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Providence, Rhode Island

Ю. С. ИЛЬЯШЕНКО

ТЕОРЕМЫ КОНЕЧНОСТИ ДЛЯ ПРЕДЕЛЬНЫХ ЦИКЛОВ

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To my family:
Lena
Serezha
Lizochka
Aleksandr

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