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Algebraic Geometry



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Algebraic Geometry

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Algebraic Geometry

**Translated by
Masayoshi Miyanishi**



American Mathematical Society

代数幾何学

DAISŪ KIKAGAKU
(Algebraic Geometry)

by Masayoshi Miyanishi

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ABSTRACT. This book covers algebraic geometry from the beginnings to an introduction of algebraic surfaces, viz., to the gate from which the classification of algebraic surfaces starts. The book has three parts. The first part provides the necessary basic results from commutative algebras and the theory of sheaves and its cohomologies. The second part is on schemes and algebraic varieties. The third part is on algebraic curves and surfaces, placing emphasis on the use of linear systems and the associated rational mappings.

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Preface to the English Edition

The interesting theory, where one first experiences the beauty in learning algebraic geometry, is perhaps the theory of algebraic curves and surfaces. The theory of algebraic surfaces is particularly important when one further studies higher-dimensional algebraic varieties.

In order to approach this theory through modern algebraic methods, one is required to have a basic knowledge of local rings, sheaves, cohomologies, projective varieties, linear systems of divisors, etc. This basic knowledge may include all results given in introductory textbooks devoted to these subjects, and it takes some time to get familiar with them.

This book grew out from the author's attempts to find a shortcut and thus, lead students effectively to an understanding of algebraic surfaces. This book, originally written in Japanese, is designed so that readers with only knowledge of algebra at an undergraduate level can start learning algebraic geometry by themselves. The description is, therefore, self-contained as much as possible.

The book consists of three parts. The first part gives background in commutative algebras, sheaf theory, and related cohomology theory. The second part introduces schemes and algebraic varieties, where one can learn the basic language of algebraic geometry. The last part is devoted to guiding the readers to the gate from which the classification of algebraic surfaces starts. It is most desirable that readers continue to further their learning through other books on algebraic surfaces.

While preparing the English translation, the author is indebted to Drs. Koji Yokogawa and Hiroyuki Ito for technical assistance in typesetting the manuscript using LaTeX.

M. Miyanishi
June 1993

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Preface

Students often say they get interested in algebraic geometry and start learning it by themselves, only to find that there are too many prerequisites; they get lost in preliminary results. The knowledge of algebra and geometry that is taught in college is not sufficient for reading through most of the serious textbooks in algebraic geometry. One needs knowledge of ring theory, field theory, local rings, and transcendental field extensions, even sheaf theory and cohomologies with coefficients in sheaves, which are certainly beyond the level of college mathematics. But if one waits to begin the study of algebraic geometry until after absorbing all the necessary knowledge, then one may have lost interest.

So it is most desirable if such basic results appear in a textbook in algebraic geometry when they are needed. In writing this book, the author tried to prove or recall results when needed.

This book, meant for senior undergraduate students and for graduate students, will guide students from the beginnings of algebraic geometry to an introduction of algebraic surfaces, viz., to the gate from which the classification of algebraic surfaces starts. The readers are advised to read higher textbooks or research papers after completing this book.

In order to keep the book to a reasonable size, some results were not included, and even the included results are not stated in full generality.

The book consists of three parts. The first part provides the necessary basic results. The second part is on schemes and algebraic varieties. The third part is on algebraic surfaces. When we refer to results stated in the same part, we refer to them in a form such as Theorem 3.1, and when we refer to results in different parts, we include the part as in Theorem II.3.1.

At the end of each chapter, some results are given as problems; brief solutions are given at the end of the book.

The author was encouraged to write this book by Professor S. Murakami of Osaka University and given frequent and warm encouragement from Mr. S. Hosoki of Shokabo Publ. Co. The author would like to express his sincere gratitude to both of them.

M. Miyanishi
June 1990

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List of Notation

$A := B$	give the set B the name A ; define A by the condition B .
$A - B$	$A \setminus (A \cap B)$ if A and B are subsets of a set S ; difference of A and B if they are elements of an abelian group.
$(I : a)$	ideal quotient; $(I : a) = \{x \in R : ax \in I\}$ if I is an ideal of a ring R .
\sqrt{I}	radical of an ideal I .
R_S, R_p	quotient ring, localization.
K-dim $R, \dim R$	Krull dimension of R .
$\mathcal{O} \supseteq \mathcal{O}'$	a local ring $(\mathcal{O}, \mathfrak{m})$ dominates a local ring $(\mathcal{O}', \mathfrak{m}')$, i.e., $\mathcal{O} \supseteq \mathcal{O}', \mathfrak{m} \cap \mathcal{O}' = \mathfrak{m}'$.
ht(\mathfrak{p})	height of a prime ideal \mathfrak{p} .
tr.deg $_k K$	transcendence degree of an extension field K/k .
Gal(K/k)	Galois group of a normal algebraic extension.
$\prod A_i, \prod_{i \in I} A_i$	direct product of sets, etc.
$\coprod A_i, \coprod_{i \in I} A_i$	direct sum of sets, etc.
$\lim_{\rightarrow i} A_i, \lim_{\leftarrow i} A_i$	inductive limit, projective limit.
$\mathcal{F} \oplus \mathcal{G}$	direct sum of sheaves of modules.
$\mathcal{A}^{(I)}, \mathcal{A}^n$	direct sum of $ I $ copies (or n copies) of a sheaf of commutative rings.
$\mathcal{F} \otimes_{\mathcal{A}} \mathcal{G}$	tensor product of \mathcal{A} -Modules.
Hom $_{\mathcal{A}}(\mathcal{F}, \mathcal{G})$	group of homomorphisms of \mathcal{A} -Modules.
Hom $_{\mathcal{A}}(\mathcal{F}, \mathcal{G})$	\mathcal{A} -Module of homomorphisms of \mathcal{A} -Modules.
\mathcal{F}^\vee	dual \mathcal{A} -Module. Hom $_{\mathcal{A}}(\mathcal{F}, \mathcal{A})$.
Ker $\varphi, \text{Coker } \varphi$	kernel, cokernel, and image of an \mathcal{A} -homomorphism
Im φ	$\varphi : \mathcal{F} \rightarrow \mathcal{G}$ of \mathcal{A} -Modules.
$f_* \mathcal{F}, f^* \mathcal{G}$	direct image, inverse image of a module of sheaves.
$\Gamma(U, \mathcal{F})$	module of sections of a sheaf of modules \mathcal{F} over an open set U .
$H^n(X, \mathcal{F})$	cohomology group defined by an injective resolution of a sheaf of modules \mathcal{F} .
$H^n(\mathcal{U}, \mathcal{F})$	Čech cohomology group of \mathcal{F} defined by an open covering \mathcal{U} .
$\check{H}^n(X, \mathcal{F})$	Čech cohomology group of \mathcal{F} : by the definition, it is $\lim_{\leftarrow \mathcal{U}} H^n(\mathcal{U}, \mathcal{F})$.
$R^n f_* \mathcal{F}$	higher direct image of \mathcal{F} .
Spec(R)	spectrum of a commutative ring R .

$V(I), D(I), D(a)$	closed set, open sets in the Zariski topology.
$\widetilde{R}, \widetilde{M}$	quasicoherent sheaves associated with R and an R -module M .
\mathfrak{p}_x	prime ideal of R corresponding to a point x of $\text{Spec}(R)$.
$k(x)$	residue field of a point x .
\mathbb{A}_k^n	n -dimensional affine space over k .
$\dim X$	Krull dimension of a scheme X .
$\text{codim}_X Z$	dodimension of a closed set Z in X .
X_{red}	reduced form of X .
$X(k)$	set of k -rational points of a k -scheme X .
$k(X)$	rational function field of a k -scheme X .
$\coprod X_i$	direct sum of schemes.
$X \times_S Y$	fiber product.
$f : X \rightarrow Y$	a morphism of schemes.
$\text{Hom}_S(X, Y)$	set of S -morphisms from an S -scheme X to an S -scheme Y .
X_y	fiber of $f : X \rightarrow Y$ over a point y of Y .
$f : X \cdots \rightarrow Y$	a rational mapping of schemes.
Γ_f	graph of a morphism (or a rational mapping) f .
$x \rightarrow x'$	specialization of points x to x' on a scheme.
A_+	irrelevant ideal of a graded ring $A = \sum_{n \geq 0} A_n$.
$\text{Proj} A$	homogeneous spectrum of A .
$V_+(E), S_+(E)$	closed set, open sets of $\text{Proj} A$.
$D_+(a)$	
$\mathcal{O}_X(\ell)$	$A(\ell) \sim$.
$\mathcal{F}(\ell)$	$\mathcal{F} \otimes_{\mathcal{O}_X} \mathcal{O}_X(\ell)$.
\mathbb{P}_k^n	n -dimensional projective space over k .
$S(\mathcal{F})$	symmetric algebra of a locally free Module.
$\mathbb{P}(\mathcal{F})$	projective bundle associated with \mathcal{F} .
$\text{Pic}(X)$	Picard group of X .
$D_1 \sim D_2$	linear equivalence of divisors.
$D_1 \approx D_2$	algebraic equivalence of divisors.
$D_1 \equiv D_2$	numerical equivalence of divisors.
$\text{Cl}(X)$	divisor class group of X .
$D \geq 0$	an effective divisor.
$\text{Supp} D$	support of a divisor.
(f)	divisor of a rational function f .
$(f)_0, (f)_\infty$	zero part, polar part of f .
$ D $	a complete linear system.
$M(L)$	k -module associated with a linear system L .
Φ_L	rational mapping attached to a linear system L .
$\Phi_{\mathcal{L}}$	rational mapping defined by $H^0(X, \mathcal{L})$ for an invertible sheaf \mathcal{L} on X .
$\text{Bs}L$	set of base points of a linear system of L .
$ D - \sum_{i=1}^n m_i P_i$	linear subsystem of $ D $ consisting of divisors on an algebraic surface V which pass through the points P_i ($1 \leq i \leq n$) with multiplicity m_i .
$X \cdot H(\sigma)$	pullback of a hyperplane $H(\sigma)$ by Φ_L .

\widehat{R}	completion of R with respect to a linear topology.
$R[[x_1, \dots, x_n]]$	formal power series ring in n variables over R .
$\Omega_{A/B}, \Omega_{X/Y}$	module of differential forms, sheaf of differential forms.
$\text{Der}_B(A, M)$	A -module of B -derivations of A .
$\text{Sing}X$	set of singular points of an algebraic variety X .
$\mathcal{F}_{X/k}$	tangent sheaf of X .
$\mathcal{N}_{Y/X}$	normal sheaf of a closed subvariety Y of X .
$\omega_{X/k}$	canonical sheaf of X .
K_X	canonical divisor of X .
$h^i(X, \mathcal{F})$	$\dim_k H^i(X, \mathcal{F})$.
$\chi(X, \mathcal{F})$	Euler-Poincaré characteristic of \mathcal{F} .
$g(C)$	(geometric) genus of an algebraic curve C .
$p(C)$	arithmetic (virtual) genus of C .
$(\mathcal{L}_1 \cdot \mathcal{L}_2), (D_1 \cdot D_2)$	intersection number of invertible sheaves (or divisors) on an algebraic surface V .
$(\mathcal{L}^2), (D^2)$	self-intersection number.
$i(D_1, D_2; P)$	local intersection multiplicity at a point P .
$\mu(C; P)$	multiplicity of an algebraic curve C on V at a point P .
$p_a(D)$	arithmetic genus of a divisor D .
$P_n, P_n(V)$	n -genus of an algebraic surface V .
$p_g \cdot p_g(V)$	geometric genus of V .
$q(V)$	irregularity of V .
$\rho(V)$	Picard number of V .
$\text{NS}(V)$	Néron-Severi group.
F_n	Hirzebruch surface of degree n .
elm_P	elementary transformation with center at a point P .

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