

Translations of

MATHEMATICAL MONOGRAPHS

Volume 87

Ideals of Identities of Associative Algebras

Aleksandr Robertovich Kemer



American Mathematical Society



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of Associative Algebras

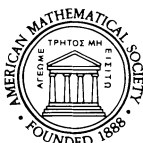
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Providence, Rhode Island

АЛЕКСАНДР РОБЕРТОВИЧ КЕМЕР

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Contents

Introduction	1
CHAPTER I. Varieties and Superalgebras	7
§1. Technical statements, utilizing the theory of representations of the symmetric group	7
§2. Grassmann hulls of superalgebras	17
§3. Semiprime varieties. Generalization of the Dubnov- Ivanov-Nagata-Higman theorem	21
CHAPTER II. Identities of Finitely-Generated Algebras	27
§1. Numerical characteristic of T_2 -ideals	28
§2. A theorem on the decomposition of T_2 -ideals	31
§3. Trace identities	45
§4. Graded identities of finitely-generated superalgebras	55
§5. Solution of Specht's problem	66
§6. On asymptotic bases of identities	68
Bibliography	75
Subject Index	81

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Bibliography

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Subject Index

- Algebra, 27
 - basis of identities, 28, 66
 - f.c.s., 28, 45, 62
 - finitely-generated, 27
- Cayley-Hamilton identity, 46, 56
- Coherent set, 57
- Decomposition, 31, 38
 - theorem, 44
 - of T_2 -ideals, 31
- Dubnov-Ivanov-Nagata-Higman theorem, 24
- Grassmann, 1, 7
 - algebra, 1, 7
 - hull, 7, 17, 64
- H -polynomial, 4, 24
- Ideal, 8
 - S , 8
 - S_2 , 11, 19
 - T -equivalent, 11
 - T , 8, 11, 19, 69
 - T_2 , 11, 19, 28
 - of generalized identities, 69
 - of graded identities, 11, 18
 - of graded identities with trace, 46
 - of identities, 8
- Identity, 68
 - unitary, 68
- Mixed element, 40, 57
- PI-algebra, 1
 - relatively free, 65
- PI-superalgebra, 4, 62, 65
 - basis of graded identities, 67
- Polynomial, 68
 - asymptotic bases of identities, 68
 - generalized, 68
 - unitary, 68
- Radical, 21
 - homogeneity, 21
- Relative complexity, 31
- Relative type, 31
- Relatively free algebra, 20
- Representations of $S(n)$, 9
- Shirshov's theorem, 2
- Specht's problem, 2, 27
 - solution of, 66
- Subspace, 11
 - graded, 11
- Superalgebra, 7
 - finite-dimensional classical, 28
 - finite-dimensional, 62
 - finitely-generated, 55
 - free with trace, 46
 - free, 11
 - semisimple, 21
 - simple, 21
- Superscript, 41
 - distinguished, 41, 58
- Surrounding element, 57
- Trace identities, 45
- Type, 31
 - of T_2 -ideal, 31
 - of f.c.s. algebra, 31
- Variety, 11
 - of associative algebras, 19
 - of superalgebras, 62
 - prime, 21
 - semiprime, 21
- \mathfrak{W} -product, 22

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