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МИНИМАЛЬНЫЕ ПОВЕРХНОСТИ И ПРОБЛЕМА ПЛАТО

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ABSTRACT. The book is an account of the current state of the theory of minimal surfaces and of one of the most important chapters of this theory—the Plateau problem, i.e. the problem of the existence of a minimal surface with boundary prescribed in advance. The authors exhibit deep connections of minimal surface theory with differential equations, Lie groups and Lie algebras, topology, and multidimensional variational calculus. The presentation is simplified to a large extent; the book is furnished with a wealth of illustrative material. Bibliography: 471 titles. 153 figures, 2 tables.

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Preface

Plateau's problem is a scientific trend in modern mathematics that unites several different problems connected with the study of *minimal surfaces*, that is, surfaces of least area or volume. In the simplest version, we are concerned with the following problem: how to find a surface of least area that spans a given fixed wire contour in three-dimensional space. A physical model of such surfaces consists of soap films hanging on wire contours after dipping them in a soap solution. From the mathematical point of view, such films are described as solutions of a second-order partial differential equation, so their behavior is quite complicated and has still not finally been studied. Soap films or, more generally, *interfaces between physical media* in equilibrium, arise in many applied problems, in chemistry, physics, and also in nature. A well-known example is that of marine organisms, Radiolaria, whose skeletons enable us to see clearly the characteristic singularities of interfaces between media and soap films that span complicated boundary contours. In applications there arise not only two-dimensional but also multidimensional minimal surfaces that span fixed closed "contours" in some multidimensional Riemannian space (manifold). It is convenient to regard such surfaces as extremals of the functional of multidimensional volume, which enables us when studying them to employ powerful methods of modern analysis and topology. We should mention that an exact mathematical statement of the problem of finding a surface of least area (volume) requires a suitable definition of such fundamental concepts as a surface, its boundary, minimality of a surface, and so on. It turns out that there are several natural definitions of all these concepts, which enable us to study minimal surfaces by different methods, which complement one another.

In the framework of a comparatively small book it is practically impossible to cover all aspects of the modern problem of Plateau, to which a vast literature has been devoted. The authors have therefore tried to construct the book in accordance with the following principle: a maximum of clarity and a minimum of formalization. Of course, it is possible to satisfy such a requirement only approximately, so in certain cases (this applies mainly to the last chapters of the book) we dwell on certain nontrivial mathematical constructions that are necessary for a concrete investigation of minimal surfaces.

The book can be conventionally split into three parts: (a) Chapter 1, which contains historical information about Plateau's problem, referring to the period preceding the 1930s, and a description of its connections with the natural sciences; (b) Chapters 2–5, which give a fairly complete survey of various modern trends in Plateau's problem; (c) Chapters 6–11, in which we give a detailed exposition of one of these trends (the homotopic version of Plateau's problem in terms of stratified multivarifolds) and the Plateau problem in homogeneous symplectic spaces (Chapter 11).

The first part is intended for a very wide circle of readers and is accessible, for example, to first-year students. The second part, accessible to second- and third-year students specializing in physics and mathematics, relies mainly on information from a standard course in geometry and topology. Here we use the elements of Riemannian geometry, differential forms, homology, and the elements of complex analysis. We recall the main concepts but without going into details. The third part is intended for specialists interested in the modern theory of minimal surfaces and can be used for special courses. Here we assume that the reader has a command of the concepts of functional analysis.

At the beginning of the book we give a brief historical survey of the sources of the modern problem of Plateau. We start our account with earlier work of the 18th century, and then we dwell in more detail on the work of the 19th century, in which the fundamental properties of minimal surfaces were discovered. We pay special attention to the famous physical experiments of Plateau (1801–1883), in which he systematized various observations about the behavior of the interface between two media. One of the results of this series of experiments was a precise formulation of the so-called *principles of Plateau*, which control both the local and the global topological behavior of interfaces between media. Together with an exposition of the mathematical and physical aspects of Plateau's problem we give information about mathematicians whose work was most closely connected with the questions under consideration. We also try to characterize the concrete historical situation that brings to life various mathematical, mechanical, and physical aspects of Plateau's problem.

In Chapter 1 we present the elements of the classical theory of minimal surfaces, mainly for the two-dimensional case. Here we try to avoid complicated calculations, referring the interested reader to the more specialized literature, of which quite a large (though not complete) list is given at the end of the book.

Chapter 2 enables the reader to become acquainted quickly with those topological concepts without which the modern theory of minimal surfaces is inconceivable.

Beginning with Chapter 3 we introduce the reader directly to the very rich world of modern ideas about minimal surfaces and their role in mechanics, physics, and mathematics. Our aim is to put into the hands of the reader a guide that will enable him to orient himself quickly in the diverse information concentrated at the forefront of modern research. We shall pay a great deal of attention to the *methods* of studying minimal surfaces and to the main results obtained by means of them.

In particular, we present the solution due to A. T. Fomenko of Plateau's spectral problem in the class of spectra of manifolds with a fixed boundary, and also the solution due to Dào Trọng Thi of Plateau's problem in the homotopy class of multivarifolds with a given boundary.

The idea of creating a book of this kind and the plan of it is due to A. T. Fomenko. Chapters 1–5 were written by A. T. Fomenko, and Chapters 6–10 by Dào Trong Thi; Chapter 11 is based on the recent results of Le Hong Van.

The structure of the book was formed as a result of A. T. Fomenko giving special courses on the theory of minimal surfaces in the Mechanics–Mathematics Department at Moscow State University. Also, the book was created under the influence of a programme of study of the connections between the topology of manifolds and the global properties of minimal surfaces developed in the research seminars "Modern Geometric Methods" and "Computer Geometry" under the supervision of A. T. Fomenko at Moscow State University. The most important results of some participants of the seminar are reflected in the book.

The book is intended for a wide circle of students, research students and mathematicians specializing in calculus of variations, topology, functional analysis, the theory of differential equations, and Lie groups and Lie algebras.

The authors thank S. P. Novikov, whose valuable support and interest stimulated the development of this scientific trend, and also the reviewer D. V. Anosov, who made a number of useful remarks and additions. The authors thank V. P. Maslov for his support.

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APPENDIX

Volumes of Closed Minimal Surfaces and the Connection with the Tensor Curvature of the Ambient Riemannian Space

The topology of a Riemannian manifold is closely connected with its Riemannian curvature. It is intuitively obvious that topologically nontrivial cycles (realized by minimal surfaces) tend to take up a position in the ambient Riemannian manifold corresponding in a certain sense to the "maximal curvature" of this manifold. We now present new results obtained by Le Hong Van and A. T. Fomenko in the course of developing Fomenko's method presented above in Chapter 4, §4.

Let $B_r(x)$ be a ball of radius r in the tangent space T_xM. The injective radius of the manifold M^n at the point x is the quantity $R(x) = \sup\{r$, where $\exp\colon B_r(x) \to M$ is a diffeomorphism$\}$. Let $R(M) = \inf R(x)$, where $x \in M$. We call $R(M)$ the injective radius of the manifold. Let $\eta_k(v, x)$ be the k-dimensional deformation coefficient of the vector field v at the point x, introduced by Fomenko (see Chapter 4, §4). Next, we recall that the quantity

$$\Omega_k^0(M) = \inf \Omega_k^0(x_0), \qquad x_0 \in M,$$

where

$$\Omega_k^0(x_0) = \gamma_k q_{x_0}(R(x_0)), \qquad q_{x_0}(r) = \exp \int_0^r \left(\max_{x \in \{f=t\}} \eta_k(v, x) \right)^{-1} dt,$$

$\gamma_k = \operatorname{vol}_k B_1^k$ is the volume of a k-dimensional unit ball, is called the geodesic deficiency of M^n if $f(x)$ is the function of distance from x to x_0 on the manifold, and $v = \operatorname{grad} f$. For the details see Chapter 4, §4. We also recall that the volume of a k-dimensional nontrivial cycle in M is not less than the number $\Omega_k^0(M)$ (see Chapter 4, Theorems 4.4.1 and 4.4.2).

Suppose that the sectional curvature of M in any two-dimensional direction does not exceed a^2, where a is real or purely imaginary. Let the injective radius of M be equal to R.

THEOREM 1 (a lower bound of the geodesic deficiency of a manifold; Le Hong Van and A. T. Fomenko).

(1) *If* $a^2 > 0$ *and* $Ra \leq \pi$, *then* $\Omega_k^0(M) \geq k\gamma_k a^{-k} \int_0^{Ra} \sin^{k-1} t\,dt$.

(2) *If* $a^2 > 0$ *and* $Ra > \pi$, *then* $\Omega_k^0(M) \geq \mathrm{vol}_k S_{1/a}^k$, *where* $S_{1/a}^k$ *is a Euclidean sphere of dimension* k *and radius* $1/a$.

(3) *If* $a = 0$, *then* $\Omega_k^0(M) \geq \gamma_k R^k$.

(4) *If* $a^2 < 0$, *then* $\Omega_k^0(M) \geq k\gamma_k |a|^{-k} \int_0^{R|a|} \sinh^{k-1} t\,dt$.

THEOREM 2 (isoperimetric inequalities; Le Hong Van and A. T. Fomenko). *Let* X^k *be a globally minimal surface passing through a point* $x \in M$, *let* $B_x(r)$ *be the geodesic ball of radius* r *and center* x *in* M, *and let* A_r^{k-1} *be the boundary of intersection of the surface with the ball* $X^k \cap B_x(r) = X_r^k$.

(1) *If* $a^2 > 0$ *and* $r \leq \min(R, \pi/a)$, *then we have* $\mathrm{vol}_{k-1} A_r \geq k\gamma_k a^{1-k} (\sin ar)^{k-1}$. *Consequently,*

$$(\mathrm{vol}_{k-1} A_r)/(\mathrm{vol}_k X_r) \geq (\sin ar)^{k-1} \Big/ \left(\int_0^r (\sin at)^{k-1}\,dt \right).$$

(2) *If* $a = 0$ *and* $r \leq R$, *then* $\mathrm{vol}_{k-1} A_r \geq k\gamma_k r^{k-1}$ (*which is equal to the volume of the standard* $(k-1)$*-dimensional sphere of radius* r). *Consequently,*

$$\mathrm{vol}\, X_r \leq kr\, \mathrm{vol}\, A_r \quad \textit{and} \quad \mathrm{vol}\, X_r \leq (\mathrm{vol}\, A_r)^{k/(k-1)} k^{-1} (k\gamma_k)^{1/(1-k)}.$$

(3) *If* $a^2 < 0$ *and* $r \leq R$, *then* $\mathrm{vol}_{k-1} A_r \geq k\gamma_k |a|^{1-k} (\sinh |a| r)^{k-1}$. *Consequently,*

$$(\mathrm{vol}\, A_r)/(\mathrm{vol}\, X_r) \geq (\sinh |a| r)^{k-1} \Big/ \left(\int_0^r (\sinh |a| t)^{k-1}\,dt \right).$$

In the general case, the estimates in these theorems are the best possible, that is, in many cases equalities are attained. The theorems confirm the conjecture that in a certain sense absolutely minimal surfaces tend to take up a position corresponding to "maximal curvature" in the ambient manifold. Nevertheless, it follows that the curvature of globally minimal surfaces "senses" the curvature of the ambient manifold.

COROLLARY 1. *Suppose that a complete noncompact Riemannian manifold* M *has nonpositive curvature, and that* X^k *is a globally minimal surface in* M. *Then the function* $V(r) = \mathrm{Vol}_k B_X(r)$, *where* $B_X(r)$ *is a geodesic ball of radius* r *in* X^k, *increases no more slowly than a polynomial of degree* k *in* r. *If* M *has negative curvature bounded below, then*

$V(r)$ increases no more slowly than the exponential function of r. In particular, it follows that at each point $x \in X^k$ there is a $v \in T_x X$ such that $R_X(v, v) < 0$, where R_X is the Ricci tensor on X at the point x.

COROLLARY 2. *If M is a compact simply-connected symmetric space with sectional curvature everywhere not greater than a, then the volume of any nontrivial k-dimensional cycle (that is, corresponding to a closed minimal surface) is not less than the volume of a k-dimensional standard sphere of curvature a.*

COROLLARY 3. *The length of a homologically nontrivial loop in a manifold M is not less than twice the injective radius of the manifold.*

COROLLARY 4 (lower bound of the volume of a manifold).
(1) *If $a^2 > 0$, then $\operatorname{vol}_n M^n \geq n\gamma_n a^{1-n} \int_0^R (\sin at)^{n-1}\, dt$.*
(2) *If $a = 0$, then $\operatorname{vol}_n M^n \geq \gamma_n R^n$.*
(3) *If $a^2 < 0$, then $\operatorname{vol}_n M^n \geq n\gamma_n |a|^{1-n} \int_0^R (\sinh |a|t)^{n-1}\, dt$.*

These estimates of the volume coincide with those that can be obtained on the basis of the Rauch-Bishop comparison theorem. Let us consider the case of symmetric spaces.

THEOREM 3 (Le Hong Van and A. T. Fomenko). *Let B be a Cartan subspace of the tangent space $T_e M$, where M is a symmetric space, $x = \exp v$, $v \in B$. We denote by α_i the root system of M with respect to B. Suppose that the roots are ordered as follows: $\alpha_1^2(v) \geq \alpha_2^2(v) \geq \cdots > \alpha_p^2(v) = 0 = \alpha_{p+1}(v) = \cdots$. Then*
(1) *for $k < p$ we have*

$$\eta_k(x) = \frac{\int_0^1 (\sin \alpha_1(v)t \cdots \sin \alpha_{k-1}(v)t)\, dt}{\sin \alpha_1(v) \cdots \sin \alpha_{k-1}(v)};$$

(2) *for $k \geq p$ we have*

$$\eta_k(x) = \frac{\int_0^1 (\sin \alpha_1(v)t \cdots \sin \alpha_{k-1}(v)t)(|v|t)^{k-p}\, dt}{(\sin \alpha_1(v) \cdots \sin \alpha_{k-1}(v))|v|^{k-p}}.$$

Thus, for example, for symmetric spaces of rank 1 we can calculate the functions $\Omega_k^0(M)$ explicitly, and hence the volumes of these spaces and the volumes of their minimal cycles.

It is well known [257], [258] that in a simply-connected irreducible compact symmetric space M there is a totally geodesic sphere of curvature a^2, where a^2 is the least upper bound of two-dimensional curvature on M. Moreover, each such sphere lies in some totally geodesic Helgason sphere of maximal dimension $i(M)$. All the Helgason spheres go into one another under the action of the group $\mathrm{Iso}(M)$, and they also have curvature a^2. Obviously, the following assertion follows immediately from Corollary 2.

PROPOSITION. *If the Helgason sphere $S(M)$ realizes a nontrivial cycle in the homology group $H_*(M, \mathbf{R})$, then it is a globally minimal surface in M.*

Below we give a list of Helgason spheres that realize nontrivial cycles in (compact) irreducible symmetric spaces.

(1) M is a simple compact group, $\dim S(M) = 3$,
(2) $M = SU_{l+m}/SU_l \times SU_m$, $\dim S(M) = 2$,
(3) $M = SO_{l+2}/SO_l \times SO_2$, $\dim S(M) = 2$,
(4) $M = SU_{2n}/Sp_n$, $\dim S(M) = 5$,
(5) $M = Sp_{m+n}/Sp_m \times Sp_n$, $\dim S(M) = 4$,
(6) $M = SO_{2n}/U_n$, $\dim S(M) = 2$,
(7) $M = F_4/\mathrm{Spin}\,9$, $\dim S(M) = 8$,
(8) $M = \mathrm{Ad}\,E_6/T^1\,\mathrm{Spin}\,10$, $\dim S(M) = 2$,
(9) $M = \mathrm{Ad}\,E_7/T^1 E_6$, $\dim S(M) = 2$.

The explicit embedding $S(M) \to M$ is described in [257].

In conclusion we give one more corollary of Theorem 1 for noncompact symmetric spaces. Clearly [258], the upper sectional curvature of such spaces is equal to zero.

COROLLARY 5. *Let N be a flat totally geodesic subspace of a noncompact symmetric space M. Then N is a globally minimal surface.*

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