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Tube Domains and the Cauchy Problem

Simon Gindikin



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Volume 111

**Tube Domains
and the Cauchy Problem**

Simon Gindikin



American Mathematical Society
Providence, Rhode Island

С. Г. ГИНДИКИН

ЗАДАЧА КОШИ И ТРУБЧАТЫЕ ОБЛАСТИ

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ABSTRACT. In the first chapter, we describe the exact spaces of functions or distributions with exponential estimates for which the Cauchy problem is solvable. In the second chapter, we investigate differential operators connecting with affine homogeneous domains.

This volume is intended for mathematicians and graduate students working in functional analysis, differential equations, and several complex variables.

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