, McGraw-Hill, 1966.

[**222**] George Springer *Introduction to Riemann surfaces*, Addison-Wesley, 1957.

[**223**] John R. Stallings, *On the loop theorem*, Ann. of Math. (2) **72** (1960), 12–19.

[**224**] ——, *On fibering certain 3-manifolds*, Topology of 3-Manifolds and Related Topics (Proc. Univ. of Georgia Inst., 1961; M. K. Fort, Jr., editor), Prentice-Hall, Englewood Cliffs, N. J., 1962, pp. 95–100.

[**225**] ——, *A topological proof of Grushko's theorem on free products*, Math. Z. **90** (1965), 1–8.

[**226**] Jacob Sturm and Meir Shinnar, *The maximal inscribed ball of a Fuchsian group*, Discontinuous Groups and Riemann Surfaces (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies,

vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 439–443.

[**227**] R. Sulanke and P. Wintgen, *Differentialgeometrie und Faserbündel*, Birkhäuser, 1972.

[**228**] Dennis Sullivan, *On the ergodic theory at infinity of an arbitrary discrete group of hyperbolic motions*, Riemann Surfaces and Related Topics: Proc. 1978 Stony Brook Conf. (I. Kra and B. Maskit, editors), Ann. of Math. Studies, vol. 97, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1981, pp. 465–496.

[**229**] Oswald Teichmüller, *Extremale quasikonforme Abbildungen und quadratische Differentiale*, Abh. Preuss. Akad. Wiss. Berlin Math.-Nat. Kl. **1939**, no. 22 (1940).

[**230**] ——, *Bestimmung der extremalen quasikonforme Abbildungen bei geschlossenen orientierten Riemannschen Flächen*, Abh. Preuss. Akad. Wiss. Berlin Math.-Nat. Kl. **1943**, no. 4.

[**231**] ——, *Veränderliche Riemannsche Flächen*, Deutsche Math. **7** (1942/44), 344–359.

[**232**] A. V. Tetenov, *Invariant components of Kleinian groups in space*, Materials Seventeenth All-Union Sci. Student Conf. Math., Novosibirsk. Gos. Univ., Novosibirsk, 1979, pp. 134–137. (Russian)

[**233**] ——, *Infinitely generated Kleinian groups in space*, Sibirsk. Mat. Zh. **21** (1980), no. 5, 88–99=Siberian Math. J. **21** (1980), 709–717.

[**234**] William Thurston, *The geometry and topology of* 3*-manifolds*, Lecture notes, Princeton Univ., Princeton, N. J., 1978.

[**235**] È. B. Vinberg, *Discrete groups generated by reflections in Lobachevsky spaces*, Mat. Sb. **72(114)** (1967), 471–488=Math. USSR Sb. **1** (1967), 429–444.

[**236**] ——, *Some examples of crystallographic groups in Lobachevsky spaces*, Mat. Sb. **78(120)** (1969), 633–639=Math. USSR Sb. **7** (1969), 617–622.

[**237**] È. B. Vinberg and O. V. Shvartsman, *Riemann surfaces*, Itogi Nauki: Algebra, Topologiya, Geometriya, vol. 16, VINITI, Moscow, 1978, pp. 191–245=J. Soviet Math. **14** (1980), no. 1, 985–1020.

[**238**] Friedhelm Waldhausen, *Eine Verallgemeinerung des Schleifensatzes*, Topology **6** (1967), 501–504.

[**239**] ——, *Gruppen mit Zentrum und* 3*-dimensionale Mannigfaltigkeiten*, Topology **6** (1967), 505–517.

[**240**] ——, *On irreducible* 3*-manifolds which are sufficiently large*, Ann. of Math. (2)**87** (1968), 56–88.

[**241**] Hermann V. Waldinger, *On the subgroups of the Picard group*, Proc. Amer. Math. Soc. **16** (1965), 1373–1378.

[**242**] André Weil, *Modules des surfaces de Riemann*, Sém. Bourbaki, 1957/58, Exposé 168, Secrétariat Math., Paris, 1958; reprint, Benjamin, 1966.

[243] J. H. C. Whitehead, *On 2-spheres in 3-manifolds*, Bull. Amer. Math. Soc. **64** (1958), 161–166.

[244] ———, *On finite cocycles and the sphere theorem*, Colloq. Math. **6** (1958), 271–281.

[245] Norbert J. Wielenberg, *Discrete Moebius groups: fundamental polyhedra and convergence*, Amer. J. Math. **99** (1977), 861–877.

[246] ———, *The structure of certain subgroups of the Picard group*, Math. Proc. Cambridge Philos. Soc. **84** (1978), 427–436.

[247] A. Wiman, *Über die algebraischen Curven von den Geschlechtern $p = 4, 5$ und 6, welche eindeutige Transformationen in sich besitzen*, Bichang Kongl. Svenska Vetenskaps-Akad. Handlingar **21** (1895/96), afd. I, no. 3.

[248] Scott Wolpert, *The finite Weil-Petersson diameter of Riemann space*, Pacific J. Math. **70** (1977), 281–288.

[249] Robert Zarrow, *Classical and non-classical Schottky groups*, Duke Math. J. **42** (1975), 717–724.

[250] I. V. Zhuravlev, *Univalent functions with quasiconformal extensions, and Teichmüller spaces*, Preprint, Inst. Mat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1979. (Russian)

Supplementary Bibliography

[251] William Abikoff, *The real analytic theory of Teichmüller space*, Lecture Notes in Math., vol. 820, Springer-Verlag, 1980.

[252] D. V. Anosov, *Geodesic flows on closed Riemannian manifolds of negative curvature*, Trudy Mat. Inst. Steklov. **90** (1967)=Proc. Steklov Inst. Math. **90** (1967).

[253] B. N. Apanasov, *Geometrically finite groups of transformations of space*, Sibirsk. Mat. Zh. **23** (1982), no. 6, 16–27=Siberian Math. J. **23** (1982), 771–780.

[254] ———, *Geometrically finite hyperbolic structures on manifolds*, Ann. Global Anal. and Geom. **1** (1983), no. 3, 1–22.

[255] ———, *Condition of conformal rigidity of hyperbolic manifolds with boundaries*, Analytic Functions (Blażejewko, 1982), Lecture Notes in Math., vol. 1039, Springer-Verlag, 1983, pp. 1–8.

[256] ———, *Discrete transformation groups and structures on manifolds*, "Nauka", Novosibirsk, 1983. (Russian)

[257] ———, *Incomplete hyperbolic manifolds and local finiteness of a tesselation of space by polyhedra*, Dokl. Akad. Nauk SSSR **273** (1983), 777–781=Soviet Math. Dokl. **28** (1983), 686–690.

[**258**] ____, *Parabolic vertices and finiteness properties for Kleinian groups in n-dimensional space*, Sibirsk. Mat. Zh. **25** (1984), no. 4, 9–27=Siberian Math. J. **25** (1984), 516–530.

[**259**] ____, *Cusp-neighbourhoods for Kleinian groups in space*, Complex Analysis and Its Applications to Partial Differential Equations (KTB, Wiss. Beiträge 1984/58 (M 35)), Martin-Luther-Univ., Halle (S), 1984, pp. 37–39.

[**260**] ____, *Cusp ends of hyperbolic manifolds*, Ann. Global Anal. and Geom. **3** (1985), no. 1, 1–12.

[**261**] ____, *Tesselations of space by polyhedra and deformations of incomplete hyperbolic structures*, Sibirsk. Mat. Zh. **27** (1986), no 4, 3–19; English transl. in Siberian Math. J. **27** (1986).

[**262**] ____, *Deformations of conformal structures on manifolds*, Abstracts Internat. Conf. Complex Anal. and Appl., Varna, 1985, p. 37.

[**263**] T. M. Bandman, *Surjective holomorphic mappings of projective manifolds*, Sibirsk. Mat. Zh. **22** (1981), no. 2, 48–56=Siberian Math. J. **22** (1981), 204–210.

[**264**] Lipman Bers, *Nielsen extensions of Riemann surfaces*, Ann. Acad. Sci. Fenn. Ser. A I Math. **2** (1976), 29–34.

[**265**] ____, *An extremal problem for quasiconformal mappings and a theorem by Thurston*, Acta Math. **141** (1978), 73–98.

[**266**] ____, *On iterates of hyperbolic transformations of Teichmüller space*, Amer. J. Math. **105** (1983), 1–11.

[**267**] Igor Dolgachev, *Integral quadratic forms: applications to algebraic geometry (after V. Nikulin)*, Sém. Bourbaki 1982/83, Exposé 611, Astérisque No. 105–106, Soc. Math. France, Paris, 1983, pp. 251–278.

[**268**] Clifford J. Earle, *On the Carathéodory metric in Teichmüller spaces*, Discontinuous Groups and Riemann Surfaces, (Proc. Conf., University Park, Md., 1973; L. Greenberg, editor), Ann. of Math. Studies, vol. 79, Princeton Univ. Press, Princeton, N. J., and Univ. of Tokyo Press, Tokyo, 1974, pp. 99–103.

[**269**] A. Fathi et al., *Travaux de Thurston sur les surfaces* (Sém. Orsay, 1976/77), Astérisque No. 66–67, Soc. Math. France, Paris, 1979.

[**270**] Jane Gilman, *On the Nielsen type and the classification for the mapping class group*, Advances in Math. **40** (1981), 68–96.

[**271**] William M. Goldman, *Conformally flat manifolds with nilpotent holonomy and the uniformization problem for 3-manifolds*, Trans. Amer. Math. Soc. **278** (1983), 573–583.

[**272**] N. A. Gusevskiĭ, *On doubly generated Kleinian groups*, Sibirsk. Mat. Zh. **23** (1982), no. 1, 183–186. (Russian)

[**273**] ____, *On the fundamental group of a manifold of negative curvature*, Dokl. Akad. Nauk SSSR **268** (1983), 777–781=Soviet Math. Dokl. **27** (1983), 140–144.

[**274**] ——, *Schottky groups in space*, Dokl. Akad. Nauk SSSR **280** (1985), 30–33=Soviet Math. Dokl. **31** (1985), 21–24.

[**275**] ——, *Uniformization of conformal structures on three-dimensional manifolds*, Abstracts, Internat. Conf. Complex Analy. and Appl., Varna, 1985, p. 80.

[**276**] Alan Howard and Andrew J. Sommese, *On the theorem of de Franchis*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **10** (1983), 429–436.

[**277**] Irwin Kra, *Deformations of Fuchsian groups*, Duke Math. J. **36** (1969), 537–546.

[**278**] ——, *On the Nielsen-Thurston-Bers type of some self-maps of Riemann surfaces*, Acta Math. **146** (1981), 231–270.

[**279**] ——, *On the vanishing of and spanning sets for Poincaré series for cusp forms*, Acta Math. **153** (1984), 47–116.

[**280**] Irwin Kra and Bernard Maskit, *Remarks on projective structures*, Riemann Surfaces and Related Topics: Proc. 1978 Stony Brook Conf. (I. Kra and B. Maskit, editors), Ann. of Math. Studies, vol. 97, Princeton Univ. Press, Princeton, N.J., and Univ. of Tokyo Press, Tokyo, 1981, pp. 343–360.

[**281**] S. L. Krushkal′, *Invariant metrics in Teichmüller spaces*, Sibirsk. Mat. Zh. **22** (1981), no. 2, 209–212. (Russian)

[**282**] ——, *Invariant metrics on spaces of closed Riemann surfaces*, Sibirsk. Mat. Zh. **26** (1985), no. 2, 108–114=Siberian Math. J. **26** (1985) (to appear).

[**283**] N. H. Kuiper, *Compact spaces with a local structure determined by the group of similarity transformations in E^n*, Nederl. Akad. Wetensch. Proc. **53** (1950), 1178–1185=Indag. Math. **12** (1950), 411–418.

[**284**] Folke Lannér, *On complexes with transitive groups of automorphisms*, Medd. Lunds Univ. Mat. Sem. **11** (1950).

[**285**] Wilhelm Magnus, *Noneuclidean tesselations and their groups*, Academic Press, 1974.

[**286**] A. I. Mal′tsev, *On the faithful representation of infinite groups by matrices*, Mat. Sb. **8(50)** (1940), 405–422; English transl., Amer. Math. Soc. Transl. (2) **45** (1965), 1–18.

[**287**] W. Mangler, *Die Klassen von topologischen Abbildungen einer geschlossenen Fläche auf sich*, Math. Z. **44** (1938/39), 541–554.

[**288**] G. A. Margulis, *Discrete groups of motions of manifolds of nonpositive curvature*, Proc. Internat. Congr. Math. (Vancouver, 1974), Vol. 2, Canad. Math. Congr., Montréal, 1975, pp. 21–34; English transl. in Amer. Math. Soc. Transl. (2) **109** (1977).

[**289**] ——, *The arithmeticity of irreducible lattices in semisimple groups of rank greater than* 1, supplement to the Russian transl. of M. S. Raghunathan, *Discrete subgroups of Lie groups*, "Mir", Moscow, 1977, pp. 277–313. (Russian)

[290] A. D. Mednykh, *On the Hurwitz problem on the number of nonequivalent coverings over a compact Riemann surface*, Sibirsk. Mat. Zh. **23** (1982), no. 3, 155–160=Siberian Math. J. **23** (1982), 415–420.

[291] ——, *On the solution of the Hurwitz problem on the number of nonequivalent coverings over a compact Riemann surface*, Dokl. Akad. Nauk SSSR **261** (1981), 537–542=Soviet Math. Dokl. **24** (1981), 541–545.

[292] ——, *Nonequivalent coverings of Riemann surfaces with a given type of branching*, Sibirsk. Mat. Zh. **25** (1984), no. 4, 120–142=Siberian Math. J. **25** (1984), 606–625.

[293] ——, *On the number of subgroups of the fundamental group of a closed nonnegative surface*, Seventeenth All-Union Algebra Conf., Abstracts of Reports, Part 1, Inst. Mat. Akad. Nauk Beloruss. SSR, Minsk, 1983, p. 125. (Russian)

[294] Jakob Nielsen, *Untersuchungen zur Topologie der geschlossenen zweiseitigen Flächen*. I, II, Acta Math. **50** (1927), 189–358; **53** (1929), 1–76.

[295] V. V. Nikulin, *On arithmetic groups generated by reflections in Lobachevsky spaces*, Izv. Akad. Nauk SSSR Ser. Mat. **44** (1980), 637–669=Math. USSR Izv. **16** (1981), 573–601.

[296] ——, *On the classification of arithmetic groups generated by reflections in Lobachevsky spaces*, Izv. Akad. Nauk SSSR Ser. Mat. **45** (1981), 113–142=Math. USSR Izv. **18** (1982), 99–123.

[297] David B. Patterson, *The fundamental group of the modulus space*, Michigan Math. J. **26** (1979), 213–223.

[298] Atle Selberg, *Recent developments in the theory of discontinuous groups of motions of symmetric spaces*, Proc. Fifteenth Scandinavian Congr. (Oslo, 1968), Lecture Notes in Math., vol. 118, Springer-Verlag, 1970, pp. 99–120.

[299] Dennis Sullivan, *Growth of positive harmonic functions and Kleinian group limit sets of zero planar measure and Hausdorff dimension two*, Geometry Sympos. (Utrecht, 1980), Lecture Notes in Math., vol. 894, Springer-Verlag, 1981, pp. 127–144.

[300] ——, *Discrete conformal groups and measurable dynamics*, Bull. (New Ser.) Amer. Math. Soc. **6** (1982), 57–74.

[301] ——, *Quasiconformal homeomorphisms and dynamics*. II: *Structural stability implies hyperbolicity for Kleinian groups*, preprint, Inst. Hautes Études Sci., Bures-sur-Yvette, 1982.

[302] William P. Thurston, *On the geometry and dynamics of diffeomorphisms of surfaces*, preprint.

[303] ——, *Three dimensional manifolds, Kleinian groups and hyperbolic geometry*, Bull. (New Ser.) Amer. Math. Soc. **6** (1982), 357–381.

[304] Pukka Tukia, *The Hausdorff dimension of the limit set of a geometrically finite Kleinian group*, Acta Math. **152** (1984), 127–140.

[**305**] ____, *Quasiconformal extension of quasisymmetric mappings compatible with a Möbius group*, Acta Math. **154** (1985), 153–193.

[**306**] È. B. Vinberg, *On groups of units of certain quadratic forms*, Mat. Sb. **87(129)** (1972), 18–36=Math. USSR Sb. **16** (1972), 17–35.

[**307**] ____, *On unimodular integral quadratic forms*, Funktsional. Anal. i Prilozhen. **6** (1972), no. 2, 24–31=Functional Anal. Appl. **6** (1972), 105–111.

[**308**] ____, *The absence of crystallographic groups of reflections in Lobachevsky spaces of large dimension*, Funktsional. Anal. i Prilozhen. **15** (1981), no. 2, 67–68=Functional Anal. Appl. **15** (1981), 128–130.

[**309**] ____, *The absence of crystallographic groups of reflections in Lobachevsky spaces of large dimension*, Trudy Moskov. Mat. Obshch. **47** (1984), 68–102=Trans. Moscow Math. Soc. **1985**, no. 1(47), 75–112.

[**310**] È. B. Vinberg and I. M. Kaplinskaya, *On the groups* $O_{18,1}(\mathbf{Z})$ *and* $O_{19,1}(\mathbf{Z})$, Dokl. Akad. Nauk SSSR **238** (1978), 1273–1275 Soviet Math. Dokl. **19** (1978), 194–197.

This page intentionally left blank

Subject Index

This page intentionally left blank

ISBN 978-0-8218-4516-5

MMONO/62