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Volume 109

Nonlinear Semigroups

Isao Miyadera

American Mathematical Society

Nonlinear Semigroups

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非線形半群

HISENKEI HANGUN (Nonlinear Semigroups)
by Isao Miyadera

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ABSTRACT. This book provides a systematic exposition of the theory of nonlinear semigroups in Banach spaces and an introduction to nonlinear evolution equations. Chapters 2 and 3 present the basic properties of dissipative operators and nonlinear contraction semigroups in Banach spaces respectively. Chapter 4 is devoted to the generation of nonlinear contraction semigroups—the Crandall-Liggett theorem and the Komura theorems, etc. Chapter 5 develops the convergence of difference approximations of Cauchy problems for ω-dissipative operators and then the Kobayashi generation theorem of nonlinear semigroups. Among the topics covered are perturbations of nonlinear semigroups and their application to nonlinear evolution equations; the convergence and approximation of nonlinear semigroups and its application to first order quasilinear equations.

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Contents

Preface

The theory of generation of semigroups of linear contractions, which is the basis of the evolution equation, was developed by Hille and Yosida in 1948. Through this theory the existence and uniqueness for a solution of Cauchy's problem for $(d/dt)u(t) = Au(t)$ with $u(0) = x \in D(A)$ were proved for an m-dissipative operator A that has a dense domain in a Banach space. The theory of linear semigroups was further deepened by results of Phillips and many other mathematicians, and the theory of linear semigroups has now secured its position as an important area in the field of analysis. In 1953 Kato extended the theory of Hille-Yosida to the case where A depends on time t. Afterward, the theory of evolution equations of parabolic type was brought forth by Kato and Tanabe, and the theory of linear evolution equations has made marked progress. Moreover, during the first half of the 1960s the semilinear evolution equation with a nonlinear perturbation term $(d/dt)u(t) = A(t) + f(t, u(t))$ was studied by Segal, Browder and Kato, who obtained excellent results.

Under this historical background, Komura attracted much attention in 1967 when he announced the theory of generation of nonlinear semigroups in a Hilbert space. The theory was immediately extended by Kato to the case of a Banach space with a uniformly convex conjugate space. Afterward, in 1971, Crandall-Liggett obtained the splendid result that "in a general Banach space, an arbitrary m-dissipative operator always generates a semigroup of (nonlinear) contractions." This, together with the work of Komura, has become the basis for the study of nonlinear evolution equations. In addition to the problem of generation, the study of convergence and perturbation of semigroups, and concrete applications of the theory to nonlinear partial differential equations have been carried out vigorously by many mathematicians within and without the country. Developments in the field have led to remarkable progress during the last decade.

This book is concerned mainly with the theory of nonlinear semigroups in a Banach space. In Chapter 1, we summarize the basic results of functional analysis that are necessary for later chapters; Chapter 2 describes the dissipative operators that are closely related to semigroups of contractions;

Chapter 3 presents several properties of the semigroup of contractions. In Chapter 4, we discuss the theory of generation of semigroups of Crandall-Liggett (§§4.1–2), its applications (§4.4), and also the theory of generation of Komura and its extension (§4.3). In Chapter 5 we examine the convergence of the difference approximation to Cauchy's problem for the nonlinear evolution equation $(d/dt)u(t) \in Au(t)$, and we describe an extension of the generation theorem of Crandall-Liggett. In first half of Chapter 6 we describe the theory of convergence and approximation of semigroups. The second half contains the perturbation theory of semigroups and its application to the evolution equation $(d/dt)u(t) = A(t)u(t)$. As an application of the theorems of convergence and approximation of the semigroups given in Chapter 6, we introduce in Chapter 7 the work of Oharu and Takahashi on Cauchy's problem for quasilinear partial differential equations of first order.

As a whole, I have emphasized the explanation of the theoretical part and attempted to prove theorems with care so that the reader with a knowledge of functional analysis at the level of a college junior is able to fully understand the material. (By doing so, I am afraid that my writing might have become wordy.) Because of this and the limitation on the size of the book and also my inability, I have not described many examples. From the standpoint of keeping the discussion focused on Banach spaces, I have not touched on the subdifferential evolution equation at all. The reader may refer to the literature quoted in the postscript of this book.

Finally, I wish to express my profound gratitude to Professors A. Ichida and S. Izumi for their guidance over the years since my school days, and my sincere thanks to Professor S. Itô of the University of Tokyo, who recommended that I write this book. I am deeply grateful also to my friends: Mr. S. Oharu, who has given me much valuable advice, and those who conducted seminars on topics related to evolution equations every Thursday at Waseda University and studied together. I also wish to thank Mr. K. Yokota of the publication division of the Kinokuniya Publishing Company for his continuous assistance in the publication of this book.

Isao Miyadera
January 1977

Appendix. The Minimax Theorem

In this appendix both X and Y are linear topological spaces with R^1 as the field of scalars. (Hence X and Y satisfy the Hausdorff axiom of separation.)

MINIMAX THEOREM. *Let X be finite dimensional, and let both $A \subset X$ and $B \subset Y$ be compact convex sets. If $K(x, y): A \times B \to R^1$ satisfies:*

(a$_1$) *for every $y \in B$, $x \mapsto K(x, y)$ is a lower semicontinuous convex function;*

(a$_2$) *for every $x \in A$, $y \mapsto K(x, y)$ is an upper semicontinuous concave function;*

then (i) *and* (ii) *hold:*

(i) $\min_{x\in A} \max_{y\in B} K(x, y) = \max_{y\in B} \min_{x\in A} K(x, y)$.

(ii) *There exist $x_0 \in A$ and $y_0 \in B$ such that for all $x \in A$ and $y \in B$*

$$K(x_0, y) \le K(x_0, y_0) \le K(x, y_0).$$

To prove this, we will use the following lemma.

LEMMA. *Let $A \subset X$ and $B \subset Y$ be compact sets. If $K(x, y): A \times B \to R^1$ satisfies*

(b$_1$) *For every $y \in B$, $x \mapsto K(x, y)$ is lower semicontinuous.*

(b$_2$) *For every $x \in A$, $y \mapsto K(x, y)$ is upper semicontinuous.*

Then (i) *and* (ii) *in the statements of the Minimax Theorem are equivalent.*

PROOF. We note that both

$$\min_{x\in A} \max_{y\in B} K(x, y) \quad \text{and} \quad \max_{y\in B} \min_{x\in A} K(x, y)$$

exist. In fact, the upper semicontinuous functions and the lower semicontinuous functions defined on compact sets have maximum and minimum values, respectively. Hence from assumption (b$_2$) it follows that for an arbitrary $x \in A$, $\max_{y\in B} K(x, y)$ exists (we write this as $g(x)$). Now from (b$_1$), $g(x)$ is lower semicontinuous on A and $\min_{x\in A} g(x) = \min_{x\in A} \max_{y\in B} K(x, y)$ exists. Similarly, $\max_{y\in B} \min_{x\in A} K(x, y)$ exists.

Next we note that

$$\max_{y\in B} \min_{x\in A} K(x, y) \le \min_{x\in A} \max_{y\in B} K(x, y) \tag{1}$$

holds.

(ii) $\Rightarrow$ (i). From the right half of the inequality in (ii), we have $K(x_0, y_0) \le \min_{x\in A} K(x, y_0) \le \max_{y\in B} \min_{x\in A} K(x, y)$. Next, from the left half of the inequality in (ii), we have $\min_{x\in A} \max_{y\in B} K(x, y) \le \max_{y\in B} K(x_0, y) \le K(x_0, y_0)$. Therefore, $\min_{x\in A} \max_{y\in B} K(x, y) \le \max_{y\in B} \min_{x\in A} K(x, y)$. From this and (1), we obtain (i).

(i) $\Rightarrow$ (ii). We set $\max_{y\in B} \min_{x\in A} K(x, y) = \min_{x\in A} \max_{y\in B} K(x, y) = \alpha$. Then there exist $x_0 \in A$ and $y_0 \in B$ with $\max_{y\in B} K(x_0, y) = \alpha = \min_{x\in A} K(x, y_0)$. Hence for arbitrary $x \in A$ and $y \in B$, $K(x_0, y) \le \alpha \le K(x, y_0)$. In particular, $K(x_0, y_0) \le \alpha \le K(x_0, y_0)$; that is, $\alpha = K(x_0, y_0)$. Hence (ii) holds. □

Proof of Minimax Theorem. From the above lemma, we need only to show (i). Since X is finite dimensional, we let the dimension be d. Then X is isomorphic with the d-dimensional Euclidean space R^d as linear topological space. When $x \ (\in X) \leftrightarrow (\xi_1, \xi_2, \dots, \xi_d) \ (\in R^d)$, we define

$$\|x\| = (\xi_1^2 + \xi_2^2 + \dots + \xi_d^2)^{1/2}.$$

Then the function $\|x\|^2 : X \to R^1$ is a strictly convex (continuous) function (that is, for arbitrary x, $x' \in X$ $(x \ne x')$ and $0 < t < 1$, $\|tx + (1-t)x'\|^2 < t\|x\|^2 + (1-t)\|x'\|^2$).

Now let $\varepsilon > 0$ and set

$$K_\varepsilon(x, y) = K(x, y) + \varepsilon\|x\|^2 \qquad (x \in A, \ y \in B).$$
$$f_\varepsilon(y) = \min_{x\in A} K_\varepsilon(x, y) \qquad (y \in B).$$

Then for each $y \in B$, since $x \mapsto K_\varepsilon(x, y)$ is a lower semicontinuous, strictly convex function, there exists a unique $x' \in A$ such that $K_\varepsilon(x', y) = \min_{x\in A} K_\varepsilon(x, y)$. Since x' depends on $y \in B$, we denote this x' by $E(y)$. Hence

$$f_\varepsilon(y) = K_\varepsilon(E(y), y) \qquad (y \in B).$$

Next for each $x \in A$, since $y \mapsto K_\varepsilon(x, y)$ is an upper semicontinuous concave function, f_ε is an upper semicontinuous concave function. From this, $f_\varepsilon(y)$ $(y \in B)$ has a maximum value. Now we choose $y^* \in B$ such that $f_\varepsilon(y^*) = \max_{y\in B} f_\varepsilon(y)$. Since $y \mapsto K_\varepsilon(x, y)$ is a concave function

$$K_\varepsilon(x, (1-t)y^* + ty) \ge (1-t)K_\varepsilon(x, y^*) + tK_\varepsilon(x, y)$$
$$\ge (1-t)f_\varepsilon(y^*) + tK_\varepsilon(x, y) \qquad (x \in A, \ y \in B, \ 0 < t < 1).$$

If we set $x = E((1-t)y^* + ty)$ in the above inequality, then

$$f_\varepsilon(y^*) \ge f_\varepsilon((1-t)y^* + ty) \ge (1-t)f_\varepsilon(y^*) + tK_\varepsilon(E((1-t)y^* + ty), y).$$

Hence we obtain

$$f_\varepsilon(y^*) \ge K_\varepsilon(E((1-t)y^* + ty), y) \qquad (y \in B, \ 0 < t < 1). \tag{2}$$

Now

$$\text{for arbitrary } y_1, y_2 \in B, \quad E((1-t)y_1 + ty_2) \to E(y_1) \text{ as } t \to 0+. \tag{3}$$

To show this, we let $y_1, y_2 \in B$ and set $x_t = E((1-t)y_1+ty_2)$ $(0 < t < 1)$. Then for every $x \in A$ and $0 < t < 1$

$$K_\varepsilon(x_t, (1-t)y_1 + ty_2) = f_\varepsilon((1-t)y_1 + ty_2) \le K_\varepsilon(x, (1-t)y_1 + ty_2). \tag{4}$$

If we take an arbitrary sequence $\{t_n\}$ such that $\lim_{n\to\infty} t_n = 0$ and $0 < t_n < 1$, then since $x_{t_n} \in A$ and A is compact, there is a subsequence $\{t_{n'}\}$ of $\{t_n\}$ and an element $z \in A$ such that $x_{t_{n'}} \to z$ as $n' \to \infty$. From (4) and the fact that $y \mapsto K(x, y)$ is a concave function, we have

$$\begin{aligned}(1-t_{n'})K_\varepsilon(x_{t_{n'}}, y_1) + t_{n'}K_\varepsilon(x_{t'_n}, y_2) &\le K_\varepsilon(x_{t'_n}, (1-t_{n'})y_1 + t_{n'}y_2) \\ &\le K_\varepsilon(x, (1-t_{n'})y_1 + t_{n'}y_2) \\ &\qquad (x \in A).\end{aligned}$$

Here if we let $n' \to \infty$, then from the lower semicontinuity of $x \mapsto K_\varepsilon(x, y)$ and the upper semicontinuity of $y \mapsto K_\varepsilon(x, y)$, for an arbitary $x \in A$ we have

$$\begin{aligned}K_\varepsilon(z, y_1) \left(\le \liminf_{n'\to\infty} K_\varepsilon(x_{t'_n}, y_1)\right) &\le \limsup_{n'\to\infty} K_\varepsilon(x, (1 - t_{n'}y_1 + t_{n'}y_2)) \\ &\le K_\varepsilon(x, y_1).\end{aligned}$$

From this, $K_\varepsilon(z, y_1) = \min_{x\in A} K_\varepsilon(x, y_1)$ and thus $z = E(y_1)$. Since we have shown that an arbitrary subsequence $\{x_{t_n}\}$ (of $\{x_t\}$) has a subsequence that converges to the same limit $E(y_1)$, it follows that $x_t \to E(y_1)$ as $t \to 0+$. Therefore, (3) is proved.

Now, letting $t \to 0+$ in (2). It follows from (3) that

$$f_\varepsilon(y^*) \left(\ge \liminf_{t\to 0+} K_\varepsilon(E((1-t)y^* + ty), y)\right) \ge K_\varepsilon(E(y^*), y) \qquad (y \in B).$$

Also from the definition of f_ε, we have that $f_\varepsilon(y^*) \le K(x, y^*)$ $(x \in A)$. Hence

$$K_\varepsilon(x^*, y) \le (f_\varepsilon(y^*) =)\ K_\varepsilon(x^*, y^*) \le K_\varepsilon(x, y^*) \qquad (x \in A,\ y \in B), \tag{5}$$

where we have set $x^* = E(y^*)$. If we set

$$M = \max\{\|x\|^2 = \sum_{i=1}^d |\xi_i|^2 : x \in A\},$$

then from the definition of K_ε we have

$$K(x, y) - M\varepsilon \le K_\varepsilon(x, y) \le K(x, y) + M\varepsilon \qquad (x \in A,\ y \in B).$$

From this and (5)

$$\begin{aligned}\min_{x\in A}\max_{y\in B} K(x, y) - M\varepsilon \le \min_{x\in A}\max_{y\in B} K_\varepsilon(x, y) &\le \max_{y\in B} K_\varepsilon(x^*, y) \le K_\varepsilon(x^*, y^*)\\ &\le \min_{x\in A} K_\varepsilon(x, y^*) \le \max_{y\in B}\min_{x\in A} K_\varepsilon(x, y)\\ &\le \max_{y\in B}\min_{x\in A} K(x, y) + M\varepsilon .\end{aligned}$$

Since $\varepsilon > 0$ is arbitrary, we have

$$\min_{x\in A}\max_{y\in B} K(x, y) \le \max_{y\in B}\min_{x\in A} K(x, y) .$$

Combining this with (1), we obtain

$$\min_{x\in A}\max_{y\in B} K(x, y) = \max_{y\in B}\min_{x\in A} K(x, y) . \quad \square$$

Postscript

One of the subjects that we have not touched upon at all in this book is the subdifferential evolution equations (in Hilbert spaces). For this, see Brezis [16] and Barbu [7], and also Watanabe [125] for recent results and for the references quoted by them. Also, [16] is an excellent expository text that summarizes the theory of nonlinear semigroups in Hilbert spaces and its applications. In the following we describe briefly the main references for each chapter and supplementary items.

Chapter 2. In §2.1 and §2.2 we summarized the results based mainly on Kato [50, 51]; see also Crandall-Liggett [36] and Oharu [102]. We referred to Brezis [16] for Example 2.2 and to Crandall [31] for Example 2.3. We sometimes call a strictly dissipative operator "a dissipative operator in the sense of Browder." In §2.3 we summarized the results of Kato [51], Brezis-Pazy [20], Brezis [14], Debrunner-Flor [41], Crandall-Pazy [38], and Minty [88, 89]. Theorem 2.20 is based on [20]; Theorem 2.23 and Corollary 2.24 on [14]. Theorems 2.25 and 2.26 and Corollary 2.27 are based on [41], [38] and [88, 89]. Example 2.6 is given by Crandall-Liggett [37].

Chapter 3. In §3.1 we summarized the results of Crandall-Pazy [38], Miyadera [94], and Kato [51]. Theorem 3.5 is based on [94] and Corollary 3.7 on [51] (see also [38]). Example 3.1 is given by Komura [64] and Example 3.2 by Webb [126]. The contents of §3.2 are based on the results of Crandall-Liggett [36], Komura [65], and Kato [52]; in particular, Theorem 3.13 is obtained by Komura [65]. Also, the proof given here is due to [52]. Theorem 3.11 is based on [36]; Lemma 3.15 of §3.3 on Miyadera-Oharu [95]. See also Brezis-Pazy [20]. Theorem 3.17 is based on Crandall-Pazy [38] for the case of a Hilbert space, and on Kobayashi [60] and Kobayasi [63] for the case of an extension to a Banach space. Theorems 3.18 and 3.19 are based on [63] (see also Miyadera [93]). Also, the estimation (3.44) used for the proof of Theorem 3.19 is given by Crandall-Liggett [36] when they prove Theorem 4.2 (generation theorem of semigroups). The way we apply induction in the proof is due to Rasmussen [115].

Chapter 4. In §4.1 and §4.2, we described generation theorems of semigroups based on Crandall-Liggett [36]. See also Miyadera [93] on Theorem 4.5 and Crandall [32] on Theorem 4.7. In §4.3 we summarized mainly those results on generation theorems of semigroups given by Komura [64, 65], Kato [50, 51], Crandall-Pazy [38], Dorroh [43], Martin [81], and Miyadera [94]. Theorems 4.13, 4.14, and 4.16 are based on [94]; Corollary 4.17 on Brezis [14]; Theorem 4.18 on [81]. Theorem 4.20 is the so-called Komura theorem. See also [38] and Komura [66]. Examples 4.5, 4.6, and 4.7 of §4.4 are based on Crandall-Liggett [36]; Examples 4.8 and 4.9 on Crandall [34]; Example 4.10 on Crandall [31]. In this book, we have not considered the question of compactness of the contraction semigroups $\{T(t)\colon t \geq 0\}$ generated by m-dissipative operators (if for each $t > 0$, $T(t)$ maps a bounded subset of $\overline{D(A)}$ into a sequentially compact set, then this semigroup is said to be compact), and we have not covered the generation of semigroups in Banach lattices. See Konishi [72] and H. Brezis, *New results concerning monotone operators and nonlinear semigroups*, Analysis of Nonlinear Problems, Res. Inst. Math. Sci., Kyoto Univ., 1974 for the characterization of compactness of semigroups. See Konishi [67, 69] and Calvert [25] for the generation of semigroups in Banach lattices.

Chapter 5. Theorem 5.1 of §5.1 is based on Brezis-Pazy [20]. The concept of the integral solution of Cauchy's problem is introduced by Benilan [9, 10]. The proof of Theorem 5.2 is quoted from Kobayashi [61]. Section 5.2 is based mainly on Benilan [10], Takahashi [121], and Kobayashi [61]. Part (i) of Theorem 5.7 (the convergence) is based on [121] and [61]. The proof (of the estimation (5.11)) given in this book is shown in [61]. Also the proof of uniqueness of the integral solution in this same theorem is due to [10]. Theorem 5.10 of §5.3 and Theorem 5.12 (generation theorem of semigroups) are based on [61]. Theorem 5.15 is due to Martin [82] (see also [61]). Section 5.4 is an application of Theorem 5.10. Theorem 5.18 is an extension of the result given in [10]. Also, Kenmochi-Oharu [57], Crandall [34], Crandall-Evans [35], Kobayashi-Kobayasi [62], and Pierre [112] are closely related to the contents of this chapter.

Chapter 6. Theorem 6.5 of §6.1, Theorems 6.8 and 6.12 of §6.2 are due to Miyadera-Kobayashi [96] (Theorem 6.12 holds even if X_n is not convex (see [96])). Corollaries 6.9 and 6.13 are summaries of the results obtained by Kurtz [76], Brezis-Pazy [19], and Goldstein [47]. Corollary 6.11 is shown by Benilan [8]. Also, its extension is given in H. Brezis, *New results concerning monotone operators and nonlinear semigroups*, Analysis of Nonlinear Problems, Res. Inst. Math. Sci., Kyoto Univ., 1974. Theorems 6.15 and 6.17 of §6.3 are due to Pierre [112]. Part (iii) of Corollary 6.19 is given by Barbu [6] (for parts (i) and (ii) of the same corollary, see also Kobayashi [61]). Also, Kato [51], Brezis [16], and Kobayashi-Kobayasi [62] are among those concerned with perturbation theorems in the form of the Corollary 6.19 (that is, the question as to whether or not $A+B$ is an m-dissipative operator, when an operator B is added to an m-dissipative operator A). $(d/dt)u(t) \in A(t)u(t)$ is one of the topics that we have not discussed in this book, even though it is related to the evolution equation $(d/dt)u(t) \in A(t)u(t)$ which we considered in §6.4. Here, the operator $A(t)$ is not necessarily single-valued (not even a continuous operator). Regarding this subject, see Kato [51], Crandall-Pazy [39], Evans [46], and also Masuda [86]. There is also the product formula of semigroups which is related to this chapter. For this, see Brezis-Pazy [18], Webb [127, 128], Kobayashi [59], and Miyadera-Kobayashi [96].

Chapter 7. Following Oharu-Takahashi [107], we explained a construction of the difference approximation of the solution of Cauchy's problem for quasilinear partial differential equations of first order. Furthermore, concrete applications of the theory of nonlinear semigroups to the partial differential equations have been carried out recently by many scholars. For example, Aizawa [1–3], Brezis-Strauss [21], Crandall [33], Konishi [69–71, 73, 74], Kurtz [76], and Oharu [105, 106] are among those who deal with nonreflexive function spaces (beside those quoted in §4.4).

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