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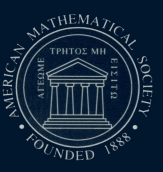
Volume 106

Conformal Mappings and Boundary Value Problems

Guo-Chun Wen



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Conformal Mappings and Boundary Value Problems

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Guo-Chun Wen

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共形映射与边值问题

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Contents

Preface

Conformal mapping and boundary value problems are two major branches of complex function theory. The former is the geometric theory of analytic functions and the latter is the analytic theory governing the close relationship between the abstract theory and many concrete problems. Moreover, there is also an intimate relationship between conformal mappings and boundary value problems.

The first three chapters of this book deal with the existence and uniqueness of conformal mappings from multiply connected domains to various canonical domains, as well as certain properties of univalent functions and explicit representation formulas of conformal mappings. The methods used to arrive at these results are diverse. For example, to get the existence of conformal mappings from multiply connected domains to canonical domains, we use the method of locating extrema, the continuity method, and the modified Dirichlet problem for analytic functions. To get the explicit representation formulas of the conformal mappings from polygons bounded by straight lines or circular arcs to the upper half-plane or unit disc, we use the method of analytic continuation. In addition, the use of the convergence properties of sequences of analytic functions is among the main ingredients in the proof of many theorems concerning conformal mappings.

The remaining three chapters give a fairly detailed introductory account of the basic boundary value problems for analytic functions on multiply connected domains (the Riemannian boundary value problem, the Hilbert boundary value problem, etc.), as well as those for harmonic functions (the first, second, and third mixed boundary value problems and the irregular oblique derivative boundary value problem). Here, we have not used the method of integral equations which is common in the study of these problems. Rather, we first use subharmonic functions of conformal mappings to prove the existence of solutions for the harmonic function Dirichlet boundary value problem on multiply connected domains. Then, on the basis of a priori estimates for the solutions of these boundary value problems, we use the continuity method and related methods to prove the solvability of the aforementioned analytic function Hilbert boundary value problem. We

also give the integral representation formulas for the solutions of these problems. As for the Riemann boundary value problem for analytic functions, we have followed the usual method of using integrals of the Cauchy type. The existence of solutions for certain boundary value problems for harmonic functions is based on the solvability of these problems for analytic functions. We also discuss a new kind of mixed boundary value problem which consists of generalizing, in the third boundary value problem, the boundary condition involving the directional derivative.

In order to make clear the connection between the theory of conformal mappings and certain boundary value problems, we have included a discussion of this connection in all the chapters. Moreover, we have written two appendices to give a brief introduction to the theory of quasiconformal mappings and the connection between boundaries and singular integral equations. We have also included some exercises for the benefit of the readers.

The writing of this book was based on the author's many years of experience in teaching and scientific research. Its content has been used many times as a text for upper division undergraduate courses as well as elementary and advanced graduate courses at Peking University. The constructive suggestions and valuable advice that the author has received over the years have resulted in many revisions. He should especially thank the following individuals for significant improvements in both content and exposition of the present text: Professors Zhuang Zi-Tai and Lu Jian-Ke, and Messieurs Dai Zhong-Wei, Chen Fang-Quan, Huang Sha, Tian Mao-Ying, and Wu Zhi-Jian. Due to the author's limitations, there are bound to be many defects in the book, and the author would welcome the criticisms of the readers.

Guo-Chun Wen
Peking University
May, 1984

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