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Volume 138

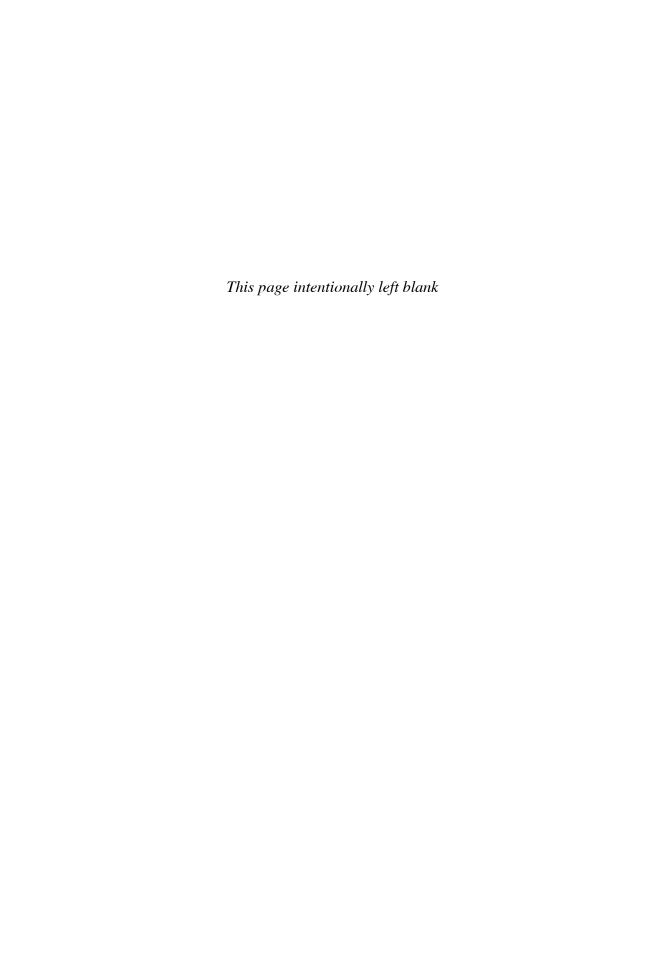
Identities of Algebras and Their Representations

Yu. P. Razmyslov

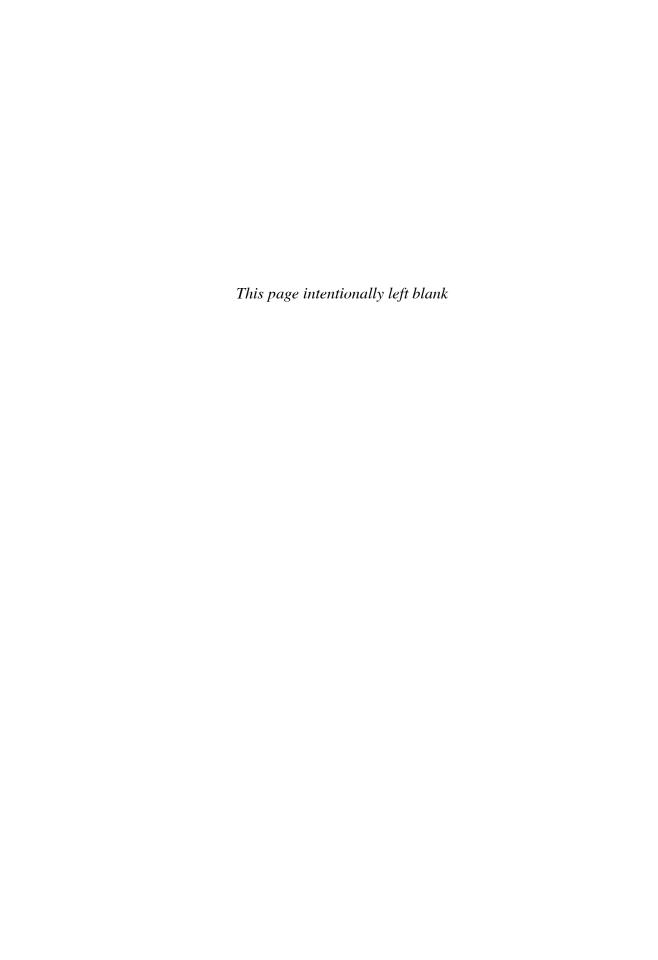


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Identities of Algebras and Their Representations

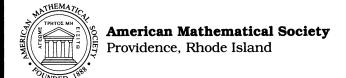


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Volume 138

Identities of Algebras and Their Representations

Yu. P. Razmyslov



Ю. П. Размыслов

ТОЖДЕСТВА АЛГЕБР И ИХ ПРЕДСТАВЛЕНИЙ

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ABSTRACT. This book is devoted to the study of an important branch of abstract algebra: analysis of identities in algebras and in representations of algebras. The main technique developed in the book is the method of 2-words and α -functions that was invented by the author in the late seventies. This method, which can be compared to the theory of group characters, enabled the author to solve a number of important problems that remained unsolved for several decades. This is the first monograph which gives a detailed account of recent achievements in the theory of identical relations in algebras and their representations. It will be useful to specialists working in algebra, mathematical logic, algebraic geometry, and other areas of mathematics.

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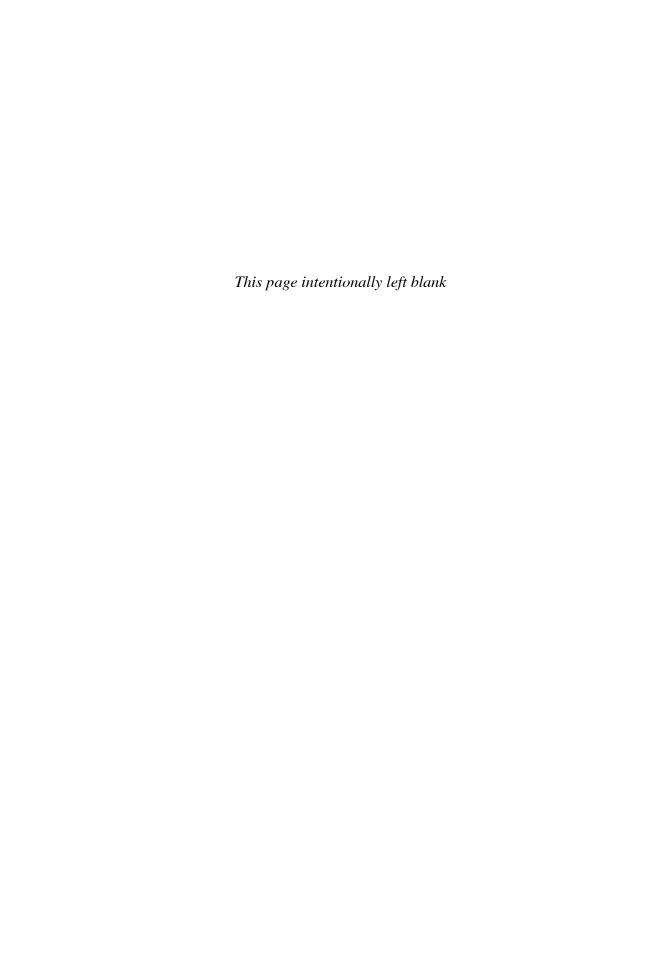
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PREFACE

During the last forty years a new trend in the theory of associative algebras, Lie algebras, and their representations has formed under the influence of mathematical logic and universal algebra—namely, the theory of varieties and identities of associative algebras, Lie algebras, and their representations. The study of two interrelated classes of problems is characteristic for this direction. The first class deals with the problems of finding identities of concrete classical algebras and their representations, while the second class includes the problems of the description of properties of algebras and their representations satisfying nontrivial identical relations, in particular, identities of certain classical algebras. The first class of problems forms a basis of the theory, and it is of interest mainly to specialists in algebra and mathematical logic, whereas the second one is the "applied" part of the theory and this can be used in various fields in mathematics and is of interest to a wide circle of experts in functional analysis, differential and algebraic geometry, superalgebras, and Lie algebras. Presently there are many papers devoted to this subject. Certain monographs have appeared; in particular. "Nauka" (Moscow) recently published a monograph of Yu. A. Bakhturin, "Identical Relations in Lie Algebras", which contains sufficiently complete information on classical problems and methods of the theory of varieties of Lie algebras.

Investigations of the last twenty years in the area of identities of algebras and their representations lead to the creation of a new method—the method of 2-words and α -functions. This original method, remotely reminiscent of the character theory in group theory, has arisen at the juncture of universal algebra and representation theory of reductive Lie algebras in the process of my attempts to solve some particular problems about identities that have remained unsolved despite prolonged efforts to apply other methods. The formation of the method was basically completed in 1976. As a result, a number of well-known principal problems of the theory of groups, rings, Lie algebras, and their representations were solved subsequently in a unified way. The results of these investigations have applications in differential and algebraic geometry, in the theory of Lie superalgebras, and in the theory of simple infinite-dimensional Lie algebras.

The possibilities of the method are far from being exhausted. However, an important period appears in its development, in which, continuing our going forward, we are to look back to sum up the applications of the method in the theory of varieties and to give a systematic exposition of contemporary achievements of the theory of identities of algebras and their representations closely related to the method of 2-words and α -functions. Understanding of this necessity made me begin writing this book and mainly determined the circle of problems on algebras and their representations treated in it.

There are seven chapters in the book.

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Chapter I is introductory; in it we analyse the three classical approaches to the concept of identity of a representation of a Lie algebra, introduce the basic notions of the theory of varieties of representations, and develop the growth theory for varieties of representations of Lie algebras. Theorems included in this chapter show that identities are important invariants of an irreducible representation of a finite-dimensional algebra and that they determine the representation up to an isomorphism.

Chapter II is an introduction to the theory of 2-words and α -functions. It forms a conceptual basis for the entire book. Here the main facts of this theory are collected and a general setting of the main problem for the method of 2-words is given. Using the examples of the nonsolvability problem for (p-2)th Engel Lie algebras and of the problem on identities of the two-dimensional representation of the simple three-dimensional Lie algebra, we show how the main notions of the method under consideration naturally arise.

The three subsequent chapters are related to the solution of the general problem of the theory of 2-words and α -functions posed in Chapter II for three partucular sets of irreducible representations of reductive Lie algebras. Their main aim is to fill the statements discussed in Chapter II with real content and to show how the method of 2-words permits us to obtain results on identities of algebras and their representations. In these chapters we discuss finite basis problems for identities of particular varieties, develop the theory of trace identities and of central polynomials for matrix algebras M_n , for matrix superalgebras, and for irreducible representations of reductive Lie algebras, and give applications of the method of 2-words to varieties of associative nil-algebras, Lie algebras with Engel property, and locally finite groups of prime period.

In Chapter VI we discuss the theory of identities of Lie algebras of Cartan type and give a classification of simple infinite-dimensional Lie algebras for which the identities of Lie algebras of Cartan type are satisfied. These results are closely related to the theory (exposed in Chapter III) of central polynomials of irreducible representations of reductive Lie algebras and also to the existence of a polynomial mapping which permits one to reconstruct the algebra of regular functions of a smooth variety from its Lie algebra of vector fields.

Each of the first six chapters is concluded by comments. We give there precise references to the authors and list the papers in which the results discussed in the chapter have been published; we also mention open problems of importance both to the theory itself and to its applications.

My intent while writing this book was to make it more accessible to a wide circle of readers, in particular, to second year students of physics and mathematics about to choose their specialization. To this end, in Chapter VII of the book I have collected more or less well-known auxiliary facts and theorems of a general nature that are constantly used in the previous chapters, but whose proofs are scattered about various textbooks and monographs. I hope that this makes Chapters I, II, III, and VI completely accessible to the reader acquainted with generalities on linear spaces over fields. Chapter IV requires the knowledge of the classical Young theory of irreducible representations of the symmetric group to the extent of several pages of §28 of the book of Curtis and Reiner [CR]. To be able to read Chapter III, the reader must be acquainted with the theory of semisimple finite-dimensional Lie algebras and their representations over algebraically closed fields of characteristic zero. A vast literature

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is presently devoted to this topic. I recommend that the reader, in case of difficulties with Chapter III, consult the excellent book [Dix] of J. Dixmier on universal enveloping algebras. Chapters I, III, IV, V, and VI are almost independent of each other, and they can be studied in an arbitrary order after Chapter II. One can learn about the main events of the development of the theory of identities from the short historical review at the end of the book.

I also wanted this book to be a reader's quick guide to the main problems of modern theory of identities of algebras and their representations. So, I shall be glad if the problems treated here help get the reader to those frontiers of the theory where the vast wide space opens up for independent creative work.



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