The Linearization Method in Hydrodynamical Stability Theory

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МЕТОД ЛИНЕАРИЗАЦИИ В ГИДРОДИНАМИЧЕСКОЙ ТЕОРИИ УСТОЙЧИВОСТИ

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ABSTRACT. The theory of the linearization method in the problem of stability of steady-state and periodic motions of continuous media is presented, and infinite-dimensional analogues of Lyapunov's theorems on stability, instability and conditional stability are proved for a large class of continuous media. Semigroup properties for the linearized Navier-Stokes equations in the case of an incompressible fluid are studied, and coercivity inequalities and completeness of a system of small oscillations are proved.

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