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Volume 141

**Qualitative Theory  
of Control Systems**

A. A. Davydov



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# Qualitative Theory of Control Systems

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A. A. Davydov

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## КАЧЕСТВЕННАЯ ТЕОРИЯ УПРАВЛЯЕМЫХ СИСТЕМ

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**ABSTRACT.** This book is devoted to the analysis of control systems using results from singularity theory and the qualitative theory of ordinary differential equations. In the main part of the book, systems with two-dimensional phase space are studied. The study of singularities of controllability boundaries for a typical system leads to the classification of normal forms of implicit first-order differential equations near a singular point. Several applications of these normal forms are indicated. The book can be used by graduate students and researchers working in control theory, singularity theory, and various areas of ordinary partial differential equations, as well as in applications.

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