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V. V. Sharko

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10.1090/mmono/131

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В. В. Шарко

ФУНКЦИИ НА МНОГООБРАЗИЯХ (АЛГЕБРАИЧЕСКИЕ И ТОПОЛОГИЧЕСКИЕ АСПЕКТЫ)

«НАУКОВА ДУМКА» КИЕВ, 1990

Translated from the Russian by V. V. Minachin
Translation edited by Simeon Ivanov

1991 *Mathematics Subject Classification.* Primary 57R45, 57R70, 58E05;
Secondary 57N20, 58B05.

Library of Congress Cataloging-in-Publication Data

Sharko, V. V. (Vladimir Vasil′evich)
[Funkt͡sii na mnogoobrazii͡akh. English]
Functions on manifolds: algebraic and topological aspects
V. V. Sharko; [translated from the Russian by V. V. Minachin].
p. cm. — (Translations of mathematical monographs, ISSN 0065-9282; v. 131)
Includes bibliographical references.
ISBN 0-8218-4578-0 (acid-free)
1. Differentiable mappings 2. Morse theory. 3. Manifolds. 4. Singularities (Mathematics)
I. Title. II. Series.
QA614.58.S5313 1993
515′.352—dc20 93-25901
CIP

This volume was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX,
the American Mathematical Society's TEX macro system.

10 9 8 7 6 5 4 3 2 1 98 97 96 95 94 93

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Preface

The idea of studying smooth manifolds by means of level curves of functions defined on them goes back to Poincaré, and even to Möbius. But its systematic development is due to Marston Morse who first observed that the number of critical points of different indices of a smoth function on a manifold can be made use of in order to study the geometric properties of the manifold. Morse proved certain inequalities connecting the number of critical points with the ranks and torsion orders of the homology groups of the manifold [102]. At various times, the development of this line of topology has been carried forward by L. A. Lyusternik, L. G. Snirel′man, G. S. Chogoshvili, L. E. Elsholz, E. Pitcher, and G. Reeb [78, 20, 21, 44, 113, 121].

In 1960, Smale showed that on any smooth simply connected manifold M^n $(n \geq 5)$ there exists a Morse function with the minimal number of critical points of each index. Among many corollaries thereto we mention the generalized Poincaré conjecture and the h-cobordism theorem [143].

Morse theory plays an important role in modern topology. Morse surgery, the theory of Smale's handles, provides flexible tools for the analysis of differentiable manifolds. The effectiveness of the theory was repeatedly demonstrated in the works of Milnor, Kervaire, S. P. Novikov, and Browder on the classification of manifolds [108]. On the other hand, despite the intensive studies of non-simply-connected manifolds made by Novikov, Wall, Farrell, Cappell, A. S. Mishchenko, Siebenmann and others, the question of the existence of a minimal Morse function on a non-simply-connected manifold remained open [105, 108, 155].

Another aspect of the Morse functions application should be mentioned. While studying families of smooth functions on simply connected manifolds, Cerf proved that the equivalence relations of isotopy and pseudo-isotopy for simply connected smooth manifolds of dimension greater than five coincide.

A new impulse was given to Morse theory in 1981 by the work of Novikov about multivalued Morse functions. The situation in this case differs qualitatively from the classical one. This area is now being investigated very actively.

The present monograph considers Morse functions on finite-dimensional smooth manifolds.

Chapter I includes the necessary material from the theory of Fréchet spaces and manifolds, which is then applied to the stratification of smooth functions on a manifold. The main result of Chapter II is a necessary and sufficient condition for two minimal Morse functions on a simply connected smooth manifold of dimension greater than 5 to be homotopic, isotopic, and conjugate. Chapter III is devoted to the algebraic technique used to construct minimal chain complexes over s-rings. It also contains a necessary and sufficient condition for the existence of a minimal chain complex in a given homotopic type.

Chapter IV considers the homotopic theory of chain complexes which is applied to the study of Morse functions on non-simply-connected cobordisms. The existence of minimal Morse functions for a wide class of non-simply-connected cobordisms is proved in Chapter V. New numerical invariants of manifolds are introduced resulting in the substantial improvement of the Morse inequalities. Chapter VI contains results on the homotopic properties of cell complexes needed for the analysis of Morse functions on closed manifolds. Our attention is centered on investigations concerning homotopic systems in the sense of Whitehead.

The question of the existence of minimal Morse functions on closed manifolds and on manifolds with a single component of the boundary is studied in Chapter VII. Chapter VIII considers so-called round Morse functions and includes recent developments in this area.

The author is sincerely grateful to S. P. Novikov for constructive advice and stimulating discussions, as well as to A. T. Fomenko and M. A. Shtanko whose counsel contributed to a deeper understanding of the problems discussed in the monograph. The author also appreciates the help and support he received in writing this book from many employees of the Institute of Mathematics of the Ukrainian Academy of Sciences.

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