

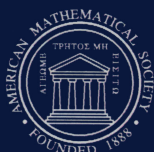
Translations of

MATHEMATICAL MONOGRAPHS

Volume 127

Tangents and Secants of Algebraic Varieties

F. L. Zak



American Mathematical Society

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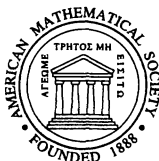
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Translations of
**MATHEMATICAL
MONOGRAPHS**

Volume 127

**Tangents and Secants
of Algebraic Varieties**

F. L. Zak



American Mathematical Society
Providence, Rhode Island

ФЕДОР ЛАЗАРЕВИЧ ЗАК
КАСАТЕЛЬНЫЕ И СЕКУЩИЕ
АЛГЕБРАИЧЕСКИХ МНОГООБРАЗИЙ

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ABSTRACT. This book is devoted to geometry of algebraic varieties in projective spaces. Among the objects considered in some detail are tangent and secant varieties, Gauss maps, dual varieties, hyperplane sections, projections, and varieties of small codimension. Emphasis is made on the study of interplay between irregular behavior of (higher) secant varieties and irregular tangencies to the original variety. Classification of varieties with unusual tangential properties yields interesting examples many of which arise as orbits of representations of algebraic groups.

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B , Borel subgroup	III, §1
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$H_z = L_z \cdot X$, Singular hyperplane section of X corresponding to a nonsingular point $z \in SX$	IV, §4
$H_u = L_u \cdot X$, Singular hyperplane section of X corresponding to a nonsingular point $u \in S^k X$	VI, §4
$I_G[V]$, Algebra of invariants for an action of group G on a vector space V	III, §1
\mathfrak{J} , Jordan algebra	IV, §4
K , Ground field	I, §1
$K^* = K \setminus 0$, The group of invertible elements of the ground field	III, §1
$k_0(X^n) = \min\{k \mid S^k X^n = \langle X \rangle = \mathbb{P}^N\}$	V, §1
κ_X	IV, §2
L_u , Tangent space to $S^k X$ at a point $u \in S^k X$	VI, §1
Λ , Highest weight	III, §1
$\lambda(X)$, (Linear) abnormality index	V, §2
$m(n, \delta) = \min\{N \mid \exists X^n \subset \mathbb{P}^N, \delta(X) = \delta\} = 2n + 1 - \delta$	V, §2
$M(n, \delta) = \max\{N \mid \exists X^n \subset \mathbb{P}^N, \langle X^n \rangle = \mathbb{P}^N, \delta(X) = \delta\}$	V, §2
M , Lowest weight	III, §1
$n(d)$	IV, §5
$\mathcal{N}_{\mathbb{P}^N/X^n}$, Normal bundle to $X^n \subset \mathbb{P}^N$	I, §2
\mathcal{N}_{A^N/X^n} , Normal bundle to $X^n \subset A^N$	I, §3
\mathfrak{N} , Cone of null-forms	III, §1
\mathbb{P}_z	IV, §2
\mathbb{P}_u	VI, §1
P_Λ , Parabolic subgroup stabilizing Kv_Λ	III, §1
\mathcal{P}_m, p_m	I, §§2, 3
p_i^Y, p_i	I, §1
$p_i^k, p_i^{a_0, \dots, a_r}$	V, §1
π_z , Rational projection $X^n \dashrightarrow \mathbb{P}^n$ with center in the subspace $\langle Y_z \rangle$	IV, §2
π_u , Rational projection $X^n \dashrightarrow \mathbb{P}^n$ with center in the subspace $\langle Y_u \rangle$	VI, §4
Q , Canonical quotient bundle over Grassmann variety	I, §2
Q_z	II, §2
Q^n , Nonsingular n -dimensional projective quadric	III, §1
R_k , Property of regularity in codimension $\leq k$	I, §1
R_m , Ramification cycle	II, §2
S , Canonical subbundle over Grassmann variety	III, §1

S^n , n -dimensional spinor variety	III, §1
S_r , Serre-Grothendieck property	I, §1
$S(Y, X)$, Relative secant variety; join of varieties Y and X	I, §1
$SX = S(X, X)$, Secant variety	I, §1
$S_X^k, S_{S^{a_0}X, \dots, S^{a_r}X}$	V, §1
$s^k(X) = \dim S^k X$	V, §1
$s(X) = s_1(X) = \dim SX$	II, §2
Σ	IV, §5
Σ_0	IV, §5
$\Sigma_x \subset \mathcal{P}_x$, Variety of hyperplane sections with "bad" singularity at x	IV, §5
T , Maximal torus	III, §1
$T_{X,x}$, Embedded (projective) tangent space to $X^n \subset \mathbb{P}^N$ at a point $x \in X$	I, §1
$T'_{X,x}$, Projective tangent star to $X^n \subset \mathbb{P}^N$ at a point $x \in X$	I, §1
$T'_{x,X}$, Projective tangent cone to $X^n \subset \mathbb{P}^N$ at a point $x \in X$	I, §1
$T'_{Y,X,y}$, Relative projective tangent star to X with respect to $Y \subset X$ at a point $y \in Y$	I, §1
$T'_{Y,X}, T'_X = T'_{X,X}$	I, §1
$T_{Y,X}, T_X = T_{X,X}$	I, §1
$T'(Y, X) = \bigcup_{y \in Y} T'_{Y,X,y}, T'X = T'(X, X),$	I, §1
$T(Y, X) = \bigcup_{y \in Y} T_{X,y}, TX = T(X, X),$	I, §1
$\Theta_{X,x}$, Zariski tangent space	II, §1
$\Theta_{X,x}$, Tangent space to a subvariety of complex torus	I, §3
$\Theta'_{X,x}$, Tangent star	II, §1
$\Theta'_{x,X}$, Tangent cone	II, §1
$\Theta'_{Y,X,y}$, Relative tangent star	II, §1
$\Theta'_{Y,X}$	II, §1
nV	III, §3
tV	III, §3
v_d , Veronese map defined by the linear system of hypersurfaces of degree d	II, §2
v_Λ , Highest weight vector	III, §1
v_M , Lowest weight vector	III, §1
W , Weyl group	III, §1
w_0 , Standard involution in W	III, §1
X_m^* , Variety of m -dimensional tangent spaces	I, §§2, 3

$X^* = X_{N-1}^*$, Dual projective variety	I, §§2, 3
$\tilde{X} = X_{/Y}$, Completion of X along $Y \subset X$	I, §2
Ξ	IV, §1
$Y_z; Y_u$	II, §2; V, §1
$Y_L = \{x \in X \mid T_{X,x} \subset L\}$	VI, §1
Y^x	IV, §3
Y_{12}	IV, §3
[], Integral part	I, §1
{ }, Fractional part	V, §2
$\langle \rangle$, Linear (vector, toroidal) span	I, §1; II, §1;
	I, §3

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