

TRANSLATIONS OF
**MATHEMATICAL
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VOLUME 64

**M. M. Lavrent'ev
V. G. Romanov
S. P. Shishat'skiĭ**

Ill-Posed Problems of Mathematical Physics and Analysis



American Mathematical Society

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American Mathematical Society
Providence, Rhode Island

НЕКОРРЕКТНЫЕ ЗАДАЧИ МАТЕМАТИЧЕСКОЙ ФИЗИКИ И АНАЛИЗА

М. М. ЛАВРЕНТЬЕВ

В. Г. РОМАНОВ

С. П. ШИШАТСКИЙ

АКАДЕМИЯ НАУК СССР
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ABSTRACT. A number of applied problems connected with the interpretation of geophysical data and leading to mathematical problems which are ill-posed in the sense of Hadamard are considered. The exposition includes the basic concepts of ill-posed problems, problems of analytic continuation from continua and discrete sets, and analogous problems of continuation of solutions of elliptic and parabolic equations, the main ill-posed boundary value problems for partial differential equations, and results on the theory of Volterra equations of the first kind, in particular, on the theory of operator Volterra equations. A very broad presentation is given of modern results on the problem of uniqueness in integral geometry and on inverse problems for partial differential equations.

The monograph is of interest to specialists in applied mathematics, physics, and geophysics.

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Preface

The theory of ill-posed problems is a direction of mathematics which has developed intensively in the last two decades and is connected with the most varied applied problems: interpretation of readings of many physical instruments and of geophysical, geological, and astronomical observations, optimization of control, management and planning, synthesis of automatic systems, etc. Development of the theory of ill-posed problems was occasioned by the advent of modern computing technology.

Various areas of the theory of ill-posed problems can be included in traditional areas of mathematics such as function theory, functional analysis, differential equations, and linear algebra.

The concept of a well-posed problem is connected with investigations by the famous French mathematician Hadamard of various boundary value problems for the equations of mathematical physics. Hadamard expressed the opinion that boundary value problems whose solutions do not satisfy certain continuity conditions are not physically meaningful, and he presented examples of such problems.

It was subsequently found that Hadamard's opinion was erroneous. It turned out that many problems of mathematical physics which are ill-posed in the sense of Hadamard and, in particular, problems noted by Hadamard himself have real physical content. It also turned out that ill-posed problems arise in many other areas of mathematics which are connected with applications. Such a classical problem of mathematical analysis as the problem of differentiation is ill-posed if it is connected with processing experimental data.

In our country several monographs have been published which are devoted to the theory of ill-posed problems and their applications (see [113], [145], [152], [164], [184] and [266]). The present state of the theory of ill-posed problems is reflected most completely in the monograph of A. N. Tikhonov and V. Ya. Arsenin [266] and in the recently published monograph of V. K. Ivanov, V. V. Vasin and V. P. Tanana [113]. However, a number of important

areas of the theory of ill-posed problems are absent or little reflected in the monograph literature noted. This refers, for example, to ill-posed problems for concrete types of differential equations, problems of analytic continuation, inverse problems for differential equations, and problems of integral geometry.

In the present monograph the authors have attempted to fill these gaps in the monograph literature on the theory of ill-posed problems, and also to give a new treatment of some of its areas.

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