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MONOGRAPHS**

Volume 117

**Uncountably
Categorical Theories**

Boris Zilber



American Mathematical Society

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Boris Zilber



American Mathematical Society
Providence, Rhode Island

Translated from the Russian by D. Louvish
Translation edited by Simeon Ivanov

1991 *Mathematics Subject Classification*. Primary 03–02, 03C35, 03C60, 05B25.

Library of Congress Cataloging-in-Publication Data

Zilber, Boris.

[Neschetno kategorichnye teorii. English]

Uncountably categorical theories/Boris Zilber.

p. cm.—(Translations of mathematical monographs, ISSN 0065-9282; v. 117)

ISBN 0-8218-4586-1

1. Categories (Mathematics) 2. Completeness theorem. 3. Model theory. I. Title.

II. Series.

QA9.67.Z5513 1993

92-31151

511.3—dc20

CIP

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Printed in the United States of America

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This publication was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$,

the American Mathematical Society's $\mathcal{T}\mathcal{E}\mathcal{X}$ macro system.

10 9 8 7 6 5 4 3 2 1 98 97 96 95 94 93

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ISBN 0-8218-4586-1



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