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Theory of Limit Cycles

Yan-Qian Ye

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Translations of

MATHEMATICAL MONOGRAPHS

Volume 66

Theory of Limit Cycles

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SUMMARY OF CONTENTS. This book belongs to the "Modern Mathematics Series." Its first edition was published in 1965. Now this is the second edition, with many substantial changes in content. It contains tremendous results obtained during the past twenty years in China, and also introduces some great achievements from abroad.

This book has eighteen sections, which can be divided into three parts. Part I (§§1–8) discusses limit cycles of general plane stationary systems, including their existence, nonexistence, stability, and uniqueness. Part II (§§9–17) discusses the global topological structure of limit cycles and phase-portraits of quadratic systems. Part III (§18) collects some important results which either could not be included under the subject matter of the previous sections or appeared in the literature very recently. This book serves as a reference book for college seniors, graduate students, and researchers in mathematics and physics.

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Contents

Preface to the Second Edition

It has been eighteen years since the publication of the first edition of this book in 1964. Over these eighteen years, there has been tremendous progress in the theory of limit cycles, especially in quadratic differential systems, in China as well as in other countries, and many new results have been obtained. Some work in this area which was considered correct in the past has turned out to be wrong; some work which was considered important in the past now seems to be not worthy of further development. It is worthwhile to mention that, owing to research developments from other fields such as biology and chemistry, research on limit cycles, especially in the theory of polynomial systems, has become more and more important. In China, there are at least ten mathematicians who have produced good work in this area over the past twenty-some years. In other countries, besides the Russian mathematicians who have had a traditional interest and made solid contributions in this area, more people in the United States and France have shown strong interest in the study of limit cycles of quadratic systems. For these reasons, the author felt that this book should be completely revised and its second edition should be published without delay.

Not having enough time to work on this project, and hoping to collect new ideas and valuable comments from others for the better presentation of the second edition, I have omitted §7, §9 and the latter part of §10 from the first edition, and concentrated on the overall rearrangement of material, final editing of the manuscripts and unification of the use of mathematical symbols in the new edition; except for the sections dealing with my own work, I entrusted the task of revision and supplementation to some of my colleagues, as follows: Wang Er-nian supplemented part of §3 and part of §6; Ma Zhi-en supplemented §3; Huang Ke-cheng supplemented §5 and wrote the new §7; Luo Ding-jun rewrote §8 and supplemented the first half of §9; Cai Sui-lin rewrote §10; Wang Ming-shu supplemented §11 and wrote the new

§15; Yang Xin-an supplemented §12 and wrote the new §17; and Chen Lan-sun supplemented the latter two-thirds of §14 and wrote the new §16.

In my final editorial work I have made very few changes in some of their manuscripts while in other cases I have made considerable changes, but they all applied their expertise in this field and spent their time and energy to make the second edition reflect more completely the current development of the theory of limit cycles. It is obvious that I could not have taken on such a task all by myself. I therefore express my heartfelt gratitude to the above eight colleagues.

Due to our limited time and ability, it is natural that there may still be some mistakes or important omissions in the new edition. We shall be grateful if our readers would kindly let us know.

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August, 1982.

Abstract

Limit cycles of plane autonomous differential systems appear in the very famous classical paper "Mémoire sur les courbes définies par une équation différentielle" of H. Poincaré (1881–1886). In the 1930s, van der Pol and A. A. Andronov showed that the closed orbit in the phase plane of a self-sustained oscillation occurring in a vacuum tube circuit is a limit cycle as considered by Poincaré. After this observation, the existence, nonexistence, uniqueness and other properties of limit cycles have been studied extensively by mathematicians and physicists. Then, from the 1950s, very many mathematical models from physics, engineering, chemistry, biology, economics, etc., were displayed as plane autonomous systems with limit cycles. Also, due to the well-known paper of I. G. Petrovskiĭ and E. M. Landis concerning the maximum number of limit cycles of all quadratic differential systems (the second part of Hilbert's 16th problem), the problem of limit cycles has become more and more important and has attracted the attention of many pure and applied mathematicians.

The purpose of this book is to bring together in one place most of the main contributions in the theory of limit cycles. Aside from the introduction (a brief historical review), it divides into three parts. §§1–8 are concerned with limit cycles of general plane autonomous systems, and §§9–17 with limit cycles and the global topological structure of phase-portraits of quadratic systems. At the end of every section, a large number of reference papers are listed. The last part, §18, has the character of an appendix, in which we mention briefly results that either could not be included in the subject matter of the foregoing sections, or have appeared in periodicals very recently. We assume that the readers have a basic knowledge of the qualitative and stability theory of ODE.

Here are the main contents of each section in the first two parts. §1 gives the fundamental concepts and examples of limit cycles, and also some criteria for the existence and nonexistence of limit cycles, including well-known ones

as well as some new ones. §2 gives criteria for the determination of the stability and multiplicity of limit cycles. Aside from the classical ones, we introduce also results of V. F. Tkachev and M. Urabe. §3 deals with the theory of rotated vector fields due to G. F. D. Duff, and also many of its extensions and applications by X. Y. Chen and Z. E. Ma. We will use this theory very often in the second part. §4 is concerned with the variation of limit cycles with the varying of a parameter in the general case; the main contents are the classical formula of H. Poincaré and contributions of M. Urabe and X. Y. Chen. §5 discusses the question of the existence of limit cycles. Aside from the well-known theorems of A. F. Filippov and A. V. Dragilëv, we present here also contributions of K. Z. Hwang, Z. J. Wu, and X. W. Zheng. This section is divided into six subsections according to the methods of proof. §6 is concerned with the problem of uniqueness of limit cycles. It also divides into seven subsections, in which we introduce methods of point-transformation due to H. Poincaré, A. A. Andronov, and E. A. Leontovich, and also results of G. Sansone, J. L. Massera, Z. F. Zhang, L. A. Cherkas, and G. S. Rychkov. §7 deals with the problem of the existence of any given number of limit cycles. The main results almost all belong to Chinese mathematicians, among which the contribution of Z. F. Zhang is preeminent, solving completely the problem of the number and position of limit cycles of the equation $\ddot{x} + \mu \sin \dot{x} + x = 0$. §8 is a short introduction to the well-known necessary and sufficient conditions for the structural stability of a plane autonomous system in a bounded domain, which we will use in the second part. It contains also some new results of G. T. dos Santos and D. J. Luo about polynomial systems. §9 deals with classical results of H. Dulac and M. Frommer on necessary and sufficient conditions for a quadratic differential system to have a center, and presents the corresponding phase-portraits due to Frommer. In this section we also give a detailed proof of an important result of N. N. Bautin concerning the maximum order of fineness of a focus of any quadratic system and the maximum number of limit cycles that can be generated from this focus. In §10, we analyze the global topological structure of phase-portraits of three types of quadratic systems without limit cycle, namely, the homogeneous systems (results of L. S. Lyagina, L. Markus, et al.), the system $\dot{x} = x + \text{h.o.t.}$, $\dot{y} = y + \text{h.o.t.}$ (result of A. N. Berlenskiĭ) and, finally, the structurally stable quadratic systems without limit cycle (results of G. T. dos Santos and S. L. Zai). §11 deals with general properties and possible relative positions of limit cycles of quadratic systems, among which results of Y. Q. Ye, C. C. Tung, Y. S. Chin, M. S. Wang, and S. L. Shi are presented. §12 introduces the classification of quadratic systems due to Y. Q. Ye, and proves a theorem on the existence, nonexistence and uniqueness of limit cycles of systems of type I, due to Y. Q. Ye, Y. H.

Deng, D. J. Luo, L. S. Chen, and X. A. Yang. §13 investigates the global topological structure of phase-portraits of a special system of type II, which contains two parameters a and m and has no limit cycle. We obtain global bifurcation curves in the (a, m) projective plane. §14 is concerned with the relative position (especially coexistence), uniqueness and the number of limit cycles of systems of type II containing only two second-degree terms in the first equation. These results are due mainly to M. S. Wang, K. T. Lee, S. X. Yu, N. D. Zhu, K. C. Chen, L. I. Zhilevich, and L. A. Cherkas. §15 discusses various interesting global properties of systems of type III; especially, we give the detailed proof of a theorem (concerning system $\mathrm{III}_{a=0}$) similar to that in §12, which was conjectured and partly proved by N. D. Zhu, and later completely proved by L. A. Cherkas, L. I. Zhilevich, and G. S. Rychkov. §16 discusses the Dulac function method used frequently by Chinese mathematicians in their research work on the qualitative investigation of quadratic systems, and uses this method to prove an interesting theorem of L. S. Chen and M. S. Wang, concerning the nesting of limit cycles surrounding just one focus. §17 introduces X. A. Yang's results on the uniqueness or nonexistence of limit cycles of bounded quadratic systems. These systems were first studied by R. J. Dickson and L. M. Perko, but the limit cycle problem remained open in their papers.

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$$dx/dt = \sum_{0\le i+k\le z} a_{ik}x^i y^k,\quad dy/dt = \sum_{0\le i+k\le z} b_{ik}x^i y^k,$$

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$$\frac{dy}{dx} = \frac{q_{00} + q_{10}x + q_{01}y + q_{20}x^2 + q_{11}xy + q_{02}y_2}{p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2}.$$

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