Translations of

MATHEMATICAL MONOGRAPHS

Volume 122

Theory of Entire and Meromorphic Functions

Deficient and Asymptotic Values and Singular Directions

Zhang Guan-Hou

American Mathematical Society

Recent Titles in This Series

122 **Zhang Guan-Hou,** Theory of entire and meromorphic functions: Deficient and asymptotic values and singular directions, 1993

121 **I. B. Fesenko and S. V. Vostokov,** Local fields and their extensions: a constructive approach, 1993

120 **Takeyuki Hida and Masuyuki Hitsuda,** Gaussian processes, 1993

119 **M. V. Karasev and V. P. Maslov,** Nonlinear Poisson brackets. Geometry and quantization, 1993

118 **Kenkichi Iwasawa,** Algebraic functions, 1993

117 **Boris Zilber,** Uncountably categorical theories, 1993

116 **G. M. Fel'dman,** Arithmetic of probability distributions, and characterization problems on abelian groups, 1993

115 **Nikolai V. Ivanov,** Subgroups of Teichmüller modular groups, 1992

114 **Seizô Itô,** Diffusion equations, 1992

113 **Michail Zhitomirskiĭ,** Typical singularities of differential 1-forms and Pfaffian equations, 1992

112 **S. A. Lomov,** Introduction to the general theory of singular perturbations, 1992

111 **Simon Gindikin,** Tube domains and the Cauchy problem, 1992

110 **B. V. Shabat,** Introduction to complex analysis Part II. Functions of several variables, 1992

109 **Isao Miyadera,** Nonlinear semigroups, 1992

108 **Takeo Yokonuma,** Tensor spaces and exterior algebra, 1992

107 **B. M. Makarov, M. G. Goluzina, A. A. Lodkin, and A. N. Podkorytov,** Selected problems in real analysis, 1992

106 **G.-C. Wen,** Conformal mappings and boundary value problems, 1992

105 **D. R. Yafaev,** Mathematical scattering theory: General theory, 1992

104 **R. L. Dobrushin, R. Kotecký, and S. Shlosman,** Wulff construction: A global shape from local interaction, 1992

103 **A. K. Tsikh,** Multidimensional residues and their applications, 1992

102 **A. M. Il'in,** Matching of asymptotic expansions of solutions of boundary value problems, 1992

101 **Zhang Zhi-fen, Ding Tong-ren, Huang Wen-zao, and Dong Zhen-xi,** Qualitative theory of differential equations, 1992

100 **V. L. Popov,** Groups, generators, syzygies, and orbits in invariant theory, 1992

99 **Norio Shimakura,** Partial differential operators of elliptic type, 1992

98 **V. A. Vassiliev,** Complements of discriminants of smooth maps: Topology and applications, 1992

97 **Itiro Tamura,** Topology of foliations: An introduction, 1992

96 **A. I. Markushevich,** Introduction to the classical theory of Abelian functions, 1992

95 **Guangchang Dong,** Nonlinear partial differential equations of second order, 1991

94 **Yu. S. Il'yashenko,** Finiteness theorems for limit cycles, 1991

93 **A. T. Fomenko and A. A. Tuzhilin,** Elements of the geometry and topology of minimal surfaces in three-dimensional space, 1991

92 **E. M. Nikishin and V. N. Sorokin,** Rational approximations and orthogonality, 1991

91 **Mamoru Mimura and Hirosi Toda,** Topology of Lie groups, I and II, 1991

90 **S. L. Sobolev,** Some applications of functional analysis in mathematical physics, third edition, 1991

89 **Valeriĭ V. Kozlov and Dmitriĭ V. Treshchëv,** Billiards: A genetic introduction to the dynamics of systems with impacts, 1991

(*Continued in the back of this publication*)

This page intentionally left blank

Theory of Entire and Meromorphic Functions

Deficient and Asymptotic Values and Singular Directions

This page intentionally left blank

10.1090/mmono/122

Translations of

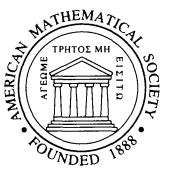
MATHEMATICAL MONOGRAPHS

Volume 122

Theory of Entire and Meromorphic Functions

Deficient and Asymptotic Values and Singular Directions

Zhang Guan-Hou



American Mathematical Society
Providence, Rhode Island

整函数和亚纯函数理论

——亏值、渐近值和奇异方向

张 广 厚 著

Translated from the Chinese by Chung-Chun Yang

1991 *Mathematics Subject Classification.* Primary 30D30, 30D35.

Library of Congress Cataloging-in-Publication Data

Zhang, Guan-Hou, 1937–1987.
[Cheng han shu ho yeh ch'un han shu li lun. English]
Theory of entire and meromorphic functions: deficient and asymptotic values and singular directions/Zhang Kuan-hua.
p. cm.—(Translations of mathematical monographs; v. 122)
ISBN 0-8218-4589-6
1. Functions, Entire. 2. Functions, Meromorphic. I. Title. II. Series.
QA353.E5Z4313 1993 93-43
515'.98—dc20 CIP

10 9 8 7 6 5 4 3 2 1 98 97 96 95 94 93

Contents

Preface

This is a monograph about the theory of entire and meromorphic functions. It sums up basically the development of this theory ever since the 1950s, with the discussion focus centered on the relationship among three main concepts, namely the deficient value, the asymptotic value and the singular direction.

In 1929, by examining some examples, R. Nevanlinna recognized that there is an intrinsic relationship between the problem of exceptional values (deficient values are exceptional value under a certain kind of implication) and the asymptotic value theory. Moreover, he anticipated that the study of their relationship might help to clarify some of the profound problems of the theory of entire and meromorphic functions [32a]. Concretely speaking, he conjectured that a deficient value is simultaneously an asymptotic value [32a]. However, this conjecture was negated later [37a, 3a]. In 1978, the author restudied this problem and found that there is a close relationship among the number of deficient values, the number of asymptotic values and the number of singular directions for a function of finite lower order. He obtained some general results [43c]. In recent years, new developments have been made for this study [43g, h, j]; for some other important kinds of functions, their corresponding related formulas are also obtained. The main content of this monograph is to introduce these studies. It is written in the sense that both systematic property and the reference of some results of recent studies are taken into consideration.

This book is divided into six chapters. The first chapter introduces Nevanlinna's fundamental theory, including mainly the famous First Fundamental Theorem, the Second Fundamental Theorem, as well as the related formulas of deficiencies. All these are the foundation of the whole book.

Chapter 2 introduces the Theory of Singular Directions, including mainly the proof of the existence of the Julia direction and the Borel direction, as well as some significant properties. In addition, this chapter includes the discussion and proof of the existence of the Nevanlinna direction under a certain meaning. This discussion is, in fact, a recent study made by Li Yu-Nian and the author himself.

Chapter 3 introduces the Deficient Theory. Deficient values are the main issue of the recent study of the Value Distribution Theory. The results obtained for this issue are particularly fruitful and wonderful. However, due to limited space, we can include in this chapter a relatively comprehensive and systematic discussion regarding merely the study of the number of deficient values. Particularly, here we include also the famous results due to Edrei-Fuchs and Weitsman, etc., with the proof of most of these results newly presented.

At the end of each of the above three chapters, we write an annotated note introducing some important results and corresponding books([1]) that have not been covered in the main text as supplements.

Chapter 4 introduces the fundamental theory of asymptotic values and some new results of the study. Included are mainly the classical Iversen Theorem and the famous result regarding the proof of Denjoy's conjecture made by Ahlfors, as well as the estimation of the growth property of a function along the asymptotic path and the estimation of the length of the asymptotic path, all attributed to the author's effort.

The first four chapters lay the foundation for, simultaneously, Chapters 5 and 6, with these last two chapters being the focus of the whole book. Chapter 5 discusses the relationship among the number of deficient values, the number of asymptotic values and the number of Julia directions of an entire function whose lower order is finite, and also the relationship among the number of deficient values, the number of asymptotic values and the number of Julia directions of an entire function which consists of finite Julia directions or with its zeros accumulating around the neighborhood of finite half straight lines. Chapter 6 takes into consideration the case of a meromorphic function and discusses the corresponding problems of a meromorphic function whose lower order is finite or having a maximal deficiency sum. However, here we have to replace the number of asymptotic values with the number of directly transcendental singularities of an inverse function.

Readers intending to peruse this book need only have the knowledge of the complex function theory and of the real function theory provided in the mathematics department of a university. This book, however, is especially suitable for those postgraduate students who study the theory of entire and meromorphic functions under their advisors' guidance.

Finally, here I shall like to express my wholehearted gratitude to Professor Chuang Chi-tai. It is under his encouragement that I became determined to write this book, and through several discussions with him, the outline of this book was drawn. I shall also like to thank Associate Professor He Yu-zan for his meticulous checking and proofreading of the manuscripts. He put

([1]) If readers want to have a relatively comprehensive understanding of the problem regarding the theory of entire and meromorphic functions in recent study and its progress, they can refer to [21d, f], [2a] and [8a].

forth lots of valuable opinions for amendments. Furthermore, I shall like to thank comrade Wu Peng-cheng who transcribed, proofread and examined the manuscripts several times. Without his assistance, surely it would have taken a longer time to finish writing this book. Finally, I shall also like to thank Associate Professor Li Yu-nian for his help. Before the publication of this book, he spared his busy time to seriously examine and proofread the entire book once.

Zhang Guan-Hou

This page intentionally left blank

Some Supplementary Results

Ever since the publication of the book, several interesting and significant results concerning the numerical relationships among the order, lower order, numbers of deficient values, asymptotic values, and singular directions of an entire or a meromorphic function and its derivatives have been obtained. The following list provides some of the more significant ones.

In 1988, L. Yang [*Deficient values and angular distribution of entire functions*, Trans. Amer. Math. Soc. **308** (1988), 583–601] gave the following definition: Let $f(z)$ be an entire-function of lower order μ, where $0 < \mu < +\infty$. A ray $\arg z = \theta_0$ $(0 \leq \theta_0 < 2\pi)$ is called a Borel direction of order $\geq \mu$ of $f(z)$, if for any positive number ε, the inequality

$$\overline{\lim_{r \to +\infty}} \frac{\log n(r, \theta_0 - \varepsilon, \theta_0 + \varepsilon, f = a)}{\log r} \geq \mu$$

holds for any finite complex value a, with possibly one exceptional value, where $n(r, \theta_0 - \varepsilon, \theta_0 + \varepsilon, f = a)$ denotes the number of zeros of $f(z) - a$ in the region $(|z| \leq r) \cap (\theta_0 - \varepsilon \leq \arg z \leq \theta_0 + \varepsilon)$ multiple zeros being counted according to their multiplicities.

Meanwhile, L. Yang also proved the following result.

THEOREM. *Let $f(z)$ be an entire function of lower order μ, where $0 < \mu < +\infty$. If $q < +\infty$ is the number of Borel directions of order $\geq \mu$ of $f(z)$ and $p_l (l = 0, -1, -2, \ldots)$ denotes the number of finite nonzero deficient values of $f^{(l)}(z)$ $(l = 0, -1, -2, \ldots, f^{(0)}(z) \equiv f(z))$, when $l < 0$, $f^{(l)}(z)$ is the primitive of order $|l|$ of $f(z)$, then we have*

$$\sum_{l=0}^{-\infty} p_l \leq 2\mu,$$

More recently, as an extension over the above result, P. C. Wu obtained the following result in "Angular distribution of entire functions and its deficient values of each order derivative", preprint.

THEOREM. *Let $f(z)$ be an entire function of lower order μ, where $0 < \mu < +\infty$. If $q < +\infty$ is the number of Borel directions of order $\geq \mu$ of $f(z)$*

and $p_l (l = 0, -1, -2, \dots)$ *denotes the number of finite nonzero deficient values of* $f^{(l)}(z) (l = 0, -1, -2, \dots f^{(0)}(z) = f(z))$, *then we have*

$$\sum_{l=0}^{-\infty} p_l < 2\mu.$$

S. J. Wu also researched this topic in his thesis "Angular distribution and Borel theorem of entire and meromorphic functions". For instance, the following two results were obtained.

THEOREM. *Suppose that* $f(z)$ *is an entire function of finite lower order* μ *and satisfies* $p = \frac{q}{2}$ *where* $p (1 \leq p < +\infty)$ *denotes the number of finite deficient values and* q *denotes the number of Borel directions of order* $\geq \mu$ *of* $f(z)$. *Then for every deficient value* a_j $(j = 1, 2, \dots, p)$, *there exists a corresponding angular domain* $\Omega(\theta_{k_j}, \theta_{k_j+1})$ *such that for every* $\varepsilon > 0$ *the inequality*

$$\log \frac{1}{|f(z) - a_j|} > A(\theta_{k_j}, \theta_{k_j+1}, \varepsilon, \delta(a_j, f))T(|z|, f)$$

holds for $z \in \Omega(\theta_{k_j} + \varepsilon, \theta_{k_j+1} - \varepsilon, r_\varepsilon, +\infty)$, *where* $A(\theta_{k_j}, \theta_{k_j+1}, \varepsilon, \delta(a_j, f))$ *is a positive constant depending only on* $\theta_{k_j}, \theta_{k_j+1}, \varepsilon$, *and* $\delta(a_j, f)$. *In particular, every deficient value of* $f(t)$ *is also its deficient value.*

THEOREM. *Under the hypothesis of the above theorem, we have*

(i) *The order* λ *of* $f(z)$ *equals* μ,

(ii) *Every asymptotic value of* $f(z)$ *is also its deficient value,*

(iii) $\sum_{a \in \mathbb{C}} \delta(a, f) \leq 1 - k(\mu)$,

where

$$k(\mu) = \begin{cases} \dfrac{|\sin \mu\pi|}{q + |\sin \mu\pi|}, & q \leq \mu \leq q + \frac{1}{2}, \\ \dfrac{|\sin \mu\pi|}{q + 1}, & q + \frac{1}{2} < \mu \leq q + 1. \end{cases}$$

References

[1] L. V. Ahlfors,

[a] *Beiträge der meromorphen Funktionen*, Congr. Math. Scand. Oslo, vol. 7 (1929), pp. 84–88.

[b] *Untersuchungen zur Theorie der konformen Abbildung und der Theorie der ganzen Funktionen*, Acta Soc. Sci. Fenn **1** (1930), 1–40.

[c] *Über die asymptotischen Werte der meromorphen Funktionen endlicher Ordnung*, Acta Acad. Aboensis. Math. et Phys. **6** (1932).

[d] *Zur Theorie der Überlagerungsflächen*, Acta Math. **65** (1935), 157–194.

[e] *Lectures on quasiconformal mapping*, Van Nostrand, Princeton, NJ, 1966.

[f] *Conformal invariants: Topics in geometric function theory*, McGraw-Hill, NY, 1973.

[2] J. M. Anderson, K. F. Barth, and D. A. Brannan,

[a] *Research problems in complex analysis*, Bull. London Math. Soc. **9** (1977), 129–162.

[3] N. V. Arakeljan,

[a] *Entire functions of finite order with an infinite set of deficient values*, Dokl. Akad. Nauk SSSR **107** (1966), 999–1002.

[4] A. Baernstein,

[a] *Proof of Edrei's spread conjecture*, Proc. London Math. Soc. **26** (1973), 418–434.

[b] *Integral means, univalent functions and circular symmetrization*, Acta Math. **133** (1974), 139–169.

[5] K. F. Barth, D. A. Brannan, and W. K. Hayman,

[a] *The growth of plane harmonic functions along an asymptotic path*, Proc. London Math. Soc. **37** (1978), 363–384.

[6] E. Borel,

[a] *Sur les zéros des fonctions entières*, Acta Math. **20** (1897), 357–396.

[7] F. Bureau,

[a] *Mémoire sur les fonctions uniformes à point singulier essentiel isolé*, Mém. Soc. Roy. Sci. Liége **17** (1932).

[8] D. M. Campbell, J. G. Clunie, and W. K. Hayman,

[a] *Research problems in complex analysis*, Aspect of Contemporary Complex Analysis (D. A. Brannan and J. G. Clunie, eds.), Academic Press, NY, 1980.

[9] H. Cartan,

[a] *Sur les systèmes de fonctions holomorphes à variétés linéaries lacunaires et leurs applications*, Ann. Sci. École Norm. Sup. **45** (1928), 255–346.

[b] *Sur la fonction de croissance attachée à une fonction méromorphe de deux variables et ses applications aux fonctions méromorphes d'une variable*, C. R. Acad. Sci. Paris **189** (1929), 521–523.

[10] Chuang Chi-tai (or Zhang Chi-tai),
[a] *Un théorème relatif aux directions de Borel des fonctions méromorphes d'ordre fini*, C. R. Acad. Sci. **204** (1937), 951–952.
[b] *Une généralisation d'une inégalité de Nevanlinna*, Sci. Sinica **13** (1964), 887–895.
[c] *Singular directions of meromorphic functions*, Science Press, 1982.

[11] A. Denjoy,
[a] *Sur les fonctions entiéres de genre fini*, C. R. Acad. Sci. Paris **145** (1907), 106–109.

[12] D. Drasin,
[a] *On asymptotic curves of functions extremal for Denjoy's conjecture*, Proc. London Math. Soc. **26** (1973), 142–166.
[b] *The inverse problem of the Nevanlinna theory*, Acta Math. **138** (1977), 83–151.
[c] *Proof of a conjecture of F. Nevanlinna concerning functions which have deficiency sum two*, Acta Math. **158** (1987), 1–94.

[13] D. Drasin and A. Weitsman,
[a] *Meromorphic functions with large sums of deficiencies*, Adv. in Math. **15** (1974), 93–126.
[b] *On the Julia directions and Borel directions of entire functions*, Proc. London Math. Soc. **32** (1976), 199–212.

[14] A. Edrei,
[a] *Sums of deficiencies of meromorphic functions*, J. Analyse Math., (I) **14** (1965), 79–107; (II) **19** (1967), 53–74.
[b] *Solution of the deficiency problem for functions of small lower order*, Proc. London Math. Soc. **26** (1973), 435–445.

[15] A. Edrei and W. H. J. Fuchs,
[a] *Valeurs déficientes et valeurs asymptotiques des fonctions méromorphes*, Comment. Math. Helv. **33** (1959), 258–295.
[b] *On the growth of meromorphic functions with several deficient values*, Trans. Amer. Math. Soc. **93** (1959), 292–328.
[c] *The deficiencies of meromorphic functions of order less than one*, Duke Math. J. **27** (1960), 233–249.
[d] *Bounds for the number of deficient values of certain classes of meromorphic functions*, Proc. London Math. Soc. **12** (1962), 315–344.
[e] *On meromorphic functions with regions free of poles and zeros*, Acta Math. **108** (1962), 113–145.

[16] W. H. J. Fuchs,
[a] *A Phragmén-Lindelöf theorem conjectured by D. J. Newman*, Trans. Amer. Math. Soc. **267** (1981), 285–293.
[b] *The development of the theory of deficient values since Nevanlinna*, Ann. Acad. Sci. Fenn. Ser. A. I. Math. **7** (1982), 33–48.

[17] W. H. J. Fuchs and W. K. Hayman,
[a] *An entire function with assigned deficiencies*, Studies in Mathematical Analysis and Related Topics Essays in Honor of George Pólya, Stanford Univ. Press, Stanford, CA, 1962, pp. 117–125.

[18] A. A. Gol'dberg,
[a] *Defects of meromorphic functions*, Dokl. Akad. Nauk SSSR **98** (1954), 893–895.
[b] *The possible magnitude of the lower order of an entire function with a finite deficient value*, Dokl. Akad. Nauk SSSR **159** (1964), 968–970.

[19] A. A. Gol'dberg and A. E. Eremenko,

[20] G. M. Golyzun,

[21] W. K. Hayman,
[a] *Multivalent functions*, Cambridge Univ. Press, 1958.

[b] *Picard values of meromorphic functions and their derivatives*, Ann. Math. **70** (1959), 9–42.
[c] *Meromorphic functions*, Oxford Univ. Press, 1964.
[d] *Research problems in function theory*, Athlone Press, London, 1967.
[e] *Angular value distribution of power series with gaps*, Proc. London Math. Soc. **24** (1972), 590–624.
[f] *Research problems in function theory*, Symposium on Complex Analysis, Canterbury, 1973 (J. G. Clunie and W. K. Hayman, eds.), Cambridge Univ. Press, NY, 1974.
[g] *On Iversen's theorem for meromorphic functions with few poles*, Acta Math. **141** (1978), 115–145.

[22] Hiong King-lai,
[a] *Sur les fonctions méromorphes et les fonctions algébroides*, Mém. Sci. Math. Fasc., vol. 139, Paris, 1957.

[23] A. Huber,
[a] *On subharmonic function and differential geometry in the large*, Comment. Math. Helv. **32** (1957), 13–72.

[24] F. Iversen,
[a] *Recherches sur les fonctions inverses des fonctions méromorphes*, Thèse de Helsingfors, 1914.

[25] G. Julia,
[a] *Sur quelques propriétés nouvelles des fonctions entières ou méromorphes*, Ann. École Norm. Sup. **36** (1919), 93–125; **37** (1920), 165–218.

[26] P. B. Kennedy,
[a] *On a conjecture of Heins*, Proc. London Math. Soc. **5** (1955), 22–47.
[b] *A class of integral functions bounded on certain curves*, Proc. London Math. Soc. **6** (1956), 518–547.

[27] J. Lewis, J. Boss, and A. Weitsman,
[a] *On the growth of subharmonic functions along paths*, Preprint.

[28] E. Lindelöf,
[a] *Sur un principle générale de l'analysde et ses applications à la théorie de la representation conforme*, Acta Soc. Sci. Fenn. **46** (1915).

[29] Lü Yi-nian and Zhang Guang-hou,
[a] *On Nevanlinna direction of a meromorphic function*, Sci. Simica **26** (1983), 607–617.

[30] H. Milloux,
[a] *Le théorème de Picard, suites de fonctions holomorphes; fonctions holomorphes et fonctions entieres*, J. de Math. **3** (1924), 345–401.
[b] *Sur une extension d'un théorème de P. Boutroux-H. Cartan*, Bull. Soc. Math. France **65** (1937), 65–75.
[c] *Extension d'un théorème de M. R. Nevanlinna et applications*, Act. Sci. et Ind., No. 888, 1940.
[d] *Sur les directions de Borel des fonctions entières, de leurs dérivées et de leurs integrales*, J. Analyse Math. **1** (1951), 244–330.

[31] F. Nevanlinna,
[a] *Über eine Klasse meromorpher Funktionen*, C. R. Congr. Math. Scand., Oslo, 1929, pp. 81–83.

[32] R. Nevanlinna,
[a] *Le théorème de Picard-Borel et la théorie des fonctions méromorphes*, Paris, 1929.
[b] *Analytic functions*, Springer, Berlin, 1970.

[33] A. Ostrowski,
[a] *Über Folgen analytischer Funktionen und einige Verschärfungen des Picardschen Satzes*, Math. Z. **24** (1926), 215–258.

[34] A. Pfluger,
[a] *Zur Defektrelation ganzer Funktionen endlicher Ordnung*, Comment. Math. Helv. **19** (1946), 91–104.

[35] A. Rauch,
[a] *Extension de théorèmes relatifs aux directions de Borel des fonctions méromorphes*, J. Math. Pures Appl. **12** (1933), 109–171.
[b] *Cas où une direction de Borel d'une fonction entière $f(z)$ d'ordre fini est aussi direction de Borel pour $f'(z)$*, C. R. Acad. Sci., Paris **199** (1934), 1014–1016.

[36] T. Shimizu,
[a] *On the theory of meromorphic functions*, Japan J. Math. **6** (1929).

[37] O. Teichmüller,
[a] *Vermutungen und sätze über die Wertverteilung gebrochener Funktionen endlicher Ordnung*, Deutsche Math. **4** (1939), 163–190.

[38] M. Tsuji,
[a] *Potential theory in modern function theory*, Maruzen, Tokyo, 1959.

[39] G. Valiron,
[a] *Lectures on the general theory of integral functions*, Edouard Privat, Toulouse, 1923.
[b] *Recherches sur le théorème de M. Morel dans la théorie des fonctions méromorphes*, Acta Math. **52** (1928), 67–92.
[c] *Sur les directions de Borel des fonctions entières*, Annali di Mat. **9** (1931), 273–285.
[d] *Directions de Borel des fonctions méromorphes*, Mém. Sci. Math., vol. 89, Paris, 1938.
[e] *Sur les valeurs déficientes des fonctions méromorphes d'ordre nul*, C. R. Acad. Sci. Paris **230** (1950), 40–42.

[40] A. Weitsman,
[a] *Meromorphic functions with maximal deficiency sum and a conjecture of F. Nevanlinna*, Acta. Math. **123** (1969), 115–139.
[b] *A growth property of the Nevanlinna characteristic*, Proc. Amer. Math. Soc. **26** (1970), 65–70.
[c] *A theorem on Nevanlinna deficiencies*, Acta Math. **128** (1972), 41–52.

[41] A. Wiman,
[a] *Sur une extension d'un théorème de M. Hadamard*, Ark. Mat. Ast. Phys. **2** (1905).

[42] Yang Lo and Zhang Guang-hou,
[a] *Sur la distribution des directions de Borel des fonctions méromorphes*, Sci. Sinica **16** (1973), 465–482.
[b] *On the total number of deficient values of an entire function*, Acta Math. Sinica **18** (1975), 35–53.
[c] *Recherches sur le nombre des valeurs déficientes et le nombre des directions de Borel des fonctions méromorphes*, Sci. Sinica **18** (1975), 23–37.
[d] *Sur la construction des fonctions méromorphes ayant des directions singulières données*, Sci. Sinica **19** (1976), 445–459.

[43] Zhang Guang-hou,
[a] *On the researches of common Borel directions of a meromorphic function and its derivatives or anti-derivatives*, Acta Math. Sinica **20** (1977; (I) 73–98; (II) 157–177; (III) 237–247).
[b] *A symptotic values of entire and meromorphic functions*, Sci. Sinica **20** (1977), 720–739.
[c] *Research of the relations among the deficient values asymptotic values, and Julia directions of entire and meromorphic functions*, Sci. Sinica add. I (1978), 1–80, Chinese.
[d] *The length of an asymptotic path of an entire function*, Sci. Sinica **22** (1979), 991–999.
[e] *On entire function extremal for Denjoy's conjecture*, Sci. Sinica, (I) **24** (1981), 885–898; (II) **11** (1982), 981–994.
[f] *Direct transcendental singularities for inverse functions of meromorphic functions and of their derivatives*, Sci. Sinica **25** (1982), 797–807.

[g] *Meromorphic functions with maximal deficiency sum*, Sci. Sinica **4** (1983), 293–305.
[h] *Entire functions with a finite number of Julia directions*, Sci. Sinica **9** (1983), 775–786.
[i] *Some general theorems of meromorphic functions and their applications*, Acta. Math. Sinica **30** (1987), no. 3, 317–354.
[j] *Entire functions with their zero and poles distributed on a finite number of rays*, Sci. Sinica (Ser. A) **10** (1986), 1009–1021.

[44] Zhang Guang-hou and Wu Peng-cheng,
[a] *On order of meromorphic functions*, Sci. Sinica Ser. A **28** (1985), no. 8, 785–800.

This page intentionally left blank

Recent Titles in This Series

(*Continued from the front of this publication*)

88 **A. G. Khovanskiĭ,** Fewnomials, 1991

87 **Aleksandr Robertovich Kemer,** Ideals of identities of associative algebras, 1991

86 **V. M. Kadets and M. I. Kadets,** Rearrangements of series in Banach spaces, 1991

85 **Mikio Ise and Masaru Takeuchi,** Lie groups I, II, 1991

84 **Đáo Trọng Thi and A. T. Fomenko,** Minimal surfaces, stratified multivarifolds, and the Plateau problem, 1991

83 **N. I. Portenko,** Generalized diffusion processes, 1990

82 **Yasutaka Sibuya,** Linear differential equations in the complex domain: Problems of analytic continuation, 1990

81 **I. M. Gelfand and S. G. Gindikin, Editors,** Mathematical problems of tomography, 1990

80 **Junjiro Noguchi and Takushiro Ochiai,** Geometric function theory in several complex variables, 1990

79 **N. I. Akhiezer,** Elements of the theory of elliptic functions, 1990

78 **A. V. Skorokhod,** Asymptotic methods of the theory of stochastic differential equations, 1989

77 **V. M. Filippov,** Variational principles for nonpotential operators, 1989

76 **Phillip A. Griffiths,** Introduction to algebraic curves, 1989

75 **B. S. Kashin and A. A. Saakyan,** Orthogonal series, 1989

74 **V. I. Yudovich,** The linearization method in hydrodynamical stability theory, 1989

73 **Yu. G. Reshetnyak,** Space mappings with bounded distortion, 1989

72 **A. V. Pogorelev,** Bendings of surfaces and stability of shells, 1988

71 **A. S. Markus,** Introduction to the spectral theory of polynomial operator pencils, 1988

70 **N. I. Akhiezer,** Lectures on integral transforms, 1988

69 **V. N. Saliĭ,** Lattices with unique complements, 1988

68 **A. G. Postnikov,** Introduction to analytic number theory, 1988

67 **A. G. Dragalin,** Mathematical intuitionism: Introduction to proof theory, 1988

66 **Ye Yan-Qian,** Theory of limit cycles, 1986

65 **V. M. Zolotarev,** One-dimensional stable distributions, 1986

64 **M. M. Lavrent'ev, V. G. Romanov, and S. P. Shishat·skiĭ,** Ill-posed problems of mathematical physics and analysis, 1986

63 **Yu. M. Berezanskiĭ,** Selfadjoint operators in spaces of functions of infinitely many variables, 1986

62 **S. L. Krushkal', B. N. Apanasov, and N. A. Gusevskiĭ,** Kleinian groups and uniformization in examples and problems, 1986

61 **B. V. Shabat,** Distribution of values of holomorphic mappings, 1985

60 **B. A. Kushner,** Lectures on constructive mathematical analysis, 1984

59 **G. P. Egorychev,** Integral representation and the computation of combinatorial sums, 1984

58 **L. A. Aĭzenberg and A. P. Yuzhakov,** Integral representations and residues in multidimensional complex analysis, 1983

57 **V. N. Monakhov,** Boundary-value problems with free boundaries for elliptic systems of equations, 1983

56 **L. A. Aĭzenberg and Sh. A. Dautov,** Differential forms orthogonal to holomorphic functions or forms, and their properties, 1983

55 **B. L. Rožděstvenskiĭ and N. N. Janenko,** Systems of quasilinear equations and their applications to gas dynamics, 1983

(See the AMS catalog for earlier titles)

ISBN 0-8218-4589-6