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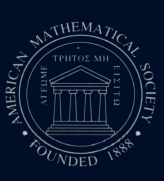
Volume 133

Nonlinear Nonlocal Equations in the Theory of Waves

P. I. Naumkin
I. A. Shishmarev



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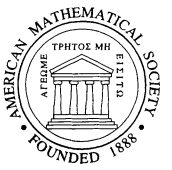

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Наумкин П. И., Шишмарёв И. А.

НЕЛИНЕЙНЫЕ НЕЛОКАЛЬНЫЕ УРАВНЕНИЯ В ТЕОРИИ ВОЛН

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ABSTRACT. Nonlinear evolutional equations of mathematical physics are studied. The major part of the book is devoted to the analysis of breaking and decay of solutions in finite time. The methods developed in the book can be applied to a wide class of conservative and dissipative nonlinear equations, both local and nonlocal. Among the important examples, the authors consider the Kolmogorov-Petrovskiĭ-Piskunov equation, the nonlinear nonlocal Schrödinger equation, the Kuramoto-Sivashinsky equation, the Korteweg-de Vries-Burgers equation, and several other important equations of mathematical physics.

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